

## Index

### a

- abel transform 514
- abelian functions 274, 275, 278
- abelian groups 358
- absolute error 477
- abstract Fourier series transformation 506
- abstract parabolic equation 556, 583
- acceptance frequency 48
- accumulation point 456, 464
- accuracy
  - computer algebra 291
  - dynamical systems 391
  - perturbation solutions 415
- Adams methods 493, 495
- Adams–Bashforth methods 493
- Adams–Moulton methods 493, 494
- addition formulas 173, 258, 276
- addition laws 254, 258, 273–275, 278, 285
- addition theorem 206
- additive noise 74, 85, 92, 94, 96
- adjacency matrix of a simple graph 154
- adjacency operator 113
- adjoint operator 457, 458, 555, 585
- adjoint system 585
- advanced algorithms 294–301
- affine reflection groups 244
- Airy functions 268, 269, 283, 495
  - turning point problem 434
- Airy's equation 495
- algebra of tensors 337, 338
- algebraic entropy 286
- algebraic equations
  - iteration methods 476–480
  - transformations 504, 505
- alpha complex 235
- Ambrosetti–Rabinowitz mountain pass theorem 554
- analytic function 10, 453, 510
- Andronov–Hopf bifurcation 393
- angle-preserving map 503
- angular momentum 171, 177, 180, 189–191, 194–197
- angular standard form 424
- annihilation operators 133, 179, 180, 198, 532
- anticommutation 179
- approximation
  - numerical 80, 93, 95, 476–480, 576
  - of continuous data 476, 487ff
- area-preserving map 503
- Arnol'd diffusion 425
- Arnol'd unfolding 444
- Arrhenius' law 103
- Askey–Wilson difference operator 284
- associated Legendre functions 160, 202, 255, 256, 260, 264
- asymptotic approximation 415, 416, 431, 444
- asymptotic matching 438
- asymptotic series 416, 444, 445
- asymptotic stability 387, 389
- asymptotic validity 415, 416
- atlas 321–324, 327, 363
  - adapted 363
- atomic shell structure 195
- attempt frequency 48
- attractor
  - chaotic 405
  - strange 228, 386, 429
- attrition problem 45
- autocovariance 75
- automorphism
  - galois theory 271, 272
  - Lie-group 358
- averaging 422–424
  - self-averaging 45

**b**

- Bäcklund maps 282, 286
- backward Fokker–Planck equation 102, 103
- backward kolmogorov equation 24, 25
- backward/forward substitution 481
- Bak–Sneppen model 17
- balance equation 12, 356
- balance laws 356–358
  - differential expression of the general balance law 358
  - nonlinear heat-transfer problem 571
- balance principle 50, 68
- ballot theorem 20
- Banach space 449–451, 551, 562
  - ordered 555
  - reflexive 553
- band-limited function 517
- banded matrices 481
- barcode 233
- basin of attraction 387
- Bernoulli distribution 35
- Bessel equation 206, 255, 262, 270, 271
- Bessel functions 202, 240, 256–258, 268, 270
  - Hankel transformation 523
- Betti number 221–223, 228, 232, 236
- betweenness centrality 143, 149, 154
- Bezout’s identity 296, 300, 301
- Bianchi identity 372
- biased sampling 45
- bifurcation point 390, 392, 394, 395, 404–407
- bifurcations 411, 418, 445
  - hopf 392–396, 399, 404, 428
  - imperfect 414
  - local 391, 392, 404
  - Pitchfork 392, 394, 396–398, 404
  - saddle-node 392–398, 406–408
- binomial distribution 35
- binomial theorem 5, 15, 285
- Biot system 545
- Biot–Savart formula 535
- Birman–Kuroda theorem 472
- Birman–Schwinger bound 469
- birth-and-death process 24
- Black–Scholes PDE 102
- Bochner space 557
- Boltzmann constant 89, 121
- Bolzano–Weierstraß theorem 553
- bond percolation 31, 32
- boole’s inequality 21
- Borel set 465
- Borel–Cantelli lemma 8, 92
- Boson annihilation operator 179
- boundary conditions 52–56, 103, 411
  - weierstraß elliptic functions 276
- boundary critical exponent 563
- boundary layer 412, 417, 426, 429–434, 436–439, 445
- boundary operator 220–225, 228, 231, 232
- boundary point 224
- boundary value problem (BVP) 400, 401, 564–566, 569–571
  - classical formulation 564, 569
- bounded operators 450, 452, 454–457, 460, 464, 547
- Bragg position 67
- branching process 7, 8, 33
- Brezis–Ekeland–Nayroles principle 557, 575
- bridge 122, 129, 154, 212
- Brillouin zone 67
  - first 170
- Brownian motion
  - approximation 80
  - geometric 86, 88, 89
  - Monte-Carlo methods 50
  - stochastic differential equations 75–80, 100
  - stochastic process 19
- Brute-force bifurcation diagrams 405
- bundle, associated 362, 364–366, 370, 372, 374
- burgers’ circuit 332
- Butcher array 494, 495

**c**

- canard 433
- canonical ensemble 47, 52, 64
- canonical transformations 426
- carathéodory mapping 563
- Cartan’s structural equation 372
- Cartan–killing metric equation 178
- Cauchy equation 272
- Cauchy problem 491, 492, 495, 496, 500, 556–559
  - doubly nonlinear 558
- Cauchy sequence 450, 552
- Cauchy’s formula 4, 356, 357, 492
- Cauchy’s postulates 355, 357
- Cayley multiplication table 167
- Čech complex 234
- cellular homology 224
- center manifolds 390, 393, 395, 397–399
- central limit theorem 9, 10, 75
- centrality measure 143, 145, 146, 149, 154
- chain groups 219, 221, 222, 224, 225, 228, 231
- Chapman–Kolmogorov equation 23, 99
- Chapman–Kolmogorov relation 17, 22, 23
- character table 188, 190, 191
- character-class duality 187
- characteristic function 10

- chart 321, 322
  - adapted 363
- Chebyshev polynomials 402
- Chebyshev–Gauss–lobatto nodes 488
- chernoff bound 21
- Christoffel symbols 184, 185, 374, 377, 380
- Ciarlet–Nečas condition 572, 584
- circular functions 273, 277, 278
- $C^k$ -differentiable structure 324
- class, group theory 168
- classical formulation of variational problem 564, 569
- classical Fourier series transformation 509
- classical mechanics 133, 160, 183, 346, 363
- classical solution 564, 577
- clique 29, 30, 114, 234
- clique complex 234
- closed walk (CW) 114, 146
- closed-graph theorem 451, 452, 458
- closeness centrality 143, 154
- cluster algorithms 49
- clustering coefficient 115, 135, 139, 140, 142, 154
- cochain 224–226
- codimension one bifurcations 392
- coefficient group 220, 221, 224, 225
- coefficients, differential 272, 507, 522
- cohomology 212, 224–226
- collocation 400–402, 406
- colored noise 77–80
- communicability 138, 147, 148, 154
- commutative groups 188
- commutativity of self-adjoint operators 472
- compact embedding theorem 563
- compact mapping 563
- compact operators 456, 464, 472
- compact set 452, 519
- compact trace operator theorem 563
- compactification 578
  - convex 578, 579
- complementarity problem 555, 574
- completeness relations 186–188
- complex coordinate transformation 509
- complex networks 111, 115, 137, 138, 140, 143, 148
- complex systems 63, 67, 111, 137, 138, 150
- complex variables 255, 279, 428, 439
- complexity 39, 137, 138, 293, 568
- composite solution 430–432, 445
- computational topology 211, 212, 230–236
- computer simulations 26, 40, 216
- condensed matter 40, 62–67, 111, 115–120, 217
- conditional expectation 36, 37
- conditional probability 11, 26, 31, 35, 37, 46
- confluent hypergeometric functions 260
- conformal symmetry 194
- conjugate
  - Hermitean 61
  - legendre 557
- conjugate gradient (CG) method 479, 480, 484, 485
- Conley index 228
- connected topological space 320
- connections 367–380
  - principal bundle 369, 370
- consensus dynamics 151
- conservation laws 51, 356, 388
- conservative forces 357
- conservative oscillator 421
- conservative systems 467
- consistency 49, 325, 365, 492, 495
  - of multistep methods 493
- constrained minimization problems 555
- constraint qualification
  - mangasarian–fromovitz 555
  - slater 556
- continuity 320
  - of probability measure 8, 16, 32
- continuous groups 165, 171, 250
- continuous maps 214, 224, 321, 322, 324, 462
- continuous symmetry 240, 250–261
- continuously differentiable 89, 504, 552, 554, 577
- contour integrals 255
- contravariant tensor algebra 338
- control theory 586
- $\Gamma$ -convergence 581, 582, 584
- convergence
  - around fixed points 478
  - higher order 477
  - linear 406, 477
  - numerical methods 92, 98
  - quadratic 477
  - strong 92, 94, 96, 98, 462
  - weak 92, 95, 96, 453, 553
  - weak\* 553, 579
- convergence analysis 477, 478
- convex mapping 556, 583
- convolution 6, 7, 512–514
  - integral 513, 514
  - theorem 6, 27, 514
- coordinate perturbations 414, 428
- corrected trapezoidal formula 489
- correlation functions 51, 59, 79
- coset 168, 209
- cotangent bundle 327, 328, 341, 342, 345, 346, 361, 364, 380
- cotangent space 326, 327, 349, 380
- coulomb potential 192, 193, 195, 196, 472
- countable topology 320

- counting function 18
  - coupled systems 266, 544, 545, 547
  - covariance 75–80, 88, 105, 184
  - covariant derivative 372, 374–376, 380
  - covariant tensor algebra 338
  - covector 326
  - covering space 216, 217, 230
  - Coxeter group 243, 244, 246
  - Cramer's rule 480
  - creation operators 179, 198, 532
  - critical point 552
  - critical slowing down 49, 55, 68
  - criticality 43, 394, 397
  - cross product 173
  - crystal field theory 190
  - cubical homology 224
  - cumulants 53, 54
  - curl 343
  - curvature form 372, 375, 376
  - curvature tensor 376, 380
  - cyclic groups 168, 188, 216, 221
  - cyclomatic number 124, 216
  - cylinders 215, 216, 256
  - cylindrical coordinates 254
- d**
- d*-tuples 19, 41
  - damping 131, 413, 414, 561
  - Darboux transformation 267
  - De Moivre–Laplace central limit theorem 9
  - De Rham cohomology 225, 226
  - De Rham's theorem 226
  - deck transformation group 217
  - deficiency indices 461, 462, 472, 473
  - deficiency subspace of  $t$  472
  - definite integrals, numerical approximation 489
  - degree distribution 138–142, 154, 155
  - Delaunay complex 235
  - delta function 78, 186
  - dense model 293, 295
  - densely defined operators 457–460, 521, 537, 538
  - density matrices 457, 472
  - density of an extensive property 355
  - deposition, diffusion, and adsorption model 27–29
  - derivative
    - covariant 374, 375, 380
    - exterior covariant 372, 376
  - descent direction method 479, 484
  - detailed balance condition 48
  - detailed balance principle 50, 68
  - determinant, Jacobian 324, 343, 353, 505
  - diffeomorphism 324, 359
    - between fibers 367
    - Lie-group isomorphism 358
  - difference operators 284
  - differentiable manifolds 323–328
  - differentiable maps 324, 328, 329, 346
  - differential
    - 0-form 342
    - $k$ -form 226, 342
    - forms 341–344
  - differential Galois theory 268, 271
  - differential topology 211, 216, 226
  - diffusion-limited aggregation (DLA) 40, 46, 47
  - digital images 224, 236
  - dilation 467, 491
  - dimension of a vector space 325
  - $\delta$ -distribution. *See* dirac- $\delta$ -distribution
  - dirac- $\delta$ -distribution 78, 79, 516, 517, 522, 531, 541
  - direct product 244, 256, 454
  - direct solutions 399
  - direct sum 186, 221, 454, 530
  - directionally differentiable 552
  - Dirichlet condition 461, 564, 569
  - Dirichlet type boundary condition 519
  - discrete Fourier transformation 508, 522
  - discrete groups 159, 164, 166–170
  - discrete Painlevé equations 286
  - discrete special functions 283, 285
  - discrete spectrum 464, 469, 470, 472
  - discrete symmetry 241, 243, 245, 247, 249, 286
  - discrete transformations 522
  - discrete-time maps 402
  - discretization 93, 97, 98, 560, 561, 576
  - disentangling theorem 164, 180, 206
  - dislocations 331, 332, 334
  - disordered series 417
  - dissipation 73, 89, 558, 560, 583
  - distribution 515–517, 531–537
    - horizontal 368–371, 374
  - divergence 343, 543
  - divided difference 487, 488
  - doubly nonlinear cauchy problem 558
  - doubly nonlinear inclusion 559
  - drift correction 90
  - dual problem 556, 576
  - dual space 170, 326, 335, 341, 451, 552, 583
  - duality pairing 524
  - Duffing equation 388, 391, 400, 419
  - Dunkl Laplacian 267
  - Dunkl operators 243, 267
  - dynamic models 68
  - dynamic richardson method 483
  - dynamical groups 160, 194–199, 207, 209
  - dynamical models 198

- dynamical similarity 159, 160, 163  
dynamical systems 11, 385ff., 418ff.
- e**
- eccentricity 114, 426, 476, 503  
edge contraction 121, 128, 154  
edge deletion 122, 128, 154  
edge-path group 230, 231  
Ehrenfest model 11, 15  
Ehresmann connection 368, 369  
eigenspace 452  
eigenvalues  
– for finite-dimensional linear systems 480–487, 498  
– iteration method for computing 478  
– of finite matrices, computing 485  
– perturbations of matrices and spectra 442–444  
Einstein’s summation convention 326  
Ekeland variational principle 584  
elasticity 528, 543, 572, 573, 581  
electrical networks 33, 111, 129  
electrodynamics 111, 182, 183, 207, 226  
electromagnetism 199, 323, 528  
embedded submanifold 370  
endomorphisms 251, 252  
energetic formulation 560  
energy equality 560  
energy levels 134, 195, 207, 262  
enrichment technique 45  
epidemics on networks 153  
epigraph convergence 581  
epimorphism 223  
equation  
– algebraic 476–479  
– cauchy 272  
– kepler’s 476  
– scalar 476, 477  
– weak 497  
equilibria 386–400, 402–406, 408, 584  
equilibrium states 131, 387  
equivalence of linear flows 389, 390  
equivalence relation 325, 366  
ergodicity 51, 68  
error  
– absolute 477  
– dynamic correlation 52–56  
– global truncation 492  
– Hermite interpolation 489  
– interpolation 487, 489  
– local truncation 491  
– –for multistep methods 493  
– multilevel Monte Carlo 97  
– residual vector 483, 484  
– sensitivity analysis 499  
– vector 478  
essential spectrum 458, 464, 470, 472, 473, 500  
essentially self-adjoint operator 472  
Euclid’s algorithm 294  
Euler characteristic 212, 222, 228, 230  
Euler–Lagrange equation 552, 554, 564, 566, 569, 570, 572, 577, 581  
Euler–Maruyama method 93–97  
event 34  
evolution operator 385, 386, 467, 470  
excited states 133  
existence and uniqueness for SDEs 84  
expanding interval 417, 421, 429, 430  
expectation 35–37  
explicit formula 413  
exponential distribution 9, 25, 46  
extended Euclidean algorithm 296  
extended Maxwell’s equations 547f.  
extensive property 355, 356  
exterior algebra 338, 339, 341  
exterior covariant derivative 372, 376  
exterior derivative 226, 343f., 350, 372, 376f.  
– covariant 372, 376  
extrema 457, 485, 551
- f**
- factorial function 259  
factorization 240, 261–268, 296–300  
– LU 481  
– table of most common methods 482  
faithful representation 185, 209  
Fast-Fourier transform (FFT) 509  
Feigenbaum’s constant 405  
Fenchel inequality 557–559  
Fermions 59–61, 179  
Ferrers formulae 255  
feynman diagrams 124  
feynman graphs 124–128  
feynman path integrals 467  
fiber 216, 327, 361, 365  
fiber bundle 327, 361–372  
fiber-preserving map 329  
Fibonacci series 41  
filtered probability space 82  
filtering 103, 104  
filters, signal processing 512  
filtration 82, 233–235  
finite groups 164, 166–169, 207, 252  
finite-dimensional linear systems 480–485  
finite-element method 489, 577, 584  
finite-size problems 52, 53, 55  
finite-size Scaling 53, 68

- first Lyapunov coefficient 394, 395
  - first variational equation 402–404
  - fixed point 8, 85, 228, 242, 402, 403, 476–478
    - iteration 476, 477
    - iteration schemes 476
    - local convergence 478
  - flavors 62
  - floating points 62, 480
  - Floer homology 229
  - Flops 480–482
  - Floquet multipliers 402–404
  - Flory–Huggins theory 64, 65
  - fluctuation-dissipation relation 89
  - fluctuations 51, 52
  - fluid mechanics 434–436
    - $^4\text{He}$  momentum distribution 59
  - flux density 356, 357
  - Fokker–Planck equation 24, 99–103, 536
  - foliations 428
  - forest 115, 123, 125, 127, 154, 155
  - 1-form 341
    - canonical 346, 373, 374
  - formulation
    - classical 564, 569, 571, 574, 577
    - de giorgi 559, 560
    - energetic 560
    - mixed 575
    - weak 564, 565, 569–571, 574, 577
  - forward Chapman–Kolmogorov equation 99
  - forward Euler method 493, 496
  - forward/backward substitution 481
  - Fourier expansion 453, 456, 472, 506, 507
  - Fourier series transformation 506–511, 522
  - Fourier sine/cosine transformation 518
  - Fourier transform 59, 79, 451
  - Fourier–laplace transformation 510–518, 521, 522, 529–531, 533, 534
  - Fourier–Plancherel operator 451, 459, 460, 464, 468, 472
  - Fourier–Plancherel transformation 504, 510–512, 515, 517, 518, 520, 522, 523
  - fourth order systems 568
  - fractals 47, 228
  - fractional power series 443
  - fractional-step method 561, 562
  - fréchet subdifferential 554, 555
  - Fredholm alternative 456
  - Fredholm integral equation 40
  - Frobenius
    - expansion 250, 280
    - method 248, 258, 259
    - norm 76, 176
    - notation 245, 246
    - theorem 370–372
  - Frobenius–Stickleberger relations 275
  - Fuchsian equations 247–250
  - function
    - analytic 10, 453, 510
    - characteristic 10
  - fundamental group 213–218, 221, 230, 231
  - fundamental solution 517, 535
  - fundamental theorem
    - for vector fields on manifolds 347
    - of algebra 301
    - of calculus 312
  - fundamental vector field 360, 369, 370, 372, 375
- g**
- Galerkin method 497, 498, 576
  - galois groups 271, 272
  - galton–watson process 7
  - gambler’s ruin 14
  - gamma function 259
  - gâteaux differential 552, 583
  - gauge theory 61, 62, 160, 199–201, 207–210, 236
  - gauges 415–417, 445
  - Gauss–Bonnet theorem 222
  - Gauss–seidel method 483
  - Gaussian distribution 9, 53, 515
  - Gaussian elimination 480, 481
  - general topology 211
  - generalized series 415–417, 420, 421, 445, 446
  - generalized weyl theorem 464
  - generating function 426
    - convolution 7, 17
    - Hermite polynomials 206, 207
    - moment generating function 9, 10, 21
    - probability generating function 5, 25
    - special functions 202, 245, 262, 263, 282, 283, 285, 286
    - stochastic process 3–10, 14–17, 21, 24, 25, 33
    - uniqueness 5, 7, 9
  - generator of a group 168
  - genus 216
  - geometric Brownian motion 86–88, 90, 91, 93, 95, 102
  - geometric realization 219
  - geometric singular perturbation 433
  - Giambelli formula 246
  - Gibbs canonical distribution 89
  - Gibbs ensemble 52
  - Gibbs random field 29–31
  - Gillespie’s algorithm 25, 26
  - girth 114, 118, 154
  - global bifurcation 392

- global flow 347
  - global minimizer 479, 553
  - global solution of a cauchy problem 491
  - global stability 495, 496
  - global truncation error 492
  - good reduction theorem 297, 298
  - google pagerank 145, 483
  - gradient flow lines 227
  - gradient method 479, 480, 484
  - gram–schmidt orthogonalization 202
  - grand canonical ensemble 52, 64
  - graph
    - bipartite 115, 117–119, 154
    - complete 114, 115, 123, 147, 148, 154
    - connected 114, 118, 119, 150, 154, 215
    - cycle 154
    - directed 112, 114
    - disconnected 115
    - Erdős–rényi 154
    - formal definition 112
    - invariant 123, 154
    - isomorphic 114, 138
    - nullity 154
    - planar 115
    - random 134–136, 138, 154, 155
    - regular 115, 138, 155
    - simple 112, 113
    - star 155
    - trivial 115, 123
    - undirected 112–114, 151
    - weighted 112, 126, 130
  - graph diameter 114, 154
  - graph theory 111–137, 147
  - greatest common divisor (GCD) 294
  - Green’s functions 133, 134, 253, 517
  - Green’s tensor 533–535
  - Green–Lagrange strain tensor 573
  - Gröbner bases 304–308
  - group action 241, 243, 358, 359, 365, 366
  - group axioms 165
  - group consistency condition 365
  - group element–matrix element duality 186, 187
  - group of boundaries 220
  - group of cycles 220
  - group theory 159–210
    - symmetries 271
- h**
- Hadamard conditions 529, 566, 583
  - Hadamard transformation 522
  - Hamilton principle 576
  - Hamilton variational principle 559
  - Hamilton–Jacobi theory 426
  - Hamiltonian equation 116, 131–133
  - Hamiltonian function 328, 347, 425
  - Hamiltonian operator 42, 47, 58, 191, 193, 467–472
  - Hamiltonian systems 346, 347, 424–426
  - Hamiltonian vector field 346
  - Hammersley–Clifford theorem 31
  - Hankel determinants 261
  - Hankel transformation 523
  - harmonic balance method 399, 400
  - harmonic functions 402
  - harmonic oscillators 131, 164, 195–197, 207, 262, 515, 532
  - harmonic polynomials 536
  - Harris–Kesten theorem 33
  - Hartley transformation 519, 520
  - Hartman–Grobman theorem 389–391, 398, 403, 404
  - Hausdorff space 320
    - topology 320
  - heat transport 545
  - heat-bath method 49, 68
  - Heisenberg group 171, 179, 181, 204
  - Heisenberg representation 180
  - helium 486
  - Hellinger–Toeplitz theorem 458
  - Hermite elements 490
  - Hermite interpolation 488, 489
  - Hermite quadrature formulae 489
  - Hermitean conjugate 61
  - Hermitean operator 454, 455, 457, 467, 472
  - hermitian matrices 486
  - heteroclinic orbits 386
  - Heun method 91, 96, 97
  - higgs mechanism 201
  - higher-dimensional transformations 521, 522
  - Hilbert space 450, 452–456, 472, 505–507, 530, 552, 562
  - Hilbert space geometry 452
  - Hilbert transformation 513, 525
  - Hilbert–Schmidt operator 456, 457, 473
  - Hilbert–Schmidt theorem 456
  - histogram method 52
  - homoclinic orbits 386, 418, 429
  - homogeneous systems 478
  - homological algebra 222–224
  - homology 218–224
  - homomorphism 166, 212, 358
  - homotopy 212ff.
  - Hook diagrams 246
  - Hopf bifurcation 392–396, 399, 404, 428
  - horizontal distribution 368–371, 374
  - Horner’s scheme 508
  - Hubbard model 118–120
  - Hückel molecular orbital method 116



- Hunziker-van Winter–Zhislin theorem 470  
hydrocarbon 117–119, 154  
hyperbolic equilibrium 387, 390, 403  
hyperelastic material 572  
hypergeometric equation 258  
hypergeometric functions 259, 260  
hyperplane 243–247  
hypersurface 425, 567
- i**
- identity element 165, 209, 214, 358  
identity transformations 359, 422, 426, 445  
IEEE standard, floating-point numbers 291  
imperfect bifurcations 414  
implicit Euler formula 560, 561, 584  
importance sampling 42, 45, 47–52, 56, 60, 62, 68, 69  
incidence matrix 113, 118, 154  
inclusion–exclusion property 222  
incremental function 491, 492  
independent events 35  
independent variables 5ff.  
independent vectors 363, 449, 462  
indeterminates 242, 292  
indicator function  $\delta_k$  555  
indicator variable 18, 36, 37  
indicial equation 248, 258  
induced rounding 52, 68  
infinite discrete groups 159, 169  
infinite groups 214, 242  
infinite-dimensional min-max principle 497  
infinitesimal rate 23  
initial-boundary value problems 540–544  
initial layer 429, 431, 436, 437, 445  
initial value problem 429, 491–496, 541, 556  
inner solution 430–435, 445  
inner-product spaces 377–379, 450, 472  
innocuous polynomials 294  
integral transform 4–10  
integration  
– by parts 87  
– in computer algebra 308–312  
– Monte Carlo methods 43, 44  
– of  $n$ -forms in  $\mathbb{R}^n$  353–355  
interacting boson model (IBM) 199  
interfaces 52, 54, 55, 57  
intermediate variable 431  
interpolation 93, 476, 487–490, 493  
– Hermite 488–490  
– Lagrange 487  
– nodes 487–490, 493  
– piecewise polynomial 489–491  
interpolation error function 487  
invariant density 101  
invariant manifolds 390, 429, 433  
invariant sets 228, 386, 387, 389, 393, 405–407  
inverse fourier transformation 517  
inverse power method 478, 486  
inverse radon transform 525  
inverse shifted power method 486  
inverse transformation 504, 505, 507, 510, 530  
inverse-mapping theorem 451  
inversely restricted sampling 45  
irreducibility 15  
irreducible representation 179, 186–192, 244, 246  
irreps 186, 188–190, 196  
Ising model 30, 55, 63, 120  
isolated point 234, 464  
isomorphic groups 231  
isomorphism 166, 358  
isospins 201  
isothermal-isobaric ensemble 52  
iteration matrix 482  
iteration stationary methods 483  
Itô formula 86–90, 92, 99, 102  
Itô integral 81–85, 88, 90, 92–94, 99, 102  
– martingale property 83  
Itô isometry 83, 88  
Itô SDEs 84, 86, 88–93, 95, 98, 101, 102
- j**
- Jacobi identity 175, 208, 251, 331  
Jacobi polynomials 261  
Jacobi–Trudi formulae 245  
Jacobian determinant 324, 343, 353  
Jacobian elliptic functions 277  
Jacobian matrix 184, 329, 387, 388, 395, 401, 402, 406, 408  
jansen formula 258  
joins 115, 228  
jordan normal form 398, 411, 442, 444  
jump rate, Markov dynamics 27
- k**
- Kalman–Bucy filter 103, 104  
Karhunen–Loève expansion 80  
Karush–Kuhn–Tucker condition 575, 583, 585  
Kato theorem 469  
Kato–Rellich theorem 459, 469  
Kelvin–Voigt model 543, 544  
Kepler’s law 160, 412  
Kirchhoff transformation 571, 584  
Kolmogorov–Arnol’d–Moser (KAM) theorem 412, 425  
Kolmogorov backward equation 24, 25  
Kolmogorov differential equations 24  
Korn inequality 573, 584



- Korteweg–de Vries (KDV) equation 276  
 Krylov–Bogoliubov–Mitropolski (KBM) method 422
- I**
- lagrange finite elements of order 1 498  
 lagrange form 487  
 lagrange interpolating polynomial 487, 488  
 lagrange interpolation 487  
 lagrangean 555, 575  
 lagrangian mechanics 328, 378, 380  
 laguerre polynomials 453  
 lamb shift 486  
 Lamé equations 277  
 Lamé system 573  
 Landau–Mignotte bound 297, 300, 301  
 Langevin equation 85, 101–103  
 Laplace equation 253, 255  
 – separation of variables 256  
 Laplace transform 10, 504  
 Laplacian matrix 126–128, 151, 152, 155  
 Laplacian operator 113, 114, 253, 255  
 large deviation theory 103  
 large deviations, random walks 10, 21, 22  
 lattice gauge theory 61, 62, 67  
 lattice sites, random selection 44  
 Lavrentiev phenomenon 567, 568, 572, 583  
 law of rare events 7  
 law of total probability 8, 13, 35  
 Lax–Ritchmyer theorem 496  
 Lebesgue space  $L^p$  557, 562, 583  
 left-invariant vector fields 361  
 legendre conjugate 557  
 legendre polynomials 203, 255, 259  
 legendre transformation 557, 559, 583  
 Legendre–Fenchel transformation 559, 583  
 legendre–Hadamard condition 566, 583  
 Lie algebra 175–182, 188–190, 205–209,  
 250–253, 331, 361  
 Lie bracket 330–334  
 Lie derivative 347–351  
 Lie group 173–175, 202, 250, 251, 358–361  
 – automorphism 358  
 – constructing 177  
 lie series 422, 445  
 lie transforms 422, 426  
 Lieb’s theorem 119, 120  
 Lieb–Thirring Inequality 469  
 limit circle 473  
 limit cycles 386, 394–396, 399–408  
 limit point 473  
 $\omega$ -limit sets 385–387  
 Lindstedt method 395, 419, 421, 422, 426, 427,  
 443, 445
- linear combination 325  
 linear differential equations 261, 271, 272, 311,  
 312, 507  
 linear groups 174, 176, 246, 358  
 linear homeomorphism 505  
 linear matrix representation 160, 185, 207, 208  
 linear multiplicative algorithm 41  
 linear multistep methods 492  
 linear operator 334–337, 449ff.  
 linear ordinary differential equations 311  
 linear PDE 528  
 linear search technique 479  
 linear solvers, Matlab and Octave 482  
 linear space 491, 551, 562, 583  
 linear standard model. *See* Poynting–Thomson  
 model  
 linear systems 480–487  
 linear transformations 172, 174, 182, 411, 442  
 linear vector space 174–176, 186, 208, 209  
 linearization  
 – constructing lie algebras 175–177  
 – of an ode 387, 402, 403  
 Liouville theorem 270  
 Liouville’s formula 404  
 Liouville’s principle 311  
 Lipschitz condition 84  
 Lipschitz continuous function 476, 491  
 Littlewood–Richardson rule 246  
 local bifurcation 391, 392, 404  
 local coordinate system 321, 370, 373  
 local truncation error 491, 493  
 long-term behavior 8, 15  
 Lorentz group 172, 176, 181–185, 207  
 Lu -factorization 481  
 Lyapunov function 388, 389  
 Lyapunov stability 387
- m**
- macromedium, differential geometry 364  
 magnetization, spontaneous 53  
 Mangasarian–Fromovitz constraint qualification  
 555  
 manifolds  
 – center 390, 393, 395, 397–399  
 – differentiable 323–328  
 – invariant 390, 429, 433  
 – of a limit cycle 403, 404  
 – of equilibria 390, 391  
 – riemannian 377–380  
 – symplectic 345, 346  
 – topological 319–324  
 Markov chain  
 – aperiodic 31  
 – continuous time 22–27, 29

- Markov chain (*contd.*)
  - discrete time 10–18
  - generator 26, 28
  - irreducible 15, 16, 31
  - positive-recurrent 16
  - random field 30, 31
  - recurrent 16, 18
  - transient 18
  - transition probabilities 12
- Markov chain Monte Carlo 31
- Markov process 10, 17, 23, 26, 27, 33, 50, 56, 68, 99
- Markov property
  - continuous time 22
  - strong 12, 13, 17, 27
- martingale property of the Itô integral 83
- master equation 23, 48, 50, 51, 55, 59, 69
- matching 118, 155, 431, 433, 434, 445, 446
- material law 539, 542–547
- Matlab and octave linear solvers 482
- matrix groups 170–173, 358
- maximization 556
- maximum dissipation principle 560
- Maxwell's equations 183, 200, 542
  - extended 547f.
- Maxwell–Dirac System 548
- Mayer–Vietoris exact sequence 223
- mean displacement of an atom (VERTEX) 155
- Mellin transformation 520
- Melnikov function 429
- metallurgy 63
- methods
  - Adams 493–495
  - Adams–Bashforth 493
  - Adams–Moulton 493, 494
  - conjugate-gradient (CG) 479, 480, 484, 576
  - descent 479, 484, 485
  - descent, for quadratic forms 479, 484
  - direct, for linear systems 480, 499
  - dynamic Richardson 483
  - explicit 492–494
  - factorization, for linear systems 480, 482
  - finite element 489, 577, 584
  - fixed point 477
  - Galerkin 497, 498, 576
  - gauss–seidel 482, 483
  - gaussian elimination 480, 481
  - gradient 479, 480, 484
  - implicit 94, 492, 494, 495
  - inverse power 478, 486
  - inverse shifted power 486
  - iteration, for eigenvalues 478
  - iteration, for linear systems 482
  - iteration, for nonlinear scalar equations 477
  - linear multistep 492
  - midpoint 492
  - multistep 492–496
  - Newton's 401, 476–479, 576
  - Newton–Simpson 478
  - numerical minimization 475, 479, 480
  - one-step 13, 14, 491, 492, 494–496
  - power 486
  - preconditioned conjugate gradient (PCG) 485
  - Runge–Kutta 494
  - Simpson's 492
  - static Richardson 483
  - stationary, for linear systems 482–484
  - successive over relaxation (SOR) 482, 483
- metric, cartan 178, 182, 205, 209
- Metropolis importance sampling 47
- micromedium, differential geometry 364
- microstructure 364, 578, 580–582
- midpoint method 492
- Milstein method 94–98
- Min-max principle 485, 486, 497
- minimal polynomials 534
- minimum-energy principle 552, 583
- minus-sign problem 60
- mixed method, perturbation methods 436
- möbius strip 403
- Möbius transformations 249
- modular algorithms 296, 297, 299
- moduli 456, 482
- molecular dynamics 51, 66, 69
- molecular dynamics method 69
- molecular Hamiltonian 155
- moment generating function 9, 10, 21
- moments 100, 101
- monodromy group 248, 249, 272, 281
- monodromy matrix 402–404, 406
- monomorphism 223
- monotone operator 553
- Monte-Carlo methods 31, 39–71, 92, 93, 98
- Monte-Carlo step 49, 50, 65, 69
- Morse function 226–230, 233, 234
  - discrete 229, 230
- Morse theory 212, 224, 226–229, 236
  - discrete 229, 236
- Morse–Bott function 228
- Morse–Smale–Witten Complex 228
- motion equation for simple harmonic motion 273
- mountain pass 554, 575
- Moutard transformation 266, 267
- multicanonical ensemble 52
- multilevel Monte Carlo method 97, 98
- multiple shooting 401
- multiple-scale method 426, 429, 438

- multiplicative noise 85, 86
- multistep methods, numerical analysis 492
- multivariate gaussian 105
- multivariate systems 304
  
- n**
- n-dimensional
  - Fourier–Laplace transformation 521
  - topological manifold 321, 323, 324
- naive expansion 419, 445, 446
- natural boundary conditions 570
- Neimark–Sacker bifurcations 404
- Nekhoroshev theorem 412, 425
- Nelson theorem 459
- Nemytskiĭ mapping theorem 563, 564
- nerve of a cover 234
- network community 155
- network motif 142, 155
- network theory 111–121
- network transitivity 115
- networks 111ff. 137–154
- Nevanlinna theory 286
- Newton divided difference formula 487
- Newton form 487
- Newton's method 476, 478, 576
- Newton–Simpson method 478
- nilpotence 251, 344
- noise
  - colored 77–80
  - thermal 74
  - white 73, 77–79, 86, 101
- nonanticipating stochastic process 82
- nonlinear heat-transfer problem 571
- nonlinear oscillations 413, 418–427, 439–441
- nonlinear scalar equations 476, 477
- nonlinear special functions 265, 272–282
- nonlinear systems 304, 390, 477–479, 494, 499
- nontrivial closed V-path 229
- nonvariational methods 585
- normal cone 555
- normal distribution 9, 77, 104
- normal form 427, 428
  - Jordan 398, 411, 442, 444
  - Smith (SNF) 231
- normal operator 455, 458, 459, 464, 473
- normed space 449–451
- nuclear shell structure 195–198
- null homotopic 213
- nullity 117–120, 154
- numerical approximation 80, 93, 95, 476, 487, 571, 576, 582
- numerical methods 92–98
  - collocation 401, 402
  - numerical continuation 405–408
  - numerical shooting 400, 401
- numerical minimization 475, 479, 480
- numerical solution of differential equations 476
- NVT ensemble 52
  
- o**
- Octave linear solvers 482
- Ogden material 572, 574, 584
- one-dimensional simple random walk 11
- one-dimensional stable manifold 391
- one-dimensional unstable manifold 391
- one-parameter
  - bifurcation diagram 393–395, 397
  - group of transformations 348, 360
  - pseudo-group of transformations 348
  - subgroups 348, 360, 375
- one-step methods 491, 492, 494–496
- Onsager's symmetry condition 558
- open-map theorem 451
- operator with pure point spectrum 473
- optimal control, relaxed 585
- order of one-step/multistep methods 402
- ordered series 417
- ordinary differential equations (ODEs)
  - equilibria 386ff.
  - Fuchsian equations 247–250
  - limit cycles 399ff.
  - numerical solution of the cauchy problem 491ff.
  - stochastic differential equations 73, 74, 86, 94
- oriented incidence matrix 113
- ornstein–Uhlenbeck (OU) process 73–75, 88, 89, 100
- ornstein–zernike relation 6
- orthogonal groups 172, 174, 572
- orthogonal polynomials 260, 261
- orthogonality of special functions 260
- orthogonality relations 186–188, 260
- orthogonalization, Gram–Schmidt 202
- oscillator
  - harmonic 131, 164, 195–197, 207, 262, 515, 532
  - quantum 133, 262, 515
- outer solution 431–435, 445, 446
- overlap domain 431, 438, 445
  
- p**
- $p$ -adic algorithms 300, 301
- PageRank algorithm 145, 483
- Painlevé equations 243, 280–283
  - discrete 286
- parabolic cylinder coordinates 256
- parallel transport 368–370, 374, 375
- parallelism 367, 376, 377

- parallelogram identity 450
- parametrization, perturbation methods 413, 414, 445
- parseval identity 453, 473
- partial differential equations (PDEs) 527–548
  - factorization 265–267
  - Fuchsian equations 247
  - perturbation theory 433–435
  - separation of variables 256
  - stochastic differential equations 98–104
- partial differential operator 531, 536
- particle path 46
- partition theorem 6, 13, 37
- partition, Monte Carlo methods 61
- passage time 13, 103
- path integral 58, 59
- Path Integral Monte Carlo (PIMC) 57–60
- percolation model 31–33
- period-doubling bifurcation 404, 405, 407
- periodic orbits 211, 386
- periodic standard form 422–424, 440, 441
- periodicity 15
- permutation group 164, 168, 169, 195
- persistence diagram 233, 234
- persistent homology 232–234
- perturbation parameter 413–416, 422, 428, 444–446
- perturbation series 415–418, 430, 431, 445
- perturbation theory 411–444
- phase factor 209
- phase space 50, 51, 185, 328, 345
- phase transitions 52–54
- Picard–Vessiot extension 271, 272
- piecewise polynomial interpolation 489–491
- piecewise polynomials 489, 490, 576
- piezoelectromagnetic model 546
- piezoelectromagnetism 546–548
- Pitchfork bifurcation 392, 394, 396–398, 404
- pivot elements 481
- pivoting technique 481
- Planck’s constant 161
- Plateau variational problem 567
- Pochhammer symbol 259, 260
- Poincaré groups 415, 419, 445
- Poincaré lemma 226
- Poincaré map 402, 403, 405
- Poincaré section 402
- Poincaré–Lindstedt method 395
- point groups 164, 169
- point sources 505
- point spectrum 452, 455, 456, 464, 473
- Poisson bracket 347
- Poisson distribution 5, 6, 25, 35, 36, 140, 141
- Poisson equation 534, 536
- Poisson process 25
- Poisson summation formula 517
- polar coordinates 243, 423
- Pólya’s recurrence theorem 18, 19
- polyconvexity 565, 566, 572
- polyhedron 219, 230
- polymer science 45, 63–67
- polynomials
  - interpolation 476, 488–490
  - numerical methods 489
- polytope 219
- pontryagin maximum principle 583, 585
- position vectors 332, 333
- potential
  - double-well 567, 578
  - of dissipative forces 558
- potential energy 58, 85, 131
- potential theory 24, 536
- Potts model 30, 48, 120–123
- power series
  - Frobenius method 248
  - lie algebra 177
  - perturbation methods 413, 415, 443
- Poynting–Thomson model 544
- preconditioned conjugate gradient (PCG) method 485
- preconditioned residual 484
- preconditioner 482
- predictable stochastic process 82, 83
- pre-dual 553, 554, 567
- preserved foliations 428
- principal axis transformation 503
- principal bundle 365–367
  - associated 362, 364
  - connections 369, 370
- principle of least dissipation 558
- principle of relativity 166, 182, 185
- probability density function (PDF) 89, 99, 102, 105
- probability generating function 5, 25
- probability mass function 5, 7, 29–31, 35
- probability space 34–36, 82, 105
- probability theory 3, 33–37
- problem
  - cauchy 491, 492, 495, 496, 500, 556–559
  - eigenvalue 40, 443, 476, 485, 498–500
  - evolution 491, 560, 577, 585
  - initial value 429, 493, 541, 556
  - linear eigenvalue 480, 498
  - spectral 476, 496, 497
- product topology 320, 324
- production rate 356, 357
- projection operators 363
- propagation 7, 153, 540, 546

- propagators 124
- pseudo-indeterminates 292
- pseudorandom numbers. *See* random numbers
- Puiseux expansions 268
- pullbacks 341–344, 349, 354, 357
- pure absorption method 436
- pure envelope method 436, 438, 441
- pure state of a quantum system 473
- pushforwards 334, 342, 349
- Pythagoras' theorem 160, 161
  
- q**
- $q$ -Hermite polynomials 285
- $Q$ -matrix, Markov chain 23, 25
- quadratic variation 77
- quadrature formulae, Hermite 489
- quantization 116
- quantum calculus 284
- quantum chromodynamics (QCD) 61
- quantum field theory 61, 124, 229, 435, 468
- quantum groups 240, 283, 284
- quantum mechanics 468–472
- quantum Monte Carlo methods 57–61
- quantum numbers 189, 191, 194, 195
- quantum oscillator 133
- quark model 62
- quasiconvex envelope 578
- quasi-leibniz rule 344
- quasi-periodic orbits 386
- quaternions 169, 174–176, 449
- quenched approximation 62
  
- r**
- radiation 46
- radioactive decay 39, 40
- Radon transformation 523–525
- random behavior 75
- random field 29–31, 80
  - Gibbs 29–31
  - Markov 29–31
- random-number generator (RNG) 40, 41, 44, 69
- random numbers 39–46, 49, 69
- random variable 3, 35–37
- random walk 18–22
  - examples 11, 12, 14, 16
  - self-avoiding (SAW) 44, 45
- rank-one convexity 565, 566, 583
- rate-independent 560
- rational functions 239, 249, 274, 294, 308–311
- rational numbers 292
- rayleigh quotient 485, 497
- real roots 303
- rectangular coordinates 243
- rectangular grids 231
- recurrence relations 202
  - special functions 240, 248, 260–263, 266, 282, 283, 285
- recurrent markov chain 16, 18
- recursion 41, 204, 205, 493, 496
- reducibility 252
- reflection groups 240, 243, 244, 246, 267
- reflection principle 19, 20
- reflections 185
- region of absolute stability 496
- regular chains 304, 308
- regular perturbations 417–419, 424, 429, 437, 445
- relative topology 320
- relaxation 577
  - by convex compactification 578
- relaxation oscillation 433, 445
- Rellich–Kondrachov theorem 563
- renewal relation 17, 21
- renormalization group (RG) method 446
- representations
  - faithful 185, 209
  - lie group 160, 169, 179, 199, 203, 208, 251, 252
- rescaled coordinate 446
- rescaled variables 414
- resistance distance 129, 130, 155
- resolvent computation 26
- resonance 425, 446
- response functions 52
- rest points 418, 419
- reversibility 12
- ricatti equations 104, 270
- Riemann  $P$ -symbol 258, 259
- Riemann integrals 80, 81, 87, 353
- Riemannian connections 379, 380
- Riemannian manifolds 377–380
- Riemannian symmetric space 159, 181, 182, 207
- Riesz lemma 452, 473
- Riesz–schauder theorem 456
- Ritz method 576, 584
- Robinson–Schensted–Knuth (RSK) correspondence 246
- Rodrigues' formula 202
- Rössler system 405–407
- rotation groups 171
- rotation symmetries 239, 522
- Rothe method 560, 584
- rounding errors 499, 505
- Rouse diffusion 66
- row echelon form 480
- ruin probability 14
- Runge's phenomenon 488
- Runge–Kutta method 494
- Runge–Lenz vector 194

## 5

- saddle equilibrium 391, 392
- saddle point 554, 575
- saddle-node bifurcation 392–398, 406–408
- sample path 105
- sampling 42–51, 55–57, 60, 62, 68, 69
- scale-free network 141, 155
- Scaling 162, 163
- scattering theory 470–472
- Schatten classes 456
- Schramm–Loewner evolution 33
- Schrödinger equation
  - gauge theory 199, 209
  - historical formulation 538
  - Monte Carlo simulation 59, 60
  - quantum mechanics 528
  - Scaling 162
  - symmetry groups 190
  - unitary groups 467
- Schrödinger operator 461, 468–470, 472, 473, 476, 531
- Schur polynomials 244–246
- Schwartz space 531–533
- Schwarzian derivative 249
- Second order systems 564–568
- Second-order differential equations 263
- Secondary hopf bifurcations 404
- self-adjoint operator 457–473
  - adjacency operator 113
- self-adjoint extensions 458, 460–462
- self-averaging 45
- self-avoiding walk (SAW) 44, 45
- self-excited oscillation 446
- self-similarity 3, 76
- semiconductors 162, 163
- semi-grand canonical ensemble 52
- semi-implicit scheme 561
- separation of variables 191, 192, 250, 256
- separatrices 390
- set theory notation 33, 34
- Shannon's sampling theorem 517
- Shielding problem 46
- Shift register generators 41
- Shifting 52–54
- shortest path 114, 129, 139, 143, 147, 155
- signal processing 80, 512
- Signorini contact 573
- similarity transformation 175, 209
- simple sampling 39, 42–45, 47, 69
- simple symmetric random walk 18, 19
- simplicial complex 218–223, 225, 229–232, 234
- simplicial homology 218, 219, 221, 224, 225
- simplification 193, 294, 300, 313, 428
- Simpson's method 492
- singular homology 218, 224, 228
- singular hopf bifurcation 395
- singular perturbations 417, 446, 578, 582
- singularities 120, 259, 269, 280
- Skew diagram 246
- Skew symmetry 331, 337–339
- Skew-self-adjoint 537, 538, 540, 543, 544
- slow–fast perturbation methods 432, 433
- small-strain tensor 573
- smith normal form (SNF) 231, 232
- smooth functions 323, 563
- smooth manifold 324
- smooth map 324
- smooth real-valued functional 552
- smooth vector fields 330, 344
- smoothness 359, 380, 565, 584
- Sobolev critical exponent 563
- sobolev space 498, 562–564, 583
- soldering form 373
- solitons 265, 267, 276
- solvability 252, 271, 272, 528, 573
- sound waves 50
- source terms 528, 548
- space group 164, 170
- spanning forest 115, 123, 125, 155
- spanning tree 115, 123, 125, 126, 155, 216, 231, 232
- Sparse matrix 304
- Sparse model 293, 295, 310
- Sparse polynomials 295
- special functions 239–286
  - group theory 202–207
- special linear groups 174, 176
- specific function spaces 562–576
- spectra 442–444
- spectral analysis 465
- spectral density 79, 135, 136
- spectral measure 462–466, 473
- spectral problems 496–499
- spectral radius 452, 482, 483
- spectral representations 507, 511, 512, 520, 523
- spectral theorem 261, 463, 464
- spectral theory 131, 462, 463, 465, 467
- spectrum of an operator 473
- sphere-valued harmonic maps 574
- spherical bessel functions 268
- spherical harmonics 203, 204, 240, 250, 255, 256, 264
- spins, Potts model 120
- spontaneous magnetization 53
- square-free decomposition 295, 309
- St. Venant–kirchhoff material 573
- stable manifolds 227, 390, 391, 399, 403, 429
- standard topology 321, 323, 353

- state space 385ff.
  - stationary covariance 76, 79
  - stationary distribution 12, 14–16, 18
  - statistical ensembles 52, 68
  - statistical errors 55, 56
  - statistical inefficiency 55
  - statistical mechanics 61–68, 146–148
  - statistical thermodynamics 47–52
  - steady-state problems 563
  - steepest descent method 485
  - stefanelli's variational principle 558
  - stochastic differential equations 73–104
  - stochastic integrals 76, 80–84, 90, 95
  - stochastic matrix 11, 14, 151
  - stochastic process 3–37, 75ff.
    - predictable 82, 83
  - stochastic trajectories 50, 51
  - Stokes phenomenon 269
  - Stokes system 575
  - Stokes' theorem 355, 357
  - Stone formula 466, 473
  - Stone theorem 467
  - stone–von neumann theorem 468
  - stopping criterion 576
  - straight-line program 293
  - straightforward expansion 419, 445
  - strange attractors 228, 386, 429
  - Stratonovich integral 81, 82, 90–92
  - strictly convex 553
  - strong convergence 92, 94, 96, 98, 462
  - strong law of large numbers 21
  - strong markov property 12, 13, 17, 27
  - structurally unstable system 390, 391
  - structure constants 175, 176, 208
    - of the gauge group 61
    - of the moving frame 334
  - Sturm–liouville equations 260
  - subdifferential 554, 555
  - subgroups, differential geometry 358
  - subset topology 320, 323
  - subspace
    - horizontal 368
    - vertical 368, 370
  - successive over relaxation (SOR) 482, 483
  - supercomputers 62
  - superposition 326, 533, 563
  - surface effects 54
  - surface physics 57, 67
  - susceptibility 53, 54
  - Suzuki–Trotter formula 59
  - sweep step 50
  - symanzik polynomial 125–129
  - symbolic computation. *See* computer algebra
  - symmetric functions 244, 245, 247, 523
  - symmetric operator 458, 459, 461, 462, 473
  - symmetric polynomials 242, 244, 245
  - symmetries
    - continuous 250–261
    - discrete 241–250
    - tensor product 337, 338
  - symmetry condition, Onsager 558
  - symmetry groups 160, 190–194, 207–209
  - symplectic geometry 229, 345
  - symplectic manifolds 345, 346
  - symplectic vector spaces 345
  - synchronization 151–153
  - system
    - confined to a box 498
    - finite-dimensional linear 480, 481, 483, 485
    - linear 480–487
    - nonlinear 304, 477–479, 494, 499
    - unconfined 499
- t**
- tangent bundle 327–330
  - tangent cone  $T_k(u)$  555
  - tangent map 329
  - tangent space 326
  - tangent vector 324, 325
  - Taylor series expansion 206, 243, 420
  - Taylor's theorem 96, 477, 479
  - tempered distributions 516, 520, 531–534
  - tensor bundles 342, 364, 372
  - tensor field over 342
  - tensor products
    - direct sums 454
    - linear operators 334–336
    - of operators 460
  - test functions 408, 515, 516
    - space of 515
  - theorem of Frobenius 370–372, 377
  - thermal equilibrium 50, 67, 68, 89
  - thermal noise 74
  - $\partial$ – $\delta$ -theory 283
  - thermodynamics 47–52
  - thermoelastic system 545, 546
  - thermoelasticity 545, 546
  - theta function 240, 278
  - Thom's lemma 304
  - Thomas factor 173
  - tight-binding model 115–117
  - time average 51, 68
  - time dependence 75, 527, 528
  - time set 385
  - time-T map 402, 403
  - tomography 523
  - topological equivalence 387, 389, 403
  - topological manifold 319–324



- topological space 319, 320
  - torsion tensor 375–377
  - total variation 77
  - trace-class operator 457, 471, 472
  - trajectories 20, 57, 470, 556
  - transformations
    - canonical 360, 426, 505
    - mathematical 503–525
    - symmetries 241, 522
  - transient markov chain 18
  - transition density 99, 101
  - transition function 22, 323, 324, 343, 361
    - between coordinate charts 321
    - of a markov chain 22
  - transition layer 411, 433, 445, 446
  - transition matrix 12–14, 56
  - transition probability 199
  - transition rate matrix 23
  - translation groups 169, 170, 216, 244, 465
  - translations 76, 360
  - transport process 42, 46
  - transpose matrix 174, 503
  - transposition 585
  - tree 115
  - trial move 49
  - triangular matrices 252
  - triangulated space 219, 222
  - trigonometric functions 311
  - triple deck 431, 434, 435, 446
  - trivial multiplier 403
  - trivializations, differential geometry 362
  - trotter formula 467
  - truncation error 491–493
  - Tschirnhaus transformation 302
  - turning point problem 434
  - tutte polynomial 121–123
  - two-dimensional Fourier–Plancherel 523
  - two-element groups 164, 166, 185
  - two-parameter bifurcation diagram 393, 394
- U**
- umbrella sampling 56
  - unbounded operators 452, 457–462
  - uncertainty 3
  - unconfined systems 499
  - uncorrelated random variables 80, 105
  - unfolding
    - Arnold 444
    - Pitchfork bifurcation 397, 398
    - universal 414, 446
  - unforced conservative oscillator 421
  - uniform approximation 420, 446
  - uniformity 416, 417
  - uniqueness property 4, 7, 9
- unitarily equivalent operators 459, 473
  - unitary Fourier series transformation 506, 508
  - unitary groups 174, 195, 467, 468, 473
  - unitary irreducible representations (UIR)
    - 188–190, 202, 203, 207
  - unitary operator 455, 467, 468, 473
  - universal enveloping algebra 251, 264, 284
  - universal unfolding 414
  - unstable manifolds 390–392, 403, 404, 429
- V**
- V-path 229, 230
  - van der pol equation 395, 396, 421, 423, 427
  - van dyke matching rule 431
  - van kampen theorem 214
  - variable-metric methods 576
  - variation of constants 88, 89, 100
  - variational convergence 581, 583
  - variational inequality 554, 555, 559, 570, 571, 574
  - variational methods 584
  - variational principle 556–560, 576, 583, 584
  - variational problems 551–584
  - varieties 244
  - vector field 330
    - discrete 229, 230
    - fundamental 360, 369, 370, 372, 375
    - gradient 229, 230
    - hamiltonian 346
  - vector space 325, 326, 449, 451
    - representations 251, 253, 257
  - vertex degree 155
  - vertical subspace 368, 370
  - vibrations 50, 130–134, 394
  - vietoris–rips complex 235
  - viscoelasticity 543, 544
  - von neumann’s saddle-point theorem 554
  - von Zeipel’s method 422, 426
  - Voronoi diagram 235
- W**
- water droplet freezing 52
  - wave equation 534, 536, 540
  - wave operator 471–473
  - weak solution 85, 565, 566, 570, 577
  - wedge product 214, 339, 340, 344
  - Weierstraß elliptic functions 272
  - Weierstraß maximum principle 579, 581, 583, 585
  - Weierstraß theorem 553
  - weight functions 60
  - wentzell–freidlin theory 103
  - Weyl alternative 461
  - Weyl group 244, 247, 282, 286

- Weyl relations 468
  - Weyl theorem 464, 473
  - white noise 73, 77–79, 86, 101
  - Whittaker functions 256
  - Wiener process. *See* Brownian motion
  - Wiener–khintchine theorem 80
  - Wigner matrices 190
  - witness complex 236
  - WKB method 434, 441
- 
- x**
  - Young diagram 244–246
  - Young measures 579–583
- 
- y**
  - z-transformation 509, 510
  - Zero Dirichlet boundary condition 103, 571
  - Zero-energy state 117, 120
  - Zero-stability 495

