Contents

Preface XV

Symbols, Operators and Coordinate Systems XVII

1 Vector Wave Equations 1

- 1.1 Maxwell Equations for Dielectric Media 1
- 1.2 Inhomogeneous Vector Wave Equations 2
- 1.3 Homogeneous Vector Wave Equations 4
- 1.4 Translation-invariant Waveguides and Propagation Modes 4
- 1.4.1 Cylindrical Polar Components 5
- 1.4.2 Cartesian Components 8
- 1.5 TE and TM modes 12
- 1.5.1 The case of *y* and *z* Invariant Planar Waveguides 12
- 1.5.2 The case of a Circularly Symmetric Refractive Index Profile n(r) 14
- 1.5.3 Concluding Remarks on TE and TM Modes 16
- 1.6 Nature of the Solutions to Vector Wave Equations 16
- 1.7 Conclusion 19 References 19

2 Fundamental Properties of Vector Modes 21

- 2.1 Reciprocity Theorems 21
- 2.2 Propagation Constant, Phase Velocity, and Writing Conventions 24
- 2.3 Orthonormality of Guided Modes 25
- 2.4 Stored Electromagnetic Energy 28
- 2.5 Poynting Vector and Power Density 29
- 2.6 Group Velocity 30
- 2.6.1 Mean Transit Time 32
- 2.6.2 Dispersion and Pulse Spreading 32
- 2.7 Expansion of the Fields onto the Basis of Guided Modes 33
- 2.8 Refractive Index Profile and Effective Index 34
- 2.9 Fraction of Modal Power in the Core 36

Guided Optics: Optical Fibers and All-fiber Components. Jacques Bures Copyright © 2009 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim ISBN: 978-3-527-40796-5

VIII Contents

2.10 Conclusion 36 References 36

3 Exact Vector Solutions for Waveguides 39

- 3.1 One-dimensional Planar Waveguides 40
- 3.1.1 Symmetrical Step-index Planar Waveguide 40
- 3.1.2 Asymmetrical Step-index Planar Waveguides 54
- 3.1.3 Multi-layered Symmetrical Planar Waveguide 59
- 3.2 Exact Solutions for Two-layer Step-index Optical Fibers 65
- 3.2.1 Choice of Solutions for Longitudinal Components e_z and h_z 66
- 3.2.2 Eigenvalue Equation 68
- 3.2.3 TE and TM Modes ($\nu = 0$) 69
- 3.2.4 Hybrid Modes HE and EH ($\nu \neq 0$) 70
- 3.2.5 Asymptotic Limits of the TE and TM Mode Eigenvalue Equations *71*
- 3.2.6 Asymptotic Limits of the HE and EH Mode Eigenvalue Equations 72
- 3.2.7 Numerical Solutions of U(V) 76
- 3.2.8 Analytical Expressions for the Fields 78
- 3.2.9 Normalization Constants 83
- 3.2.10 Fraction of Power Guided in the Core 84
- 3.2.11 Group Velocity 90
- 3.2.12 Polarization of the Transverse Electric and Magnetic Fields 91
- 3.2.13 Modal Power Density of the Hybrid Modes 95
- 3.2.14 Radial Distribution of the Hybrid Mode Field Components 97
- 3.3 Exact Solutions for Multi-layer Step-index Optical Fibers 97
- 3.3.1 Matrix Method 100
- 3.3.2 Layer by Layer Method 101
- 3.3.3 The Case of the TE and TM Modes 103
- 3.3.4 Numerical Example for the SMF28[™] Fiber 104
- 3.3.5 Effective Index Curves 105
- 3.3.6 Group Velocity and Intramodal Dispersion of HE₁₁ and EH₁₁ in the SMF28[™] Fiber 106
- 3.3.7 Fundamental Mode of Multi-layered Step-index Fibers 106
- 3.4 Exact Solutions for Graded-index Optical Fibers 110
- 3.5 Conclusion 112
 - References 112

4 Scalar Mode Theory 115

- 4.1 Scalar Wave Equation 116
- 4.2 Two-layer Step-index Fibers 118

- 4.2.1 Eigenvalue Equation 119
- 4.2.2 Limit Values of U(V) 120
- 4.2.3 Nomenclature of the Modes 122
- 4.2.4 Polarization of the LP Modes 122
- 4.2.5 Universal Graph U(V) 124
- 4.2.6 Effective Index Graph $n_{\text{eff}}(V)$ 125
- 4.2.7 Normalization of the Modes 125
- 4.2.8 Numerical Examples of Radial Profiles $\Psi(r)$ 127
- 4.2.9 Fraction of Power Guided in the Core 129
- 4.2.10 Group Velocity 131
- 4.3 Multi-layer Step-index Fibers 133
- 4.3.1 Eigenvalue Equations 133
- 4.3.2 Numerical Examples 135
- 4.3.3 Normalization of the Modes 138
- 4.3.4 Group Velocity 138
- 4.3.5 Fraction of Power Guided in the Core 139
- 4.4 Graded-index Fibers 139
- 4.4.1 Solving the Scalar Wave Equation 140
- 4.4.2 Other Calculations 141
- 4.4.3 Numerical Example 141
- 4.5 Conclusion 142 References 143

5 Degeneracy of the Vector Modes 145

- 5.1 Degenerate Vector Modes in the Weakly Guiding Regime (Two-layer Fiber) 146
- 5.1.1 Degenerate Forms of the Eigenvalue Equation 146
- 5.1.2 Degenerate Forms of the Field Components 147
- 5.1.3 Polarization Degeneracy 150
- 5.1.4 Combinations of Degenerate Modes to form LP Modes 152
- 5.1.5 Generalization to Multi-layer and Graded-index Fibers 156
- 5.2 Polarization Corrections for Two-layer Fibers 156
- 5.2.1 General Formula 157
- 5.2.2 Approximation of Nearly Identical Fields 159
- 5.2.3 Circularly Symmetric Fibers 159
- 5.3 Polarization Corrections for other Circularly Symmetric Fibers 165
- 5.3.1 Example 1: The *N*-layer Step-index Fiber 165
- 5.3.2 Example 2: The Graded-index Fiber 166
- 5.3.3 Example 3: The Composite Profile Fiber 166
- 5.4 Conclusion 167 References 168

- X Contents
 - 6 Mode Coupling and Bragg Gratings 169
 - 6.1 General Mode Coupling Equations 169
 - 6.1.1 Coupling Equation for a Forward-propagating Mode *j* 171
 - 6.1.2 Coupling Equation for a Backward-propagating Mode -j 172
 - 6.1.3 General Coupled Equations 173
 - 6.1.4 Energy Conservation 174
 - 6.1.5 Coupling between Two Modes due to a Periodic Perturbation and Bragg Grating 175
 - 6.2 Coupling between two Codirectional Modes 176
 - 6.2.1 Solving the Coupled Equations 176
 - 6.2.2 Frequency Response of Transmission Bragg Gratings 178
 - 6.3 Coupling between Two Counterdirectional Modes 181
 - 6.3.1 Solving the Coupled Equations 181
 - 6.3.2 Frequency Response of Bragg Reflectors 186
 - 6.4 Experimental Realization of Bragg Gratings 188
 - 6.4.1 Reflection Bragg Grating Obtained by a Stationary Wave 189
 - 6.4.2 Reflection Bragg Grating Written with a Phase Mask 195
 - 6.4.3 Long-period Bragg Gratings Obtained by Electric Discharges 200

 - 6.5 Conclusion 204 References 205

7 Tapered Fibers 207

- 7.1 Local Modes 208
- 7.1.1 Normal Modes of a Local Uniform Waveguide 208
- 7.1.2 Local Modes of a Tapered Fiber 209
- 7.1.3 Orthonormality of the Local Modes 209
- 7.1.4 Decomposition on the Basis of Local Modes 210
- 7.2 Coupled Equations for Local Modes 211
- 7.2.1 Coupled Equations and the First Form for the Coefficients 211
- 7.2.2 Symmetry of the C_{jm} 212
- 7.2.3 Second Form of the Coupling Coefficients 213
- 7.2.4 Alternate Form of the Coupled Equations 215
- 7.3 Case of Circularly Symmetric Step-index Waveguides 217
- 7.3.1 Vector Modes 217
- 7.3.2 Scalar Modes in the Weakly Guiding Regime 218
- 7.4 Modal Behavior of Tapered Fibers 219
- 7.4.1 Characteristic regions of the waveguide 220
- 7.4.2 Modal interferometer 221
- 7.5 Experimental Study of a Tapered Fiber 223
- 7.5.1 Transmitted Power During the Tapering Process 224
- 7.5.2 Spectral response 224
- 7.5.3 Response as a Function of the Index of the External Medium 225

Contents XI

- 7.6 Technological Applications of Tapered Fibers 226
- 7.6.1 Temperature Sensor 226
- 7.6.2 Displacement and Curvature Sensor 226
- 7.6.3 $Pass(\lambda_p)/Stop(\lambda_s)$ Spectral Filter 228
- 7.6.4 Pass-band Spectral Filter 231
- 7.6.5 Power Concentrator 232
- 7.6.6 Adiabaticity: Very Strong or Very Weak Slopes of Tapered Fibers 240
- 7.7 Conclusion 242 References 242

8 Fiber Splices 245

- 8.1 Reflection and Transmission at a Splice 245
- 8.1.1 Calculation of the Amplitudes of the Reflected and Transmitted Modes 246
- 8.1.2 Calculation of the Overlap Integrals I_{jk} 248
- 8.1.3 Numerical example 249
- 8.2 Reflection Modal Interferometer 250
- 8.3 Transmission Bi-modal Interferometer 253
- 8.4 Reflection and Transmission on the Fiber Endface 255
- 8.4.1 Numerical Example 258
- 8.5 Conclusion 260 References 260

9 2 × **2** Fiber Couplers 261

- 9.1 Coupling via the Field Overlap Between the Fibers 265
- 9.1.1 Coupled Equations and Coupling Coefficients 267
- 9.1.2 Single-mode Fibers and the Adiabatic Coupler 269
- 9.1.3 Physical Interpretation of the Quantity $n^2(x, y, z) \overline{n}^2(x, y, z)$ 270
- 9.1.4 Expression for $C_{1,1}^{12}$ in the Case of Two Identical Step-index Fibers with n_{co} and n_{cl} 271
- 9.1.5 Expression of $C_{1,1}^{11}$, Correction of $\overline{\beta}_{01}$ of the LP₀₁ Mode of Fiber 1 273
- 9.1.6 Numerical Calculation of $C_{1,1}^{12}$ and $C_{1,1}^{11}$ 274
- 9.1.7 Solving the Coupled Equations in the case of an Adiabatic Coupler with Identical Fibers 274
- 9.1.8 Transfer Matrix 276
- 9.1.9 Coupler with $b_1^{(2)}(0) = 0$ and $b_1^{(1)}(0) = 1$ 277
- 9.1.10 Modeling of a Coupler and Numerical Examples 277
- 9.1.11 Expression of the Coupling Coefficient $C_{j,k}^{12}$ in the Case of Two Different Fibers 281
- 9.1.12 Phase Mismatch and Coupling Between Non-codirectional Modes 284

XII Contents

- 9.2 Coupling via Beating Between Supermodes 284
- 9.2.1 Illustration of the First Two Supermodes of a 2 \times 2 Coupler of Identical Fibers $\qquad 285$
- 9.2.2 Coupling via the Beating of the First Two Supermodes 286
- 9.2.3 Calculation of the Supermodes 289
- 9.2.4 Modeling a Partially Fused Structure 291
- 9.2.5 Finding the Eigenvalues 292
- 9.3 Numerical Comparison of the Two Methods 294
- 9.4 Experimental Results 296
- 9.4.1 Presentation and Analysis of the Results 296
- 9.4.2 Modeling the Effects of Polarization 300
- 9.5 Special Couplers 301
- 9.5.1 Wavelength Independent Power Dividing Couplers 301
- 9.5.2 Couplers with Abrupt Slopes in the Wavelength Response 304
- 9.5.3 Couplers with a Broadened Wavelength Response 306
- 9.5.4 Looped Couplers 307
- 9.5.5 Mode-separating Couplers 312
- 9.6 Conclusion 314 References 314

Appendix A: Bessel Functions and Modified Bessel Functions of Integer Order 317

- A.1 Differential Equations of the Bessel and Modified Bessel Functions 317
- A.2 Graphs of the First Three Orders of Bessel and Modified Bessel Functions 318
- A.3 Expansion Series of the Bessel and Modified Bessel Functions 318
- A.4 Symmetry Relations 319
- A.5 Recurrence Relations 320
- A.6 Derivatives 320
- A.7 Wronskians 320
- A.8 First Terms of the Asymptotic Forms when $x \rightarrow 0$ 321
- A.9 First Terms of the Asymptotic Forms when $x \to \infty$ 321
- A.10 Integrals of Bessel and Modified Bessel Functions 322
- A.11 Addition Formula 323
- A.12 Roots of both $J_{\ell}(x)$ and its Derivatives $J'_{\ell}(x) = 324$

Appendix B: Proof of the Identity used to Establish the Group Velocity Formula 325

References 327

Contents XIII

Appendix C: Distinguishing between the HE and EH Vector Modes of a Multi-layered Fiber 329

- C.1 Effective Index Curves and the Inversion of Correspondences 330
- C.2 Polarization of Electric and Magnetic Fields as a Function of the Reduction Ratio *R* 330
- C.3 Method for Distinguishing between the $EH_{\nu m}$ and $HE_{\nu m+1}$ Modes 332 References 335

Appendix D: Definitions of the Refractive Indices 337

- D.1 Absolute Index of Air 337
- D.2 Relative and Absolute Indices of Silica 337
- D.3 Relative and Absolute Refractive Indices of GaAs between $\lambda=0.8$ and 1.65 μm -338
- D.4 Relative and Absolute Refractive Indices of AlAs between $\lambda=0.65$ and $1.6\,\mu m 338$
- D.5 Relative Index Curves of AlAs and GaAs 339 References 339

Index 341