

1 Introduction

1.1 Systems

Figures 1.1 to 1.6 show six examples of systems that have NME: powdered aluminum, thermal barrier coating, sandstone, cement, ceramic, and soil. For each figure there is a scale bar or caption that makes it clear that the systems of interest have noticeable inhomogeneities on a length scale smaller than the sample size, say $100\ \mu\text{m}$, but much larger than the microscopic scale, $0.1\ \text{nm}$. We imagine the physical systems that possess NME to have *very approximately* a *bricks-and-mortar* character. The bricks [quartz grains in the case of rocks, packets of crystallites (quartz, feldspar, ...) with clay particles in the case of soils, single crystals of aluminum in the case of powdered aluminum, ...] interface with one another across a distinctive, elastically different system, the mortar (a system of asperities in the case of rocks, a system of fluid layers and fillets in the case of (wet) soil, a layer of defective material in the case of aluminum powder, etc.). We are interested in these systems on a length scale that is large compared to that of their bricks. Systems built up to this length scale have important elastic features conferred by the geometry of the system that are strikingly different from those of their bricklike constituents.

For example, in the case of a Berea sandstone, the typical elastic modulus is an order of magnitude smaller than the corresponding modulus of quartz, that is, the

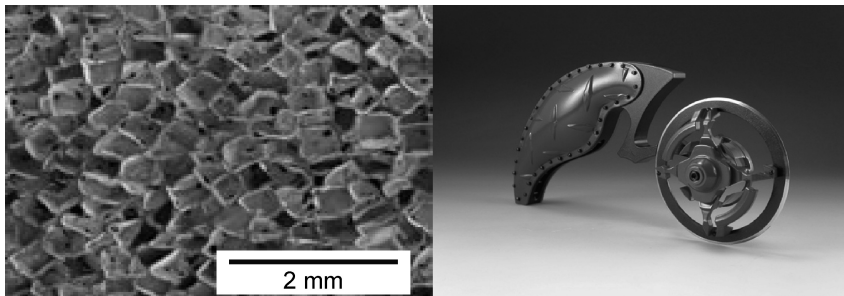


Fig. 1.1 Porous aluminum powder [9]. (Please find a color version of this figure on the color plates)

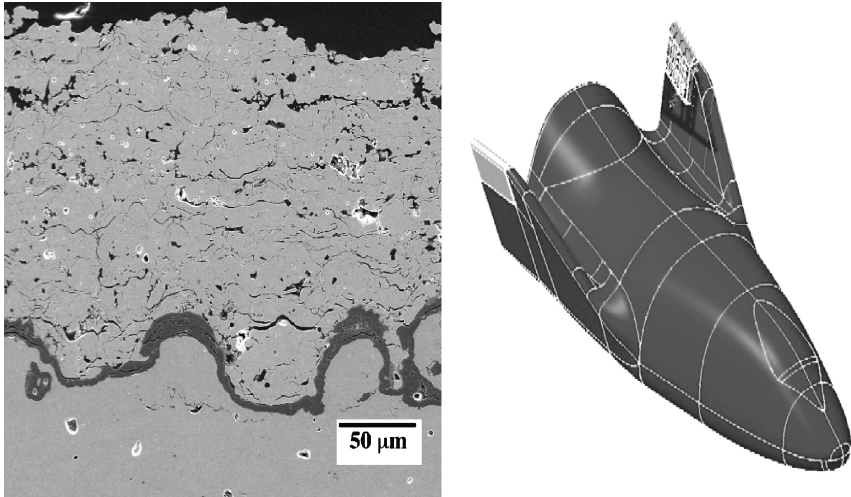


Fig. 1.2 Thermal barrier coating [10, 11]. (Please find a color version of this figure on the color plates)

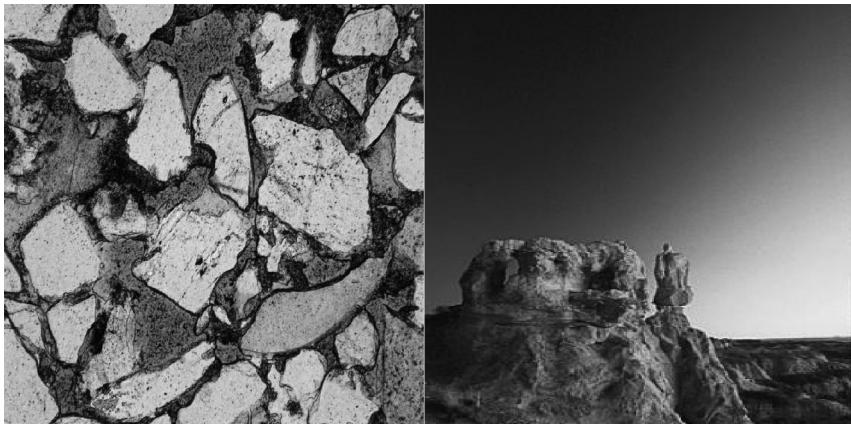


Fig. 1.3 Sandstone (typical grain size $100\ \mu\text{m}$) [12]. (Please find a color version of this figure on the color plates)

bricks. This means that a given force, say across a sample, produces ten times as much displacement as it would if applied across the quartz alone. This displacement must reside in the mortar as the assembly process could not have altered the stiffness of the bricks. The mortar is a minor constituent of the whole comprising, perhaps, 10% of the volume. Ten times as much displacement due to 10% of the volume means that the mortar is very soft and that it carries strains approximately two orders of magnitude greater than those in the bricks. Accompanying the inhomogeneity in the structure is an inhomogeneity in the strain. There is a further important point. Ten percent by volume of soft material randomly distributed in otherwise hard material could not markedly modify the response of the assembly.

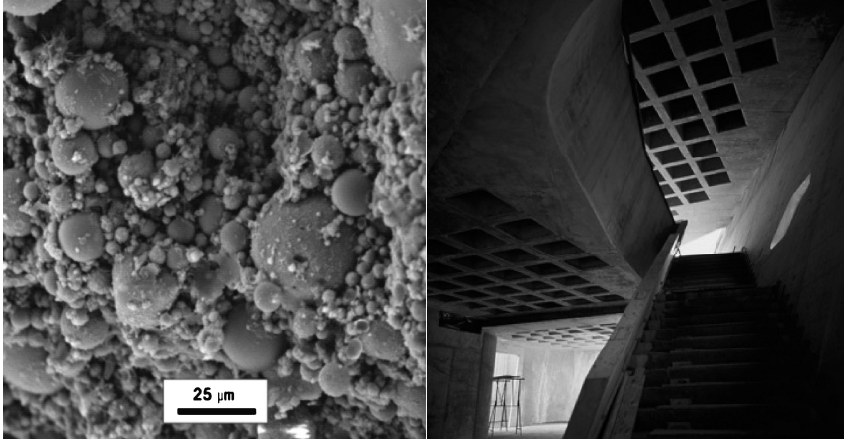


Fig. 1.4 Cement [13]. (Please find a color version of this figure on the color plates)

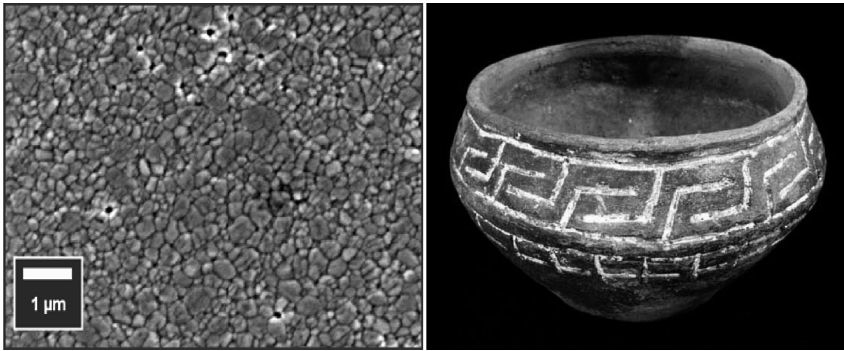


Fig. 1.5 Ceramic [14]. (Please find a color version of this figure on the color plates)



Fig. 1.6 Soil (sieved, typical grain size 1 mm) [15, 16]. (Please find a color version of this figure on the color plates)

The bricks-and-mortar picture captures an essential aspect of the way in which NME materials are constructed, that is, in such a way that the minority component (by volume) can effectively shunt the behavior of the majority component.

In identifying systems of interest with these simple ideas we cast a net that includes ceramics, soils, rocks, etc. But we do not pretend in any way to do justice to the disciplines of ceramic science, soil science, concrete science, . . . , or even to elasticity in ceramics, soils, concretes, . . . These are highly developed fields comprised of many subdisciplines. The discussion we present will be relevant more or less as dictated by the specific types of soil/ceramic/concrete/...

1.2

Examples of Phenomena

In Figure 1.7 we illustrate schematically eight examples of elastic behavior that we associate with NME. These include behavior that is quantitatively different from the usual behavior, behavior that is qualitatively different from the usual behavior, behavior that brings to the fore the importance of time scale and behavior in auxiliary fields. Not all NME materials possess these behaviors to the same degree. We sketch what is being illustrated schematically in each panel below. In the figure caption, information is given that locates an example of these experiments and characterizes them quantitatively.

1. The velocities of sound, c , of a sandstone are a factor of 2 to 4 less than those of the major constituent, for example, a quartz crystal. Thus the elastic constants of NME materials, K , $K \propto c^2$, might be less than the elastic constants of the parent material by an order of magnitude (even more for a soil).
2. When the pressure, P , is changed from 1 bar to 200 bar, the velocity of sound of a sandstone changes by a factor of 2. The same pressure change produces a 1% change in the velocity of sound in quartz (water, other homogeneous materials). Thus elastic nonlinearity, measured by $\gamma_c = d \ln(c)/d \ln(P)$, is very large for NME materials, often several orders of magnitude larger than that of the parent material.
3. When a sandstone (soil) is taken through a pressure loop, the strain that results is a hysteretic function of the pressure. In addition, when there are minor pressure loops within the major loop, the strain at the endpoints of the minor loop is “remembered”. NME materials can have hysteretic quasistatic equations of state with endpoint memory.
4. A sample is subjected to a step in stress. Accompanying that step is a prompt step in strain followed by a slow further strain increase that evolves approximately as $\log(t)$. Recovery from the release of the step stress has a similar prompt step in strain and $\log(t)$ further reduction in strain. NME materials exhibit *slow dynamics* in response to transient loading.
5. The resonance of a bar of NME material is swept over at a sequence of fixed drive amplitudes. As the drive amplitude is increased, the resonant frequency shifts (to a lower frequency) and the effective Q of the system, measured by the amplitude at resonance, decreases. In a plot of the detected amplitude per unit drive, this is seen as a shift in the resonance peak accompanied by

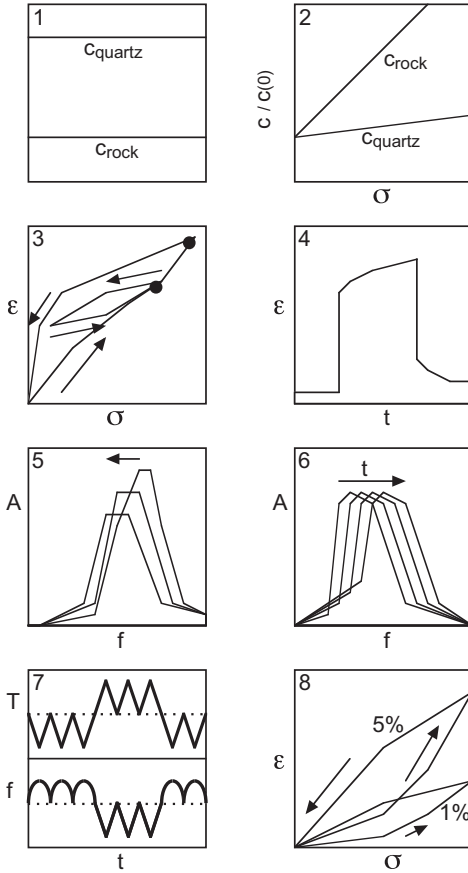


Fig. 1.7 Eight experiments. The eight experiments of interest are: (1) The velocity of sound, hence elastic constants, of a sandstone is a factor of 2 to 4 less than that of the major constituent, for example, a quartz crystal [1]. (2) When the pressure is changed, the velocity of sound of a sandstone changes by a factor of 2 for the application of 200 bar, whereas the same pressure change produces a 1% change in the velocity of sound in quartz (water, other homogeneous materials) [2]. (3) When a sandstone (soil) is taken through a pressure loop, the strain that results is a hysteretic function of the pressure and exhibits elastic endpoint memory [3]. (4) Accompanying the step in stress is a step in strain followed by a slow further strain response, that is, more strain, that evolves as $\log(t)$. Recovery from the release of the step stress has a similar strain step and $\log(t)$ further

strain [4]. (5) The resonance of a bar of material is swept over at a sequence of fixed drive amplitudes. As the drive amplitude increases, the resonant frequency shifts (to lower frequency) and the effective Q of the system decreases [5]. (6) The slow evolution of the elastic state, brought about by an AC drive (compare to panel 4), can be seen in experiments in which the elastic state, once established, is probed by a low drive sweep over a resonance [6]. (7) When the temperature is changed slightly, the elastic response to that change involves a broad spectrum of time scales (compare to panels 4 and 6), suggesting $\log(t)$ behavior. In addition, the elastic response to temperature is asymmetric in the sign of the temperature change [7]. (8) A stress/strain loop similar to that in panel 3 is changed markedly by the configuration of fluid in the pore space [8].

a reduction of the amplitude at resonance. This behavior, which follows the fast motion of the drive, is an example of *fast dynamics*.

6. A bar of NME material is brought to steady state in response to a large-amplitude AC drive. The AC drive is turned off and the subsequent elastic state of the bar is probed with a low-amplitude drive that is swept over a resonance. The resonance, initially with resonance frequency shifted to a lower frequency as in panel 5, evolves back to a higher frequency approximately as $\log(t)$. The elastic state of the bar, established by a *fast dynamics drive*, relaxes once that drive is turned off by *slow dynamics*.
7. When the temperature of an NME material is changed slightly, the elastic response to that change, brought about by the temperature-induced internal forces, involves a broad spectrum of time scales (compare to panels 4 and 6), suggesting $\log(t)$ behavior at the longest times. In addition, the elastic response to temperature is asymmetric in the sign of the temperature change.
8. When an NME material is subjected to the internal forces of fluid configurations, a stress/strain loop similar to that in panel 3 is changed markedly. Much like a sponge, a rock is softer when wet.

The sequence of experiments sketched here call attention to the physical variables that are involved in the description of NME systems. The nature of a probe, the pressure, the temperature, the fluid configurations, the probe size, the duration of a probe, and the aftereffect of a probe having been present must all be considered and examined.

1.3

The Domain of Exploration

NME materials are probed in the complex phase space illustrated in Figure 1.8, that is:

1. Length. There are three length scales associated with NME materials, the microscopic scale (interatomic spacing) $a = 0.1$ nm, the scale of inhomogeneity $b \approx 1\text{--}100$ μm , and the sample size $L \gg b$. A quasistatic measurement is at $k \rightarrow 0$ ($k = 2\pi/\lambda$), whereas a resonant bar experiment is at wavelengths related to the sample size, $b \ll \lambda < L$.
2. Strain. There are judged to be two strain values of importance. At strains $\varepsilon < 10^{-7}\text{--}10^{-6}$, the nonlinear effects are small and have a more or less traditional behavior. At strains $\varepsilon > 10^{-3}\text{--}10^{-2}$ irreparable damage is done to a sample. The middle ground $10^{-7} < \varepsilon < 10^{-3}$ is the strain domain of NME.
3. Force. The standard for the strength of forces is the pressure given by a typical elastic constant, $K \approx \rho c^2$, where ρ is the density and c is the speed of sound, $K \approx 10^{11}$ dyne/cm² = 10^4 MPa for a sandstone (1 atmosphere is 10^6 dyne/cm² = 10 MPa). NME materials may be subject to a wide range of forces – applied forces, forces delivered to the interior of the systems from

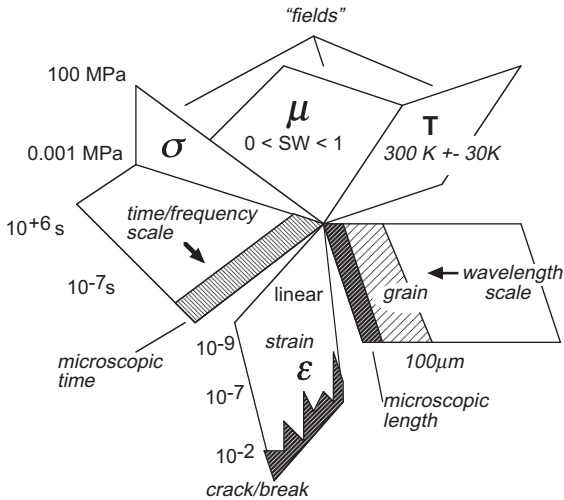


Fig. 1.8 Phase space. The materials of interest are probed on different time scales, length scales, and strain scales and with a variety of applied “fields”.

the complex thermal response of constituents, or forces delivered to the interior of the system from arrangements of fluid in the pore space. The approximate strain consequence of a force (pressure) is found using $\epsilon \approx P/K$, where P is the pressure. The strain range given above, $10^{-7} < \epsilon < 10^{-2}$, implies $10^{-3} \text{ MPa} < P < 10^2 \text{ MPa}$.

4. Time. The fastest time scale relevant to NME materials is approximately the time for sound to cross an inhomogeneity, $\tau \geq 100 \mu\text{m}/c \approx 10^{-7} \text{ s}$. A resonant bar measurement is typically at 10^3 – 10^4 Hz (this scale is set by sample size L), a quasistatic measurement of stress/strain may last 10 min, and the strain response to a change in temperature may develop over a week. The range of time scales is enormous, 10^{-7} to 10^6 s .

All of these scales – length, time, and force – are far removed from the corresponding microscopic scales, for example, 0.1 nm is the microscopic length scale, 10^{12} Hz (a typical Debye frequency) is the microscopic time scale, and a microscopic energy per microscopic volume (say $0.1 \text{ eV}/(0.1 \text{ nm})^3 \approx 10 \text{ GPa}$) is the microscopic force scale (stated here in terms of pressure since force alone means little).

1.4 Outline

Our interest is in the nonlinear elasticity of mesoscopically inhomogeneous materials. We will discuss the theoretical apparatus that is used to describe these mate-

rials, the phenomenology of the experiments conducted, and the large body of data that illustrates the behavior that characterizes these materials.

In Part I, Chapters 1–5, we give a theoretical introduction to traditional linear and nonlinear elasticity. We begin the discussion at the microscopic level. It is here that the basic structure of linear and nonlinear elasticity is established and the numbers that determine the magnitude of almost all quantities of interest are set. It is a short step from a microscopic description to the continuum description that corresponds to the traditional theory of linear/nonlinear elasticity. These topics are covered in Chapter 2, which is followed, in Chapter 3, by a series of illustrations of the consequences of the theory. To get to the domain of elasticity of mesoscopically inhomogeneous materials we must jump a gap. Across this gap, where we will work, we start with a theoretical apparatus, having the same form as the traditional theory of linear/nonlinear elasticity, to which we will add a collection of *ad hoc* ingredients that have no immediate source in the domain we have left behind. A variety of mesoscopic elastic elements, contacts, interfaces, etc. are described in Chapter 4. So also is an effective medium scheme for turning mesoscopic elastic elements into elastic constants suitable for a theory of elasticity. The coupling of the elastic field to auxiliary fields, particularly temperature and saturation, is taken up in Chapter 5.

In Part II, Chapters 6–9, we introduce hysteretic elastic elements, or strain elements with an elaborate stress response, Chapter 6. The dynamics of elastic systems carrying these elastic elements can be complex because of an internal field that responds to stress slowly in time. A discussion of the resulting *fast* and *slow* dynamics is given in Chapter 7. A set of practical matters related to data analysis and modeling of data sets is taken up in Chapter 8. This is followed by a description in Chapter 9 of a wide variety of considerations that relate to using data on elastic systems for characterization (spectroscopy) and for location (tomography).

In Part III, Chapters 10–13, we discuss experiments. Quasistatic measurements, including coupling to auxiliary fields, are described in Chapter 10. Dynamic measurements, dynamic/quasistatic to dynamic/dynamic, are described in Chapter 11. The current picture of fast/slow dynamics is given a full airing. In Chapter 12, field experiments that touch on NME are described. The final chapter, Chapter 13, contains a description of a wide variety of nondestructive evaluation applications of NME.

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