

Figure 2.4 (a) Circuit with a voltage source for nodal analysis. (b) Transformed network for using nodal analysis using matrix formulation.

But, $V(s) = I_1(s)R_2$. On substituting and bringing it to the left side, one gets

$$\begin{bmatrix} R_1 + R_2 & 0 \\ g_m R_2 R_3 & R_3 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_s(s) \\ 0 \end{bmatrix} \quad (2.2)$$

Equation 2.2 is the loop matrix formulation for Figure 2.3a.

Example 2.2. Figure 2.4a shows a circuit with a voltage source. Develop the nodal matrix formulation for the network.

Figure 2.4b shows the equivalent circuit redrawn after applying source transformation to $v_s(t)$ and transforming the network using the representation for a capacitor as shown in Figure 2.2b. Since admittances are to be used, letting $G = 1/R$ for the conductance, one can write

$$\begin{bmatrix} G_1 + G_2 + sC & 0 \\ 0 & G_3 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} I_s(s) + \cancel{Cv_1(0_-)} \\ -g_m V_1(s) \end{bmatrix} \quad (2.3)$$

where $I_s(s) = G_1 V_s(s)$. On substituting and bringing $V_1(s)$ to the left side, the final formulation becomes

$$\begin{bmatrix} G_1 + G_2 + sC & 0 \\ g_m & G_3 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_s(s)G_1 \\ 0 \end{bmatrix} + \begin{bmatrix} \cancel{Cv_1(0_-)} \\ \cancel{0} \end{bmatrix} \quad (2.4)$$

It is observed that Eq. (2.4) is in the form $W(s)X(s) = F(s) + h(s)$.

2.4

Network Functions

If we study the relationships developed in connection with nodal and loop analyses, we discover a general format, namely, $W(s)X(s) = F(s) + h(s)$, where $W(s)$ can be either an admittance or an impedance matrix, $X(s)$ a nodal voltage vector or a loop current vector, $F(s)$ the vector of independent sources, and $h(s)$ the vector of initial conditions. Using this equation, one can easily arrive at $X(s) = W^{-1}(s)F(s) + W^{-1}(s)h(s)$. The *first* part of the solution on the right-hand side (RHS) is the complete solution if the initial values were zero (i.e., $h(s) = 0$). This is called the *zero (initial)-state response*. The *second* part of the solution on the RHS is the complete solution if the forcing functions were zero (i.e., $F(s) = 0$). This is known as the *zero-input* or *natural response*.

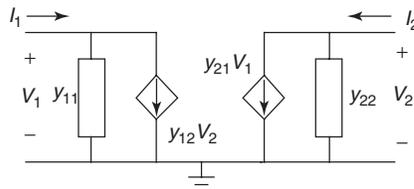


Figure 2.7 Equivalent circuit model for two-port admittance matrix.

2.5.2.2 Impedance Matrix Parameters

If V_1 and V_2 are chosen as the independent variables, then the two-port may be characterized by

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or

$$[z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.7}$$

where the parameters z_{11}, z_{12}, z_{21} , and z_{22} may be determined by the equations $z_{11} = [V_1/I_1]_{I_2=0}, z_{12} = [V_1/I_2]_{I_1=0}, z_{21} = [V_2/I_1]_{I_2=0}$, and $z_{22} = [V_2/I_2]_{I_1=0}$. Since $I_1 = 0, I_2 = 0$ implies open (to AC) circuit conditions, the parameters z_{11}, z_{12}, z_{21} , and z_{22} are called *open circuit impedance parameters*, and $[z]$ the *open circuit impedance matrix* (or simply, impedance matrix) of the two-port. It is obvious from Eqs. (2.6) and (2.7) that

$$[y] = [z]^{-1} \tag{2.8}$$

The AC equivalent circuit model is shown in Figure 2.8.

2.5.2.3 Chain Parameters (Transmission Parameters)

If we consider V_2 and I_2 as the independent variables, then we may write

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [a] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{2.9}$$

The matrix $[a]$ is called the *chain matrix* or *transmission matrix* of the two-port. The parameters A, B, C , and D are called the *chain or transmission parameters*. Note that $-I_2$ instead of I_2 is used to imply a current flowing outward at port 2. It is seen that the parameters A, B, C , and D may be defined as

$$\frac{1}{A} = \frac{V_2}{V_1} \Big|_{I_2=0}, \quad \frac{1}{B} = \frac{-I_2}{V_2} \Big|_{V_2=0}, \quad \frac{1}{C} = \frac{V_2}{I_1} \Big|_{I_2=0}, \quad \frac{1}{D} = \frac{-I_2}{I_1} \Big|_{V_2=0} \tag{2.10}$$

suffix 2 should change to suffix 1

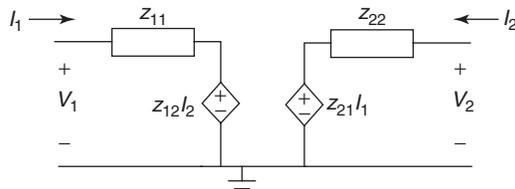


Figure 2.8 Equivalent circuit model for two-port impedance matrix.

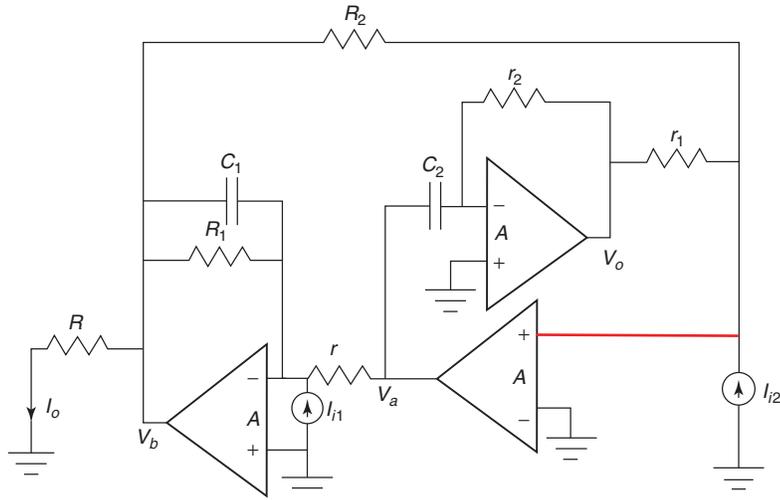


Figure P2.10

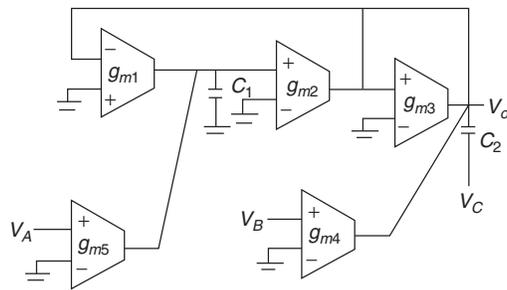


Figure P2.11

$$\frac{I_o}{I_{i2}} = \frac{-\frac{1}{rR}}{\frac{1}{rR_2} + \left(\frac{1}{Ar} + \left(1 + \frac{1}{A} \right) \left(\frac{1}{R_1} + sC_1 \right) \right) \left(\frac{\frac{sC_2}{r_1}}{\frac{1}{r_2} + sC_2} + \frac{\frac{1}{r_1} + \frac{1}{r_2}}{A} \right)}$$

2.11 Using the ideal small signal model for each OTA in Figure P2.11, derive that

$$V_o = \frac{g_{m2}g_{m5}V_A - sg_{m4}C_1V_B + s^2C_1C_2V_C}{s^2C_1C_2 + sg_{m3}C_1 + g_{m1}g_{m2}}$$

2.12 Figure P2.12 shows a twin-Tee network frequently used in connection with design of equalizers. Write a subset of two-port parameters which could be used very easily to derive the overall two-port parameters of the system.

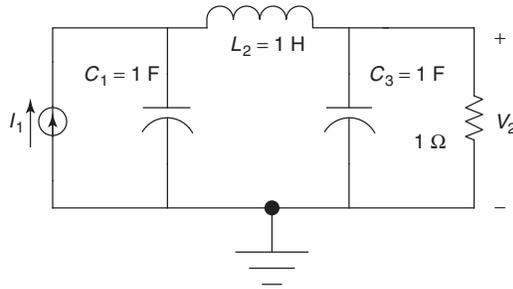


Figure 3.2 An LC filter with a normalized load resistance of 1Ω .

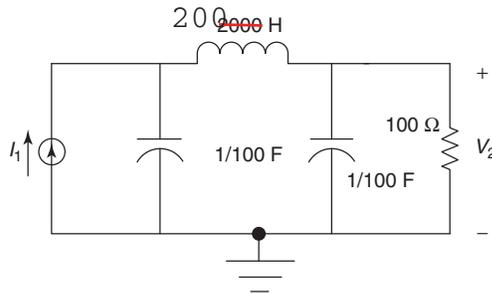


Figure 3.3 The LC filter corresponding to that of Figure 3.2 for a load of 100Ω .

The system function given by $V_2/I_1 = Z_{21}$ is

$$Z_{21} = \frac{1}{s^3 C_1 L_2 C_3 + s^2 L_2 C_1 + s(C_1 + C_3) + 1} = \frac{1}{\cancel{2}s^3 + \cancel{2}s^2 + 2s + 1}$$

To obtain the filter corresponding to a 100Ω resistance at the load we perform the following scaling:

$$R_i^* = 100R_i = 100 \Omega, L_2^* = 100L_2 = 200 \text{ H}, C_1^* = C_2^* = \frac{1}{100} \text{ F}$$

Hence,

$$Z_{21}^* = 100Z_{21} = \frac{100}{\cancel{2}s^3 + \cancel{2}s^2 + 2s + 1}$$

The scaled circuit is shown in Figure 3.3.

3.3 Dual and Inverse Networks

3.3.1 Dual and Inverse One-Port Networks

It is well known that if two one-port RLC networks N_1 and N_2 have their impedances z_1 and z_2 related by

$$z_1 z_2 = K \tag{3.7}$$

3.7

Frequency Scaling

Consider a general LTI network N . If we change the complex frequency s to s/b , b being a constant, it is clear that in any of the system functions, whether it be the TIF $Z_{21}(s)$, the TAF $Y_{21}(s)$, the VTF $T_V(s)$, or the CTF $T_I(s)$, the frequency s is changed to s/b . Hence, whatever was the magnitude or phase of a system function $F(s)$ at a frequency ω , the new system function $F(s/b)$ would have the same magnitude and phase at the frequency ω/b , that is, the frequency has been scaled by a factor $1/b$. Such an operation is called *frequency scaling* by the factor $1/b$. As a consequence of the frequency scaling, it is clear that an inductance of value L and a capacitor of value C would now have reactances of sL/b and $1/(sC/b)$, respectively. Thus, these will become, respectively, an inductor of value L/b and a capacitor of value C/b , while a resistor will remain unchanged. It is also seen that the ideal controlled sources also remain unaltered. Frequency scaling is used to obtain the response of a given filter in a given frequency band scaled up or down to a different frequency band.

Example 3.4. The Sallen and Key structure of Figure 3.1 realizes the LP TF

$$\frac{V_2}{V_1} = T_V(s) = \frac{2.9/4}{s^2 + 0.05s + 0.25}$$

According to the standard notations for a second-order filter, namely,

$$T_V(s)|_{\text{LP}} = \frac{H_{\text{LP}} \Omega_p^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

the above filter has a pole frequency $\omega_p = \sqrt{0.25} = 0.5$.

(a) Find the TF if ω_p is to be 1000 rad s^{-1} .

(b) If in addition, we want to use 1 K resistors in the circuit, how is the TF altered. Also, how do the values of the components change in the LP structure?

Solution: (a) The frequency scaling factor is $b = 1000/0.5 = 2000$. Hence, s is changed to $s/2000$ in the expression for $T_V(s)$, giving the new TF to be

$$T'_V(s) = \frac{(2.9/4)(2000)^2}{s^2 + 0.05(2000)s + 0.25(2000)^2}$$

This does not alter the values of the resistors, but the capacitance values become 0.001.

(b) If all the resistors have to be 1 K, we have to scale all the impedances by the factor $a = 1000$. Since impedance scaling does not alter a VTF, the TF remains $T'_V(s)$. However, the new values of the components are $R_1 = R_2 = 1000 \Omega$ and $C_1 = C_2 = 1 \text{ } \cancel{\mu\text{F}}$, with the gain $K = 2.9$ of the voltage amplifier remaining unchanged. micro-farad

3.8

Types of Filters

Filters are categorized depending on the type of filtering function that they perform. If the primary consideration is the magnitude or attenuation characteristic, then

low-pass (LP)

we classify them as **LP**, high-pass (HP), band-pass (BP), and **bandstop** (BS) or band-reject (BR). However, if our main concern is the phase or delay specification, then the type of filters are all-pass (AP) and delay equalizers. In all these cases, the TF to be realized is of the form

$$H(s) = \frac{N(s)}{D(s)} = k \frac{s^m + \dots + a_1s + a_0}{s^n + \dots + b_1s + b_0}, \quad m \leq n \quad (3.28)$$

An LP filter is to pass low frequencies from DC to a desired frequency ω_c , called the *cutoff frequency*, and to attenuate frequencies beyond the cutoff. An ideal LP filter would have a magnitude for $H(j\omega)$ that is constant from zero to ω_c and zero beyond ω_c . The frequency band from DC to ω_c is called the *passband*. However, such a brick-wall characteristic is impossible to realize in practice, and the LP filter specifications are always given in terms of its cutoff frequency ω_c , an **SB** frequency ω_s , maximum loss (A_p) allowed in the PB, and minimum SB attenuation (A_s). The

stop band (SB)

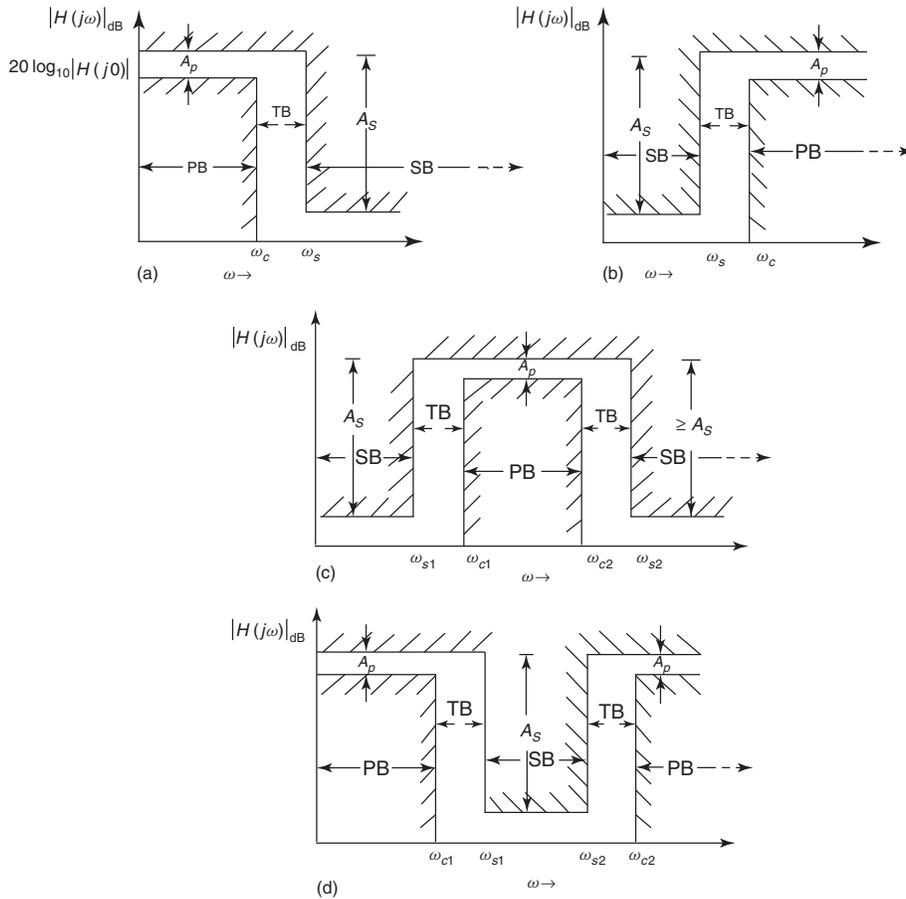


Figure 3.9 Typical specifications of (a) **LP** filter, (b) **HP** filter, (c) a BP filter, and (d) a BR filter.

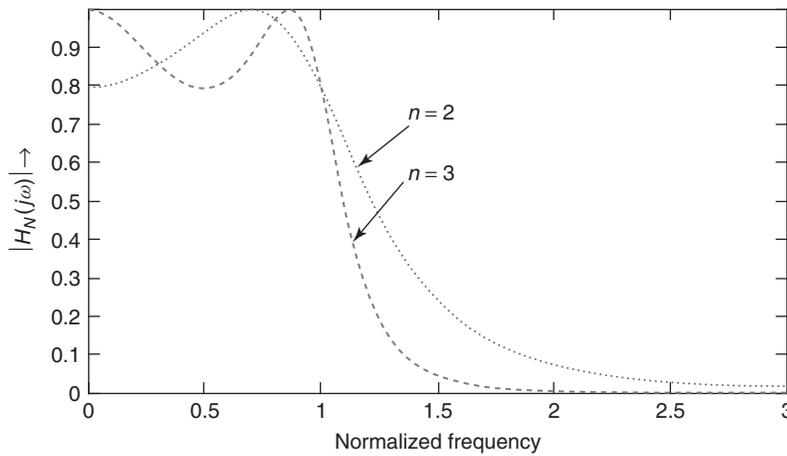


Figure 3.12 Magnitude response of the normalized CHEB filter for $n = 2$ and 3 with $\varepsilon = 0.5089$.

Given the parameter A_p in decibels, we can find the ripple factor ε using

$$\varepsilon = \sqrt{10^{0.1A_p} - 1} \quad (3.49)$$

Also, at the edge of the SB, that is ω_s , the loss in decibels is A_s . Hence,

$$-A_s = 10 \log[1/[1 + \varepsilon^2 C_n^2(\omega_s)]]$$

Using Eqs. (3.47) and (3.49), we get the order of the filter, n , to be the lowest integer when

$$n \geq \frac{\cosh^{-1} \sqrt{\eta}}{\cosh^{-1} \omega_s} \quad (3.50a)$$

where

$$\eta = \frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1} \quad (3.50b)$$

An alternate approximate expression is (Schaumann, Ghausi, and Laker, 1990)

$$n \geq \frac{\ln \sqrt{4(10^{0.1A_s} - 1)/\varepsilon^2}}{\ln(\omega_s + \sqrt{\omega_s^2 - 1})} \quad (3.51)$$

3.9.2.1 CHEB Filter Transfer Function

Just as in the case of the MFM approximation, we start with

$$D(s)D(-s) = 1 + \varepsilon^2 C_n^2(\omega)$$

where $\omega = s/j$, and obtain the roots of $1 + \varepsilon^2 C_n^2(\omega) = 0$, and associate the LH plane roots with those of the poles of $D(s)$. The above equation can be rewritten as

$$\cos\{n \cos^{-1}(-js)\} = \pm j/\varepsilon \quad (3.52)$$

3.11.1

Phase Characteristics of a Transfer Function

The phase function associated with a signal is easily recognized by considering the phasor representation for a time-varying signal. Thus, in polar coordinates $V = V_M \exp(-j\varphi)$ represents a signal of magnitude V_M having a phase delay of φ radians. For a system, such as a filter, the frequency-dependent TF can be similarly expressed in polar form with a magnitude part and a phase part. The magnitude approximation problem discussed earlier is associated with this magnitude part. An understanding about the phase part can be obtained by writing the TF in the form

$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{m_1(j\omega) + jn_1(j\omega)}{m_2(j\omega) + jn_2(j\omega)} \quad (3.74)$$

For m_1, n_1, m_2, n_2
read $(j\omega)$ as (ω)
only, i.e., instead of $m_1(j\omega)$
read $m_1(\omega)$..

Read $N(\omega)$ as
 $N(j\omega)$

where m_1 and n_1 are the even and odd parts (i.e., terms involving even and odd powers in ω) of $N(\omega)$, and m_2 and n_2 are the even and odd parts of $D(\omega)$. Read $D(\omega)$ as $D(j\omega)$
The magnitude of $H(j\omega)$ is given by $\sqrt{(m_1^2 + n_1^2)/(m_2^2 + n_2^2)}$, while the phase angle is given by $\phi(j\omega) = \arctan(n_1/m_1) - \arctan(n_2/m_2)$. Using basic trigonometric identities, $\phi(j\omega)$ can be expressed as

$$\phi(j\omega) = \arctan \left[\frac{n_1 m_2 - n_2 m_1}{m_1 m_2 + n_1 n_2} \right] \quad (3.75)$$

In phase approximation problems, this phase function $\phi(j\omega)$ will be required to have a desirable characteristic.

3.11.2

The Case of Ideal Transmission

For digital/video signals, the goal of signal processing is very different from the concept of frequency-selective filtering (i.e., rejection of certain frequencies relative to other frequencies). The signal transmission has to be ideal, that is, the magnitude can be changed only by a constant factor K irrespective of the frequency, and there can only be a constant delay in time so that the relative timing (phase) among the various frequency contents of the signal remains unchanged. Analytically, if $f(t)$ is a given signal, the processed signal can be $f(t) = Kf(t - t_0)$, where t_0 is a fixed delay. Taking Laplace transform and setting $s = j\omega$, the processed signal would be $F'(j\omega) = K F(j\omega)\exp(-j\omega t_0)$. Thus the transfer characteristic of the processing function is required to be $H(j\omega) = F'(j\omega)/F(j\omega) = K \exp(-j\omega t_0)$. Since $\exp(-j\omega t_0)$ represents only a delay, the TF $H(j\omega)$ has a magnitude equal to K , which implies an AP characteristic in the frequency domain. The phase $\varphi = -\omega t_0$ implies a constant delay $\tau = t_0$. The phase is a linear function of frequency ω leading to a constant delay in the time domain.

$$4) \frac{2}{3}s \left(\frac{1}{6}s \leftarrow \text{capacitor, } C = \frac{1}{6}\text{F} \right)$$

$$\frac{2}{3}$$

$$\frac{2}{3}$$

$$\frac{2}{3}$$

Thus,

$$Z_{LC}(s) = s + \frac{1}{\frac{1}{3}s + \frac{9}{\frac{1}{3}s + \frac{1}{\frac{1}{6}s}}}} \tag{4.10}$$

The above $Z_{LC}(s)$ may be realized in the Cauer-I form as shown in Figure 4.1a. The Cauer-II form is obtained by CF expansion of $Z_{LC}(s)$ around $s = 0$ and is given below.

$$2s + s^3 \left(4 + 5s^2 + s^4 \left(2/s \leftarrow \text{capacitor, } C = 1/2\text{F} \right) \right)$$

$$\frac{4 + 2s^2}{3s^2 + s^4} \left(\frac{2}{3}s \leftarrow \text{inductor, } L = 3/2\text{H} \right)$$

$$\frac{2s + \frac{2}{3}s^3}{\frac{1}{3}s^3} \left(\frac{9}{s} \leftarrow \text{capacitor, } C = 1/9\text{F} \right)$$

$$\frac{3s^2}{s^4} \left(\frac{1}{3}s^3 \left(\frac{1}{3}s \leftarrow \text{inductor, } L = 1/3\text{H} \right) \right)$$

$$\frac{1}{3}s^3$$

$$\frac{1}{3}$$

Hence,

$$Z_{LC}(s) = \frac{2}{s} + \frac{1}{\frac{2}{3}s + \frac{9}{\frac{1}{3}s + \frac{1}{\frac{1}{6}s}}}} \tag{4.11}$$

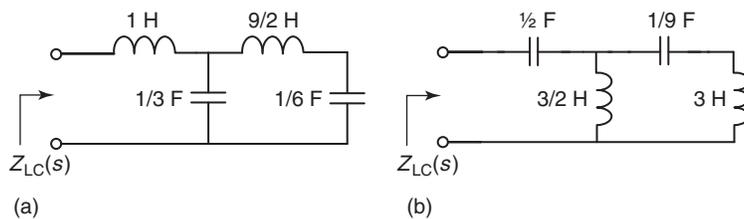


Figure 4.1 Realization of $Z_{LC}(s)$ of Example 4.1: (a) Cauer-I form and (b) Cauer-II form.

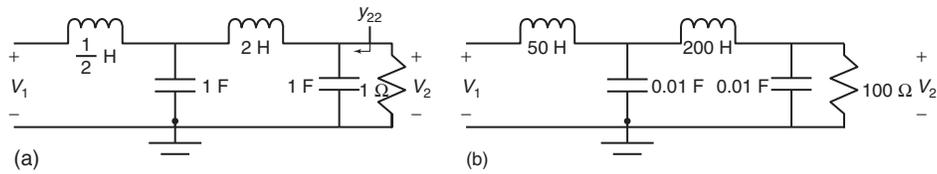


Figure 4.4 Realization of $H(s)$ of Example 4.2 when (a) $R_L = 1 \Omega$ and (b) $R_L = 100 \Omega$.

Hence, the realization of $H(s)$ is as shown in Figure 4.4a. At $s = 0$, all the inductors are short circuits and the capacitors are open circuits, and hence, $V_2/V_1 = 1 = H(0) = (k/2)$; thus $k = 2$. Since the given termination is 100Ω , we now scale all the impedances by 100Ω , which of course will not alter the transfer function. The scaled network is shown in Figure 4.4b.

Example 4.3. The function $H(s) = \frac{k}{s^3 + 2s^2 + 2s + 1}$ represents a third-order normalized LP Butterworth filter function. Realize $H(s)$ by a singly terminated network when the network is driven (a) by an ideal voltage source and (b) by an ideal current source, the terminating load resistance being 1Ω . (c) If the load resistance is 100Ω and the cutoff frequency of the filter is 100 rad s^{-1} , find the corresponding realizations.

(a) In this case, $m(s) = 2s^2 + 1$ and $n(s) = s^3 + 2s$. Since the degree of $m(s)$ < the degree of $n(s)$, we have to perform the CF expansion on $n(s)/m(s)$ around $s = \infty$ to realize $(1/\gamma_{22})$. The CF expansion is as follows:

$$\begin{aligned}
 & 2s^2 + 1) \overline{s^3 + 2s} \left(\frac{s}{2} \leftarrow \text{inductor, } L = \frac{1}{2} \text{H} \right. \\
 & \quad \underline{s^3 + \frac{1}{2}s} \\
 & \quad \quad \frac{3}{2}s) \overline{2s^2 + 1} \left(\frac{4}{3}s \leftarrow \text{capacitor, } C = \frac{4}{3} \text{F} \right. \\
 & \quad \quad \quad \underline{2s^2} \\
 & \quad \quad \quad \quad 1) \overline{\frac{3}{2}s} \left(\frac{3}{2}s \leftarrow \text{inductor, } L = \frac{3}{2} \text{H} \right. \\
 & \quad \quad \quad \quad \quad \underline{\frac{3}{2}s} \\
 & \quad \quad \quad \quad \quad \quad -
 \end{aligned}$$

The realization of $H(s)$ as a VTF with $R_L = 1 \Omega$ is shown in Figure 4.5a. The value of $k = 1$, since $H(0) = 1$.

(b) As discussed in Section 4.1, we may now realize the given $H(s)$ at the TIF, $Z_{21}(s)$, of a singly terminated network driven by an ideal current source by simply replacing the network N by N_D , and the corresponding realization is shown in Figure 4.5b.

(c) If the terminating resistance is changed to 100Ω , we shall use impedance scaling. Thus scale all $L \rightarrow 100L$ and all $C \rightarrow C/100$.

If the cutoff frequency is 100 rad s^{-1} , that is, $\omega_c = 2\pi \times 100 \text{ rad s}^{-1}$, then frequency scale all $L \rightarrow L/(2\pi \times 100)$, and all $C \rightarrow C/(2\pi \times 100)$.

5.1

Some Basic Building Blocks using an OA

In Chapter 2, we discussed how an OA could be used as a summer or integrator. There are a number of other simple building blocks, including bilinear filters, which are useful in the study of higher-order filters using OAs. These are tabulated in Table 5.1; these may be easily derived using the model of an ideal OA introduced in Chapter 2 and are left to the reader to derive them.

5.2

Standard Biquadratic Filters or Biquads

A second-order transfer function of the form

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_2s^2 + b_1s + b_0}{s^2 + a_1s + a_0} \quad (5.1)$$

is called a *biquadratic function*. Even though, in general, the poles and zeros may lie on the negative real axis, we will assume them to be complex conjugates, since poles and zeros on the negative real axis can be realized using passive-RC circuits (Van Valkenburg, 1960). In such a case, we may express

$$H(s) = \frac{N(s)}{D(s)} = H_0 \frac{(s+z)(s+z^*)}{(s+p)(s+p^*)} = H_0 \frac{s^2 + (\omega_z/Q_z)s + \omega_z^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \quad (5.2)$$

where

$$\omega_z^2 = (\operatorname{Re} z)^2 + (\operatorname{Im} z)^2, Q_z = \omega_z/2\operatorname{Re}(z) \quad (5.3a)$$

$$\omega_p^2 = (\operatorname{Re} p)^2 + (\operatorname{Im} p)^2, Q_p = \omega_p/2\operatorname{Re}(p) \quad (5.3b)$$

and H_0 could be positive or negative. ω_p and Q_p are called the *pole frequency* and *pole Q*; sometimes ω_p is called the *undamped natural frequency* since there will be resonance at $s = j\omega_p$, if the s -term is not present in the denominator of Eq. (5.2). The poles are given by

$$p_{1,2} = -\frac{\omega_p}{2Q_p} \pm j\frac{\omega_p}{Q_p} \sqrt{4Q_p^2 - 1} \quad (5.4)$$

It is clear that, in order to have complex poles, $Q_p > 0.5$. For a highly selective filter, Q_p should be large, that is, the real part of the poles should tend to zero, and hence the poles will be close to the imaginary axis. The following properties can be observed from Eqs. (5.2) and (5.3):

- 1) The dc gain is $[H_0(\omega_z^2/\omega_p^2)]$.
- 2) The gain at $\omega = \infty$ is $|H_0|$.
- 3) The maximum value of $|H(j\omega)|$ occurs approximately at ω_p if $Q_p \gg 1$. This is particularly true if $\omega_z \gg \omega_p$ or $\omega_z \ll \omega_p$; otherwise, it is slightly moved away from ω_p .
- 4) The minimum value occurs at approximately $\omega = \omega_z$ for $Q_z \gg 1$.

IC technology, it is easier to maintain ratios of capacitors or resistors more accurately than to maintain their absolute values precisely. However, the disadvantage is that the spread in the resistor value is rather large for large Q_p , since the ratio of the resistors is Q_p^2 . Also, observe that gain at $\omega = 0$ is always 2. Let us consider an example.

Example 5.1. Design an LP filter with a pole Q of 4 and a pole frequency of 10^4 rad s^{-1} , using the above design procedure.

The transfer function of the LP filter is given by

$$H_{LP}(s) = H_0 \frac{\omega_p^2}{s^2 + \left(\frac{\omega_p}{Q}\right)s + \omega_p^2} \quad (5.16)$$

where the values of ω_p and Q_p are given by $\omega_p = 10^4$ rad s^{-1} and $Q_p = 4$, respectively. From Eq. (5.15), we see that $R_1 = 16R_3$, $C = \frac{1}{(4(10^4)R_3)}$, $K = 2$, and hence $H_0 = 2$. Assuming $R_3 = 1 \Omega$, we get $R_1 = 16 \Omega$ and $C = 25 \mu\text{F}$. Impedance scaling by 1000, we have the component values as

$$R_3 = 1 \text{ k}\Omega, R_1 = 16 \text{ k}\Omega, C_1 = C_2 = 0.025 \mu\text{F}, \text{ and } K = 2.$$

Case 2: Equal-resistor and equal-capacitor design

Let $R_1 = R_3 = R$ and $C_2 = C_4 = C$. This leads to the design equations

$$C = 1/(R\omega_p^2) \text{ and } K = 3 - (1/Q_p) \quad (5.17)$$

The advantage of this design is the resistor as well as the capacitors are of equal value, but it should be noted that the value of the gain K is dependent on Q_p .

Example 5.2. Design an LP filter for the same specifications as those in Example 5.1 using the above design equations.

Using Eq. (5.17), we get

$$C_2 = C_4 = C = 1/(10^8 R), K = 3 - \left(\frac{1}{4}\right) = 2.75$$

Choosing $R = 100 \Omega$, we get the component values to be

$$R_1 = R_3 = 100 \Omega, C_2 = C_4 = 100 \text{ pF} \text{ and } K = 2.75$$

Case 3: Unity-gain design

Hence, $K = 1$. Let

$$m = C_4/C_2 \text{ and } n = R_3/R_1. \quad (5.18a)$$

and Q_p of the filters are related to the network components as follows:

$$\omega_p = \sqrt{\frac{R_6/R_5}{R_1 R_2 C_1 C_2}} \quad (5.37)$$

$$\frac{1}{Q_p} = \frac{1 + R_6/R_5}{1 + R_4/R_3} \sqrt{\frac{R_5 R_2 C_2}{R_6 R_1 C_1}} \quad (5.38)$$

However, the values of H_0 for the three filters are different and are given by

$$H_0(\text{LP}) = \frac{1 + R_6/R_5}{1 + R_3/R_4} \frac{R_5}{R_6} \quad (\text{at } \omega = 0) \quad (5.39a)$$

$$H_0(\text{BP}) = -\frac{R_4}{R_3} \quad (\text{at } \omega = \omega_p) \quad (5.39b)$$

$$H_0(\text{HP}) = \frac{1 + R_6/R_5}{1 + R_3/R_4} \quad (\text{at } \omega = \infty) \quad (5.39c)$$

Example 5.5. Design a BP filter with $Q_p = 10$ and $\omega_p = 10^3 \text{ rad s}^{-1}$, using the KHN biquad structure.

Equations (5.34), (5.37), (5.38), and (5.39b) show that there are eight variables ($R_i, i = 1, \dots, 6$ and C_1, C_2), while there are only two specifications. Hence, we have sufficient flexibility in assigning arbitrary values to six of the components. An examination of Eqs. (5.37) and (5.34) reveals that the ratio R_4/R_3 occurs only in the expression for Q_p . Therefore, we determine this to satisfy the value of specified Q_p . We now assume that $R_1 = R_2 = R$, $C_1 = C_2 = C$, and $R_5 = R_6 = R_x$. Then we have two simple expressions for ω_p and Q_p :

$$\omega_p = \frac{1}{RC} \implies R = \frac{1}{\omega_p C} \quad (5.40)$$

$$\frac{1}{Q_p} = \frac{2}{1 + (R_4/R_3)} \implies R_4 = (2Q_p - 1)R_3 \quad (5.41)$$

Assuming $C = 1 \text{ F}$, we get $R = 10^{-3} \Omega$. Impedance scaling by 10^8 , we have $C = 0.01 \mu\text{F}$, $R_1 = R_2 = R = 100 \text{ k}\Omega$. Further, since for R_3, R_4, R_5 , and R_6 , only the ratio $(R_6/R_5) = 1$ and $(R_4/R_3) = 19$ are important, we can choose them to be $R_5 = R_6 = R_3 = 1 \text{ k}\Omega$, and $R_4 = 19 \text{ k}\Omega$. The value of H_0 , given by Eq. (5.39b), is $H_0 = -(R_4/R_3) = -19$. It is noted that the same circuit gives LP and HP outputs at V_{o1} and V_{o3} for which $H_0 = (2Q_p - 1)/Q_p$, that is, $H_0 = 1.9$.

5.6.2

Tow–Thomas Biquad

If one examines the KHN biquad, one finds that the signal is fed at only one node of the system, while filters of different characteristics (*viz.*, LP, HP, and BP) are obtained at distinct output nodes. Such a system is commonly referred to as a *single-in, multi-out* (SIMO) system. In contrast, there could be a system where the input signal is fed to several nodes in the system while only one node delivers the desired output. Filters of different characteristics are obtained by special choice of

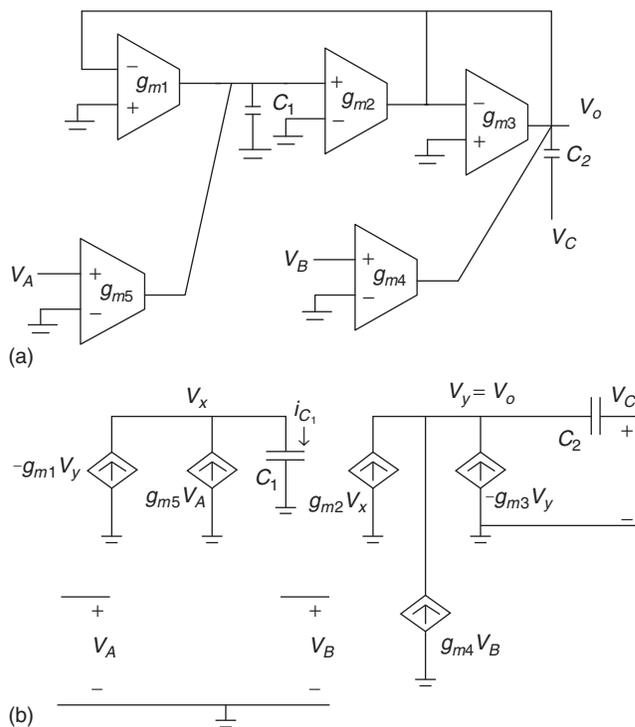


Figure 5.19 (a) A universal biquad filter using five OTAs and (b) equivalent circuit of (a).

and

$$Q_p = \frac{1}{g_{m3}} \sqrt{\frac{g_{m1}g_{m2}C_2}{C_1}} \tag{5.75b}$$

On examining Eq. (5.74), it is obvious that, by appropriately choosing the signal voltages V_A , V_B , or V_C , we can realize different transfer functions. Thus

For an LP filter: choose $V_B = V_C = 0$ (5.76)

For an HP filter: choose $V_A = V_B = 0$ (5.77)

For a BP filter: choose $V_A = V_C = 0$ (5.78)

For a notch filter: choose $V_B = 0, V_A = V_C$ (5.79)

For an AP filter: choose $V_A = V_B = V_C = V_i, g_{m1} = g_{m5}$, and $g_{m2} = \frac{g_{m4}}{3}$ (5.80)

5.10 Technological Implementation Considerations

In the early stages of active-RC filters, the filters were made using OAs and discrete RC elements mounted together on a printed circuit board (PCB). As the impetus

in z -domain can be written down from the transfer function of the RC-active filter simply by replacing “ s ” by the function of “ z ,” as given by the $s \leftrightarrow z$ transformation relation Eq. (6.10). It must also be understood that the output of each SC integrator in the filter must be sampled and transferred to the following stage in the filter, during the clock phase ϕ_2 , so that the transformation in Eq. (6.10) is applicable. If, on the other hand, the output of each SC integrator in the filter is sampled during clock phase ϕ_1 , the pertinent transformation will be $s \leftrightarrow \frac{1}{T}(z - 1)$. One can derive this transformation relation by comparing the transfer function of Eq. (6.2) with the z -domain transfer function given by Eq. (6.8b). Two other important $s \leftrightarrow z$ transformations can be derived by considering an alternative implementation of a resistance by an SC network, while replacing an RC integrator with an SC integrator. Table 6.2 presents the various $s \leftrightarrow z$ transformations. For more details regarding the various transformations, one may refer to Mohan, Ramachandran, and Swamy (1982, 1995) and Allen and Sanchez-Sinencio (1984).

6.2.2

Frequency Domain Characteristics of Sampled-Data Transfer Function

By now, it is clear that a given continuous-time transfer function $H(s)$, when implemented using SC network, produces a transfer function $H(z)$ in the discrete-time domain. Corresponding to a given $H(s)$, there could be more than one possible $H(z)$ because of the presence of more than one clock signal in the SC network. For frequency domain characterization of $H(z)$, we use the relation $z = \exp(j\Omega T)$, where Ω is the sampled-data angular frequency. This is identical with the real physical frequency used to test and simulate a sampled-data system. Since $\exp(j\Omega T)$ is a periodic function, it is obvious that $H(z)$ will be a periodic function. This is an important distinction between the continuous-time frequency domain transfer function $H(s)$ and the sampled-data (i.e., discrete-time) frequency domain transfer function $H(z)$. One may visualize the situation as though the given $H(s)$, when implemented as an SC filter function, causes $H(s)$ to be modulated in frequency domain (multiplied in time domain) by the clock frequency $f_s = 1/T$ of the clock signals. Thus, corresponding to an LP $|H(j\omega)|$, we get a periodic train for $|H(e^{j\Omega T})|$, which will consist of lobes of $|H(j\omega)|$ around zero frequency (i.e., DC) and around $\pm 2\pi f_s, \pm 4\pi f_s, \dots, \pm 2n\pi f_s$. This is illustrated in Figure 6.4, where $f_s = 1/T$.

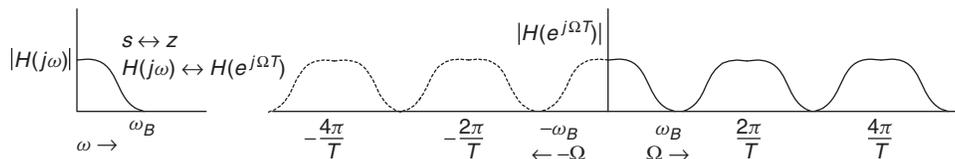


Figure 6.4 Sampling operation on $|H(j\omega)|$ and the sampled spectrum $|H(e^{j\Omega T})|$.

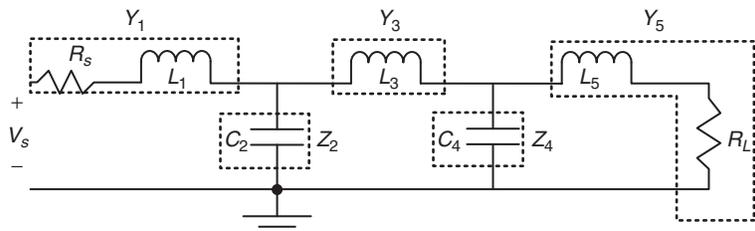


Figure 7.16 Realization of a doubly terminated fifth-order all-pole LC filter using the method of operational simulation.

structure discussed above. For this, we first identify the impedance and admittance elements. The scaled VTFs (i.e., $Y_1 R, Z_2/R, \dots$) assuming $R = 1$ are as follows:

$$\begin{aligned}
 T_1(s) &\rightarrow Y_1 = \frac{1/L_1}{s + R_s/L_1} \\
 T_2(s) &\rightarrow -Z_2 = -\frac{1}{sC} \\
 T_3(s) &\rightarrow Y_3 = \frac{1}{sL_3} \\
 T_4(s) &\rightarrow -Z_4 = -\frac{1}{sC_4} \\
 T_5(s) &\rightarrow Y_5 = \frac{1/L_5}{s + R_L/L_5}
 \end{aligned}
 \tag{7.17}$$

Of the above transfer functions, T_2 and T_4 can be realized by conventional inverting integrators. T_3 can be implemented by a noninverting integrator as shown in Figure 7.17a or a conventional integrator followed by an inverting amplifier of gain unity. The transfer functions T_1 and T_5 can be implemented by noninverting lossy integrators; the noninverting lossy integrator can be realized using an inverting amplifier followed by an inverting lossy integrator, shown in Figure 7.17b.

In Fig. 7.16
 R=1, in Fig
 7.17 the R
 is a diff.
 not equal
 to 1!

In order to make the transfer function in Figure 7.17b to comply with T_1 , for example, one has to equate the DC gains $1/R_s = R_x/R$, giving $R_x = R/R_s$. Similarly, $1/R_x C_x \rightarrow R_s/L_1$, leading to $C_x = L_1/R$. An implementation of the LP filter of Figure 7.16 using the architecture of Figure 7.14 is shown in Figure 7.18. In Figure 7.18, the noninverting integrator has been realized as a cascade of a resistance, inverting amplifier and an inverting integrator.

7.2.3
Systematic Steps for Designing Low-Pass Leapfrog Filters

The design procedure for an LP leapfrog filter may be summarized as follows:

- 1) Design the normalized LP prototype either by using filter tables or by the methods described in Chapter 4.
- 2) Select one of the general system diagrams from Figure 7.14 or 7.15.

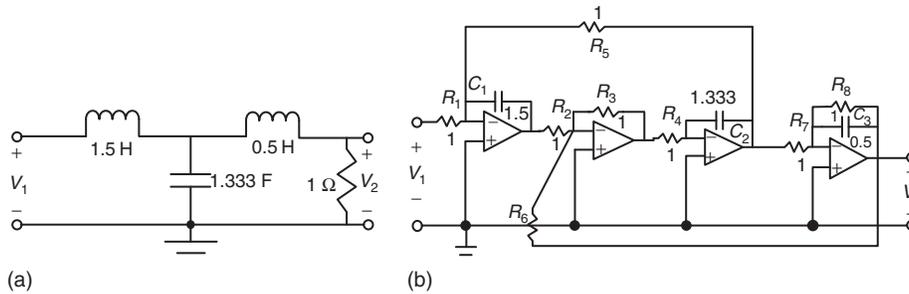


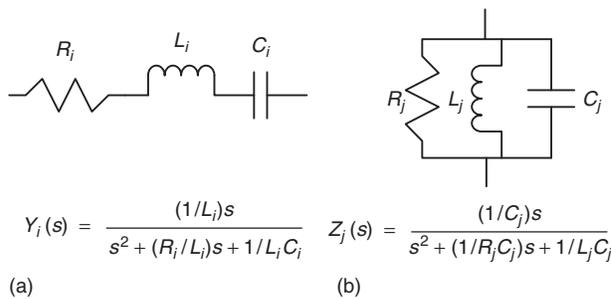
Figure 7.19 (a) A third-order low-pass LC filter and (b) operational simulation of the filter in (a).

7.2.4
Leapfrog Band-Pass Filters

The leapfrog technique discussed above is also applicable to BP filters with zeros at the origin and at infinity. This includes, for example, a series resonance network in the series arm and/or a parallel resonance network in the shunt arm of the ladder filter. Figures 7.20a and 7.20b show these two cases with the corresponding Y and Z functions. In the intermediate locations of the ladder, we have $R_i \rightarrow 0$ and $R_j \rightarrow \infty$.

Thus, instead of first-order RC transfer function networks, as was in the case of an LP filter, we have to use second-order RC-active filter sections in this case to implement the normalized admittance and impedance functions. The second-order RC network must have the capability to produce $Q_p \rightarrow \infty$ as will be required for $R_i \rightarrow 0$ or $R_j \rightarrow \infty$. A Tow-Thomas network with the summing capability at the input, which will also afford this special condition (namely, $Q_p \rightarrow \infty$), is shown in Figure 7.21.

The condition $R_i \rightarrow 0$ in $Y_i(s)$, and $R_j \rightarrow \infty$ in $Z_j(s)$ can be realized from this network by setting $R_1 = \infty$, that is, an open circuit. The leapfrog realization of a prototype BP filter (Figure 7.22a), using the above biquad as a building block, is illustrated in Figure 7.22b.



~~Figure 7.20 (a) A third-order low-pass LC filter and (b) operational simulation of the filter in (a)~~
Figure 7.20 (a) The admittance function of a series RLC-circuit, and (b) admittance function of a shunt RLC-circuit

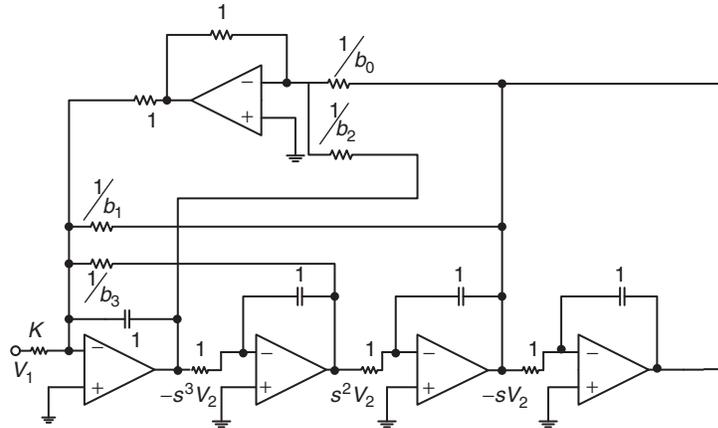


Figure 7.29 Alternative FLF realization of the fourth-order LP filter given by Eq. (7.27).

Comparing Eq. (7.30) with the all-pole LP transfer function given by Eq. (7.26), we have

$$\begin{aligned}
 f_{n-1} &= b_{n-1} - n\alpha \\
 f_{n-2} &= b_{n-2} - (n-1)\alpha f_{n-1} - \{n(n-1)/2\}\alpha^2 \\
 &\vdots \\
 f_0 &= b_0 - \alpha f_1 - \alpha^2 f_2 - \dots - \alpha^{n-1} f_{n-1} - \alpha^n
 \end{aligned} \tag{7.31}$$

We may choose a suitable value for α , and find the feedback coefficients $f_{n-1}, f_{n-2}, \dots, f_1$, and f_0 , successively from Eq. (7.31), in terms of $b_{n-1}, b_{n-2}, \dots, b_1, b_0$, and α . We can now realize Eq. (7.30) using the structure of Figure 7.26, wherein the function $T_i(s) = 1/(s + \alpha)$ is realized using lossy integrators. Since we will be using inverting integrators, we have to associate proper signs to $f_{n-1}, f_{n-2}, \dots, f_1$, and f_0 in realizing Eq. (7.30). We illustrate this case for a Butterworth filter of order 3. The same procedure could be used for a higher-order filter.

Example 7.7. Realize the third-order LP filter given by

$$H(s) = \frac{V_2}{V_1} = \frac{K}{s^3 + 2s^2 + 2s + 1} \tag{7.32}$$

using lossy integrators of the type $-1/(s + 1)$.

Using Eq. (7.31), we have
 minus 1

$$f_2 = 2 - 3 = -1, \quad f_1 = 2 - 4 + 3 = 1 \quad \text{and} \quad f_0 = 1 - 2 + 2 - 1 = 0$$

Hence, the given transfer function can be written as

$$\frac{V_2}{V_1} = \frac{K}{[(s + 1)^3 - (s + 1)^2 + (s + 1)]}$$