

## Contents

**Preface** *XI*

**List of Contributors** *XIII*

<b>1</b>	<b>Functional Complexity Based on Topology</b>	<b>1</b>
	<i>Hildegard Meyer-Ortmanns</i>	
1.1	Introduction	1
1.2	A Measure for the Functional Complexity of Networks	3
1.2.1	Topological Equivalence of LCE-Graphs	3
1.2.2	Vertex Resolution Patterns	5
1.2.3	Kauffman States for Link Invariants	6
1.2.4	Definition of the Complexity Measure	8
1.3	Applications	9
1.3.1	Creation of a Loop	10
1.3.2	Networks of Information	10
1.3.3	Transport Networks of Cargo	10
1.3.4	Boolean Networks of Gene Regulation	12
1.3.5	Topological Quantum Systems	12
1.3.6	Steering Dynamics Stored in Knots and Links	13
1.4	Conclusions	14
	References	15
<b>2</b>	<b>Connections Between Artificial Intelligence and Computational Complexity and the Complexity of Graphs</b>	<b>17</b>
	<i>Ángel Garrido</i>	
2.1	Introduction	17
2.2	Representation Methods	18
2.3	Searching Methods	20
2.4	Turing Machines	22
2.5	Fuzzy Logic and Fuzzy Graphs	24
2.6	Fuzzy Optimization	26
2.7	Fuzzy Systems	27
2.8	Problems Related to AI	27
2.9	Topology of Complex Networks	28
2.10	Hierarchies	30

2.10.1	Deterministic Case	30
2.10.2	Nondeterministic Case	31
2.10.3	Alternating Case	31
2.11	Graph Entropy	32
2.12	Kolmogorov Complexity	34
2.13	Conclusion	37
	References	38
<b>3</b>	<b>Selection-Based Estimates of Complexity Unravel Some Mechanisms and Selective Pressures Underlying the Evolution of Complexity in Artificial Networks</b>	<b>41</b>
	<i>Hervé Le Nagard and Olivier Tenaillon</i>	
3.1	Introduction	41
3.2	Complexity and Evolution	42
3.3	Macroscopic Quantification of Organismal Complexity	43
3.4	Selection-Based Methods of Complexity	44
3.5	Informational Complexity	44
3.6	Fisher Geometric Model	46
3.7	The Cost of Complexity	48
3.8	Quantifying Phenotypic Complexity	49
3.8.1	Mutation-Based Method: Mutational Phenotypic Complexity (MPC)	49
3.8.2	Drift Load Based Method: Effective Phenotypic Complexity (EPC)	50
3.8.3	Statistical Method: Principal Component Phenotypic Complexity (PCPC)	50
3.9	Darwinian Adaptive Neural Networks (DANN)	52
3.10	The Different Facets of Complexity	54
3.11	Mechanistic Understanding of Phenotypic Complexity	56
3.12	Selective Pressures Acting on Phenotypic Complexity	57
3.13	Conclusion and Perspectives	57
	References	59
<b>4</b>	<b>Three Types of Network Complexity Pyramid</b>	<b>63</b>
	<i>Fang Jin-Qing, Li Yong, and Liu Qiang</i>	
4.1	Introduction	63
4.2	The First Type: The Life's Complexity Pyramid (LCP)	64
4.3	The Second Type: Network Model Complexity Pyramid	67
4.3.1	The Level-7: Euler (Regular) Graphs	68
4.3.2	The Level-6: Erdős–Rényi Random Graph	68
4.3.3	The Level-5: Small-World Network and Scale-Free Models	69
4.3.4	The Level-4: Weighted Evolving Network Models	70
4.3.5	The Bottom Three Levels of the NMCP	71
4.3.5.1	The Level-3: The HUHPNM	72
4.3.5.2	The Level-2: The LUHNM	72
4.3.5.3	The Level-1: The LUHNM-VSG	73
4.4	The Third Type: Generalized Farey Organized Network Pyramid	78

4.4.1	Construction Method of the Generalized Farey Tree Network (GFTN)	78
4.4.2	Main Results of the GFTN	80
4.4.2.1	Degree Distribution	80
4.4.2.2	Clustering Coefficient	81
4.4.2.3	Diameter and Small World	82
4.4.2.4	Degree–Degree Correlations	83
4.4.3	Weighted Property of GFTN	85
4.4.4	Generalized Farey Organized Network Pyramid (GFONP)	87
4.4.4.1	Methods	87
4.4.4.2	Main Results of GFONP	90
4.4.4.3	Brief Summary	95
4.5	Main Conclusions	96
	Acknowledgment	96
	References	96
<b>5</b>	<b>Computational Complexity of Graphs</b>	<b>99</b>
	<i>Stasys Jukna</i>	
5.1	Introduction	99
5.2	Star Complexity of Graphs	100
5.2.1	Star Complexity of Almost All Graphs	104
5.2.2	Star Complexity and Biclique Coverings	107
5.3	From Graphs to Boolean Functions	107
5.3.1	Proof of the Strong Magnification Lemma	111
5.3.2	Toward the $(2 + c)n$ Lower Bound	114
5.4	Formula Complexity of Graphs	116
5.5	Lower Bounds via Graph Entropy	121
5.5.1	Star Complexity and Affine Dimension of Graphs	125
5.6	Depth-2 Complexity	126
5.6.1	Depth-2 with AND on the Top	128
5.6.2	Depth-2 with XOR on the Top	130
5.6.3	Depth-2 with Symmetric Top Gates	131
5.6.4	Weight of Symmetric Depth-2 Representations	134
5.7	Depth-3 Complexity	138
5.7.1	Depth-3 Complexity with XOR Bottom Gates	141
5.8	Network Complexity of Graphs	145
5.8.1	Realizing Graphs by Circuits	148
5.9	Conclusion and Open Problems	150
	References	151
<b>6</b>	<b>The Linear Complexity of a Graph</b>	<b>155</b>
	<i>David L. Neel and Michael E. Orrison</i>	
6.1	Rationale and Approach	155
6.2	Background	157
6.2.1	Adjacency Matrices	157

6.2.2	Linear Complexity of a Matrix	158
6.2.3	Linear Complexity of a Graph	159
6.2.4	Reduced Version of a Matrix	160
6.3	An Exploration of Irreducible Graphs	161
6.3.1	Uniqueness and Prevalence	163
6.3.2	Structural Characteristics of the Irreducible Subgraph	164
6.4	Bounds on the Linear Complexity of Graphs	164
6.4.1	Naive Bounds	165
6.4.2	Bounds from Partitioning Edge Sets	166
6.4.3	Bounds for Direct Products of Graphs	167
6.5	Some Families of Graphs	168
6.5.1	Trees	168
6.5.2	Cycles	169
6.5.3	Complete Graphs	169
6.5.4	Complete $k$ -partite Graphs	170
6.5.5	Johnson Graphs	171
6.5.6	Hamming Graphs	173
6.6	Bounds for Graphs in General	173
6.6.1	Clique Partitions	173
6.7	Conclusion	174
	References	175
<b>7</b>	<b>Kirchhoff's Matrix-Tree Theorem Revisited: Counting Spanning Trees with the Quantum Relative Entropy</b>	<b>177</b>
	<i>Vittorio Giovannetti and Simone Severini</i>	
7.1	Introduction	177
7.2	Main Result	178
7.3	Bounds	181
7.4	Conclusions	188
	Acknowledgments	189
	References	189
<b>8</b>	<b>Dimension Measure for Complex Networks</b>	<b>191</b>
	<i>O. Shanker</i>	
8.1	Introduction	191
8.2	Volume Dimension	192
8.3	Complex Network Zeta Function and Relation to Kolmogorov Complexity	193
8.4	Comparison with Complexity Classes	194
8.5	Node-Based Definition	195
8.6	Linguistic-Analysis Application	196
8.7	Statistical Mechanics Application	198
8.8	Function Values	201
8.8.1	Discrete Regular Lattice	201
8.8.2	Random Graph	202

8.8.3	Scale-Free Network and Fractal Branching Tree	202
8.9	Other Work on Complexity Measures	204
8.9.1	Early Measures of Complexity	205
8.9.2	Box Counting Dimension	205
8.9.3	Metric Dimension	206
8.10	Conclusion	206
	References	206
<b>9</b>	<b>Information-Based Complexity of Networks</b>	<b>209</b>
	<i>Russell K. Standish</i>	
9.1	Introduction	209
9.2	History and Concept of Information-Based Complexity	210
9.3	Mutual Information	212
9.4	Graph Theory, and Graph Theoretic Measures: Cyclomatic Number, Spanning Trees	213
9.5	Erdos–Renyi Random Graphs, Small World Networks, Scale-free Networks	215
9.6	Graph Entropy	216
9.7	Information-Based Complexity of Unweighted, Unlabeled, and Undirected Networks	216
9.8	Motif Expansion	218
9.9	Labeled Networks	218
9.10	Weighted Networks	219
9.11	Empirical Results of Real Network Data, and Artificially Generated Networks	220
9.12	Extension to Processes on Networks	220
9.13	Transfer Entropy	222
9.14	Medium Articulation	223
9.15	Conclusion	225
	References	225
<b>10</b>	<b>Thermodynamic Depth in Undirected and Directed Networks</b>	<b>229</b>
	<i>Francisco Escolano and Edwin R. Hancock</i>	
10.1	Introduction	229
10.2	Polytopal vs Heat Flow Complexity	231
10.3	Characterization of Polytopal and Flow Complexity	233
10.3.1	Characterization of Phase Transition	233
10.4	The Laplacian of a Directed Graph	236
10.5	Directed Heat Kernels and Heat Flow	238
10.6	Heat Flow–Thermodynamic Depth Complexity	239
10.6.1	Definitions for Undirected Graphs	239
10.6.2	Extension for Digraphs	241
10.7	Experimental Results	241
10.7.1	Undirected graphs: Complexity of 3D Shapes	241
10.7.2	Directed Graphs: Complexity of Human Languages	244

10.8	Conclusions and Future Work	245
	Acknowledgments	246
	References	246
<b>11</b>	<b>Circumscribed Complexity in Ecological Networks</b>	<b>249</b>
	<i>Robert E. Ulanowicz</i>	
11.1	A New Metaphor	249
11.2	Entropy as a Descriptor of Structure	250
11.3	Addressing Both Topology and Magnitude	251
11.4	Amalgamating Topology with Magnitudes	252
11.5	Effective Network Attributes	253
11.6	Limits to Complexity	253
11.7	An Example Ecosystem Network	255
11.8	A New Window on Complex Dynamics	257
	References	258
<b>12</b>	<b>Metros as Biological Systems: Complexity in Small Real-life Networks</b>	<b>259</b>
	<i>Sybil Derrible</i>	
12.1	Introduction	259
12.2	Methodology	261
12.3	Interpreting Complexity	264
12.3.1	Numerically	267
12.3.1.1	Scale-free	267
12.3.1.2	Small World	268
12.3.1.3	Impacts of Complexity	269
12.3.2	Graphically	271
12.4	Network Centrality	274
12.4.1	Centrality Indicators	275
12.4.1.1	Degree Centrality	275
12.4.1.2	Closeness Centrality	275
12.4.1.3	Betweenness Centrality	276
12.4.2	Network Centrality of Metro Networks	277
12.4.2.1	Degree Centrality	277
12.4.2.2	Closeness Centrality	278
12.4.2.3	Betweenness Centrality	279
12.5	Conclusion	282
	References	283

**Index** 287