

2. Electron-Capture Subshell Ratios

Electron capture subshell ratios can be calculated as described below,¹ using decay-scheme information and the squared amplitudes of the bound-state electron radial wavefunctions given in Table 1.

The electron-capture transition probability per unit time from all atomic shells is

$$\lambda = \frac{G_\beta^2}{2\pi^3} \sum_x n_x C_x F_x, \quad (1)$$

where G_β is the fundamental weak coupling constant, n_x is the relative occupation number for partially filled shells ($n_x=1$ for closed shells), C_x contains the nuclear matrix elements, and the summation extends over all atomic subshells x from which electrons can be captured. The function F_x , which corresponds to the integrated Fermi function of β decay, is given by

$$F_x = \frac{\pi}{2} q_x^2 \beta_x^2 B_x. \quad (2)$$

Here $q_x (=W_0+W_x)$ is the neutrino energy (neglecting the atomic recoil energy), $W_0 (=Q_\epsilon-E_f)$ is the electron-capture transition energy, Q_ϵ is the electron-capture decay energy (the difference between the atomic masses of the parent and daughter nuclei), E_f is the energy of the level populated in the daughter nucleus, $W_x (=1-|E'_x|)$ is the energy of the bound electron in the parent atom, E'_x is the electron binding energy in the parent atom, β_x is the Coulomb amplitude of the bound-state electron radial wavefunction, and B_x is the associated electron exchange and overlap correction. W_0 , W_x , and E'_x are in units of the electron rest-mass energy m_0c^2 .

After a power series expansion of the wavefunctions, the (L-1)-forbidden-unique electron-capture transition probability, λ , from all atomic shells becomes

$$\lambda = M_L \frac{(2L-2)!!}{(2L-1)!!} \sum_x \frac{n_x p_x^{2(k_x-1)} q_x^{2(L-k_x+1)} \beta_x^2 B_x}{(2k_x-1)![2(L-k_x)+1]!}, \quad (3)$$

where L is the electron-capture transition angular momentum, $p_x (=1-W_x^2)^{1/2}$ is the bound electron linear momentum, M_L contains the nuclear matrix elements, and

$$k_x = \begin{cases} 1 & \text{for capture from the } K, L_1, L_2, M_1, M_2, \dots \text{ atomic shells} \\ 2 & \text{for capture from the } L_3, M_3, M_4, \dots \text{ atomic shells} \\ 3 & \text{for capture from the } M_5, N_5, N_6, \dots \text{ atomic shells} \end{cases}$$

Table 1 gives the squared amplitudes $\beta_x^2 B_x p_x^{2(k_x-1)}$ of the bound-state electron radial wavefunctions for $Z=1-102$, derived from calculations by Bambynek *et al.*¹ These are used to calculate electron-capture subshell ratios as follows:

For allowed decay, the terms for $k_x=1$ in equation (3) predominate, and the transition probability becomes

$$\lambda^{(k_x=1)} = M_L^2 [n_K q_K^2 \beta_K^2 B_K + n_{L_1} q_{L_1}^2 \beta_{L_1}^2 B_{L_1} + n_{L_2} q_{L_2}^2 \beta_{L_2}^2 B_{L_2} + \dots]. \quad (4)$$

The electron-capture subshell ratios are then easy to derive, e.g.,

$$\frac{\lambda_{L_1}}{\lambda_K} = \frac{n_{L_1} q_{L_1}^2 \beta_{L_1}^2 B_{L_1}}{n_K q_K^2 \beta_K^2 B_K}. \quad (5)$$

For first forbidden unique transitions ($L=2$), the contribution of subshells with $k_x=1,2$ leads to

$$\lambda = \lambda^{(k_x=1)} + \lambda^{(k_x=2)}, \quad (6)$$

¹W. Bambynek, H. Behrens, M. H. Chen, B. Crasemann, M. L. Fitzpatrick, K. W. D. Ledingham, H. Genz, M. Mutterer, and R. L. Intemann, *Rev. Mod. Phys.* **49**, 77 (1977).

where

$$\lambda^{(k_x=1)} = M_2^2 [n_K q_K^4 \beta_K^2 + n_{L_1} q_{L_1}^4 \beta_{L_1}^2 B_{L_1} + \dots], \quad (7)$$

and

$$\lambda^{(k_x=2)} = M_2^2 [n_{L_3} q_{L_3}^2 p_{L_3}^2 \beta_{L_3}^2 B_{L_3} + n_{M_3} q_{M_3}^2 p_{M_3}^2 \beta_{M_3}^2 B_{M_3} + \dots]. \quad (8)$$

The L_1/K ratio is then

$$\frac{\lambda_{L_1}}{\lambda_K} = \frac{n_{L_1} q_{L_1}^4 \beta_{L_1}^2 B_{L_1}}{n_K q_K^4 \beta_K^2 B_K}. \quad (9)$$

Expressions for the L_2/K , M_1/K , L_2/L_1 , and M_1/L_1 capture ratios may be derived in an analogous manner. The L_3/L_1 ratio, on the other hand, is given by

$$\frac{\lambda_{L_3}}{\lambda_{L_1}} = \frac{n_{L_3} p_{L_3}^2 q_{L_3}^2 \beta_{L_3}^2 B_{L_3}}{n_{L_1} q_{L_1}^4 \beta_{L_1}^2 B_{L_1}}, \quad (10)$$

as are the other $k_x=2$ to $k_x=1$ electron-capture subshell ratios.

Table 1. Squared Amplitudes of the Bound-State Electron Radial Wavefunctions ($\beta_x^2 B_x p_x^{2(k_x-1)}$)

	₁ H	₂ He	₃ Li	₄ Be	₅ B	₆ C	₇ N	₈ O	₉ F	₁₀ Ne	₁₁ Na	₁₂ Mg
K	1.023×10 ⁻⁶	6.975×10 ⁻⁶	2.877×10 ⁻⁵	7.579×10 ⁻⁵	1.576×10 ⁻⁴	2.844×10 ⁻⁴	4.670×10 ⁻⁴	7.147×10 ⁻⁴	0.001038	0.001448	0.001955	0.002574
L ₁			3.778×10 ⁻⁶	8.519×10 ⁻⁶	1.692×10 ⁻⁵	2.783×10 ⁻⁵	4.185×10 ⁻⁵	6.072×10 ⁻⁵	8.630×10 ⁻⁵	1.198×10 ⁻⁴	1.688×10 ⁻⁴	2.227×10 ⁻⁴
L ₂							4.508×10 ⁻⁹	1.016×10 ⁻⁸	2.045×10 ⁻⁸	3.785×10 ⁻⁸	7.431×10 ⁻⁸	1.343×10 ⁻⁷
L ₃											2.881×10 ⁻⁷	5.180×10 ⁻⁷
	₁₃ Al	₁₄ Si	₁₅ P	₁₆ S	₁₇ Cl	₁₈ Ar	₁₉ K	₂₀ Ca	₂₁ Sc	₂₂ Ti	₂₃ V	₂₄ Cr
K	0.003319	0.004205	0.005246	0.006461	0.007865	0.009475	0.01132	0.01342	0.01578	0.01843	0.02142	0.02477
L ₁	2.952×10 ⁻⁴	3.830×10 ⁻⁴	4.882×10 ⁻⁴	6.127×10 ⁻⁴	7.587×10 ⁻⁴	9.273×10 ⁻⁴	0.001121	0.001339	0.001595	0.001885	0.002214	0.002585
L ₂	2.287×10 ⁻⁷	3.416×10 ⁻⁷	5.343×10 ⁻⁷	8.061×10 ⁻⁷	1.180×10 ⁻⁶	1.684×10 ⁻⁶	2.353×10 ⁻⁶	3.229×10 ⁻⁶	4.357×10 ⁻⁶	5.796×10 ⁻⁶	7.609×10 ⁻⁶	9.888×10 ⁻⁶
L ₃	8.747×10 ⁻⁷	1.293×10 ⁻⁶	2.013×10 ⁻⁶	3.022×10 ⁻⁶	4.402×10 ⁻⁶	6.249×10 ⁻⁶	8.667×10 ⁻⁶	1.179×10 ⁻⁵	1.580×10 ⁻⁵	2.085×10 ⁻⁵	2.716×10 ⁻⁵	3.496×10 ⁻⁵
M ₁	2.425×10 ⁻⁵	3.723×10 ⁻⁵	5.279×10 ⁻⁵	7.161×10 ⁻⁵	9.398×10 ⁻⁵	1.199×10 ⁻⁴	1.567×10 ⁻⁴	1.973×10 ⁻⁴	2.430×10 ⁻⁴	2.937×10 ⁻⁴	3.505×10 ⁻⁴	4.080×10 ⁻⁴
M ₂			3.110×10 ⁻⁸	5.450×10 ⁻⁸	8.930×10 ⁻⁸	1.392×10 ⁻⁷	2.309×10 ⁻⁷	3.609×10 ⁻⁷	5.178×10 ⁻⁷	7.217×10 ⁻⁷	9.842×10 ⁻⁷	1.287×10 ⁻⁶
M ₃									1.874×10 ⁻⁶	2.578×10 ⁻⁶	3.463×10 ⁻⁶	4.491×10 ⁻⁶
M ₄												5.799×10 ⁻¹⁰
	₂₅ Mn	₂₆ Fe	₂₇ Co	₂₈ Ni	₂₉ Cu	₃₀ Zn	₃₁ Ga	₃₂ Ge	₃₃ As	₃₄ Se	₃₅ Br	₃₆ Kr
K	0.02848	0.03263	0.03723	0.04230	0.04791	0.05407	0.06086	0.06827	0.07642	0.08538	0.09516	0.1058
L ₁	0.003001	0.003466	0.003983	0.004563	0.005202	0.005906	0.006697	0.007573	0.008533	0.009586	0.01075	0.01202
L ₂	1.270×10 ⁻⁵	1.618×10 ⁻⁵	2.042×10 ⁻⁵	2.559×10 ⁻⁵	3.184×10 ⁻⁵	3.927×10 ⁻⁵	4.812×10 ⁻⁵	5.866×10 ⁻⁵	7.115×10 ⁻⁵	8.585×10 ⁻⁵	1.031×10 ⁻⁴	1.233×10 ⁻⁴
L ₃	4.447×10 ⁻⁵	5.603×10 ⁻⁵	6.994×10 ⁻⁵	8.663×10 ⁻⁵	1.064×10 ⁻⁴	1.298×10 ⁻⁴	1.571×10 ⁻⁴	1.892×10 ⁻⁴	2.266×10 ⁻⁴	2.699×10 ⁻⁴	3.198×10 ⁻⁴	3.776×10 ⁻⁴
M ₁	4.851×10 ⁻⁴	5.649×10 ⁻⁴	6.534×10 ⁻⁴	7.527×10 ⁻⁴	8.525×10 ⁻⁴	9.833×10 ⁻⁴	0.001126	0.001289	0.001473	0.001680	0.001913	0.002172
M ₂	1.732×10 ⁻⁶	2.245×10 ⁻⁶	2.875×10 ⁻⁶	3.646×10 ⁻⁶	4.508×10 ⁻⁶	5.705×10 ⁻⁶	7.195×10 ⁻⁶	9.039×10 ⁻⁶	1.128×10 ⁻⁵	1.398×10 ⁻⁵	1.724×10 ⁻⁵	2.113×10 ⁻⁵
M ₃	5.951×10 ⁻⁶	7.654×10 ⁻⁶	9.728×10 ⁻⁶	1.224×10 ⁻⁵	1.505×10 ⁻⁵	1.884×10 ⁻⁵	2.348×10 ⁻⁵	2.910×10 ⁻⁵	3.589×10 ⁻⁵	4.398×10 ⁻⁵	5.361×10 ⁻⁵	6.498×10 ⁻⁵
M ₄	1.061×10 ⁻⁹	1.602×10 ⁻⁹	2.355×10 ⁻⁹	3.387×10 ⁻⁹	4.358×10 ⁻⁹	6.621×10 ⁻⁹	9.770×10 ⁻⁹	1.407×10 ⁻⁸	1.989×10 ⁻⁸	2.760×10 ⁻⁸	3.770×10 ⁻⁸	5.077×10 ⁻⁸
M ₅							3.604×10 ⁻⁸	5.180×10 ⁻⁸	7.279×10 ⁻⁸	1.004×10 ⁻⁷	1.365×10 ⁻⁷	1.828×10 ⁻⁷
N ₁									1.378×10 ⁻⁴	1.748×10 ⁻⁴	2.167×10 ⁻⁴	2.638×10 ⁻⁴
	₃₇ Rb	₃₈ Sr	₃₉ Y	₄₀ Zr	₄₁ Nb	₄₂ Mo	₄₃ Tc	₄₄ Ru	₄₅ Rh	₄₆ Pd	₄₇ Ag	₄₈ Cd
K	0.1174	0.1301	0.1441	0.1592	0.1756	0.1934	0.2129	0.2339	0.2568	0.2817	0.3087	0.3376
L ₁	0.01342	0.01497	0.01665	0.01853	0.02057	0.02279	0.02522	0.02791	0.03085	0.03396	0.03752	0.04131
L ₂	1.469×10 ⁻⁴	1.745×10 ⁻⁴	2.065×10 ⁻⁴	2.437×10 ⁻⁴	2.866×10 ⁻⁴	3.363×10 ⁻⁴	3.937×10 ⁻⁴	4.595×10 ⁻⁴	5.350×10 ⁻⁴	6.219×10 ⁻⁴	7.212×10 ⁻⁴	8.341×10 ⁻⁴
L ₃	4.435×10 ⁻⁴	5.194×10 ⁻⁴	6.056×10 ⁻⁴	7.042×10 ⁻⁴	8.155×10 ⁻⁴	9.423×10 ⁻⁴	0.001086	0.001246	0.001427	0.001631	0.001858	0.002111
M ₁	0.002463	0.002783	0.003138	0.003525	0.003960	0.004438	0.004965	0.005547	0.006188	0.006886	0.007666	0.008505
M ₂	2.579×10 ⁻⁵	3.131×10 ⁻⁵	3.786×10 ⁻⁵	4.557×10 ⁻⁵	5.463×10 ⁻⁵	6.519×10 ⁻⁵	7.753×10 ⁻⁵	9.189×10 ⁻⁵	1.086×10 ⁻⁴	1.278×10 ⁻⁴	1.502×10 ⁻⁴	1.760×10 ⁻⁴
M ₃	7.827×10 ⁻⁵	9.380×10 ⁻⁵	1.119×10 ⁻⁴	1.327×10 ⁻⁴	1.568×10 ⁻⁴	1.844×10 ⁻⁴	2.160×10 ⁻⁴	2.522×10 ⁻⁴	2.933×10 ⁻⁴	3.400×10 ⁻⁴	3.930×10 ⁻⁴	4.528×10 ⁻⁴
M ₄	6.761×10 ⁻⁸	8.903×10 ⁻⁸	1.163×10 ⁻⁷	1.505×10 ⁻⁷	1.934×10 ⁻⁷	2.458×10 ⁻⁷	3.106×10 ⁻⁷	3.892×10 ⁻⁷	4.850×10 ⁻⁷	6.003×10 ⁻⁷	7.401×10 ⁻⁷	9.075×10 ⁻⁷
M ₅	2.422×10 ⁻⁷	3.175×10 ⁻⁷	4.120×10 ⁻⁷	5.294×10 ⁻⁷	6.745×10 ⁻⁷	8.532×10 ⁻⁷	1.071×10 ⁻⁶	1.336×10 ⁻⁶	1.657×10 ⁻⁶	2.043×10 ⁻⁶	2.503×10 ⁻⁶	3.050×10 ⁻⁶
N ₁	3.333×10 ⁻⁴	4.145×10 ⁻⁴	4.982×10 ⁻⁴	5.895×10 ⁻⁴	6.800×10 ⁻⁴	7.881×10 ⁻⁴	9.186×10 ⁻⁴	1.072×10 ⁻³	1.255×10 ⁻³	1.468×10 ⁻³	1.712×10 ⁻³	2.000×10 ⁻³
N ₂	2.306×10 ⁻⁶	3.229×10 ⁻⁶	4.253×10 ⁻⁶	5.467×10 ⁻⁶	6.768×10 ⁻⁶	8.525×10 ⁻⁶	1.076×10 ⁻⁵	1.310×10 ⁻⁵	1.602×10 ⁻⁵	1.928×10 ⁻⁵	2.349×10 ⁻⁵	2.851×10 ⁻⁵
N ₃			1.263×10 ⁻⁵	1.611×10 ⁻⁵	1.988×10 ⁻⁵	2.448×10 ⁻⁵	3.033×10 ⁻⁵	3.604×10 ⁻⁵	4.323×10 ⁻⁵	5.090×10 ⁻⁵	6.104×10 ⁻⁵	7.296×10 ⁻⁵
N ₄						1.658×10 ⁻⁸	2.596×10 ⁻⁸	3.246×10 ⁻⁸	4.380×10 ⁻⁸	5.370×10 ⁻⁸	7.585×10 ⁻⁸	1.042×10 ⁻⁷

Table 1. Squared Amplitudes of the Bound-State Electron Radial Wavefunctions ($\beta_x^2 B_x \rho_x^{2(k_x-1)}$) (continued)

	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	55 Cs	56 Ba	57 La	58 Ce	59 Pr	60 Nd
K	0.3691	0.4030	0.4399	0.4793	0.5231	0.5696	0.6204	0.6747	0.7341	0.7986	0.8685	0.9441
L ₁	0.04547	0.04994	0.05488	0.06014	0.06609	0.07238	0.07937	0.08691	0.09531	0.1044	0.1144	0.1252
L ₂	9.645×10 ⁻⁴	0.001112	0.001280	0.001471	0.001691	0.001937	0.002219	0.002539	0.002902	0.003315	0.003783	0.004310
L ₃	0.002397	0.002713	0.003065	0.003458	0.003894	0.004375	0.004910	0.005504	0.006158	0.006882	0.007683	0.008560
M ₁	0.009439	0.01045	0.01156	0.01277	0.01411	0.01556	0.01718	0.01893	0.02087	0.02300	0.02535	0.02790
M ₂	2.058×10 ⁻⁴	2.400×10 ⁻⁴	2.793×10 ⁻⁴	3.242×10 ⁻⁴	3.764×10 ⁻⁴	4.355×10 ⁻⁴	5.036×10 ⁻⁴	5.809×10 ⁻⁴	6.698×10 ⁻⁴	7.714×10 ⁻⁴	8.876×10 ⁻⁴	0.001019
M ₃	5.204×10 ⁻⁴	5.964×10 ⁻⁴	6.821×10 ⁻⁴	7.780×10 ⁻⁴	8.864×10 ⁻⁴	0.001007	0.001142	0.001292	0.001460	0.001648	0.001858	0.002090
M ₄	1.108×10 ⁻⁶	1.347×10 ⁻⁶	1.630×10 ⁻⁶	1.964×10 ⁻⁶	2.361×10 ⁻⁶	2.826×10 ⁻⁶	3.373×10 ⁻⁶	4.012×10 ⁻⁶	4.760×10 ⁻⁶	5.634×10 ⁻⁶	6.651×10 ⁻⁶	7.826×10 ⁻⁶
M ₅	3.698×10 ⁻⁶	4.464×10 ⁻⁶	5.367×10 ⁻⁶	6.424×10 ⁻⁶	7.665×10 ⁻⁶	9.108×10 ⁻⁶	1.079×10 ⁻⁵	1.274×10 ⁻⁵	1.499×10 ⁻⁵	1.762×10 ⁻⁵	2.067×10 ⁻⁵	2.414×10 ⁻⁵
N ₁	0.001966	0.002230	0.002524	0.002848	0.003216	0.003614	0.004066	0.004565	0.005116	0.005682	0.006263	0.006933
N ₂	3.459×10 ⁻⁵	4.183×10 ⁻⁵	5.031×10 ⁻⁵	6.023×10 ⁻⁵	7.197×10 ⁻⁵	8.556×10 ⁻⁵	1.015×10 ⁻⁴	1.200×10 ⁻⁴	1.416×10 ⁻⁴	1.649×10 ⁻⁴	1.901×10 ⁻⁴	2.203×10 ⁻⁴
N ₃	8.699×10 ⁻⁵	1.033×10 ⁻⁴	1.223×10 ⁻⁴	1.441×10 ⁻⁴	1.693×10 ⁻⁴	1.982×10 ⁻⁴	2.309×10 ⁻⁴	2.680×10 ⁻⁴	3.103×10 ⁻⁴	3.540×10 ⁻⁴	3.994×10 ⁻⁴	4.529×10 ⁻⁴
N ₄	1.402×10 ⁻⁷	1.854×10 ⁻⁷	2.422×10 ⁻⁷	3.123×10 ⁻⁷	3.986×10 ⁻⁷	5.034×10 ⁻⁷	6.312×10 ⁻⁷	7.856×10 ⁻⁷	9.730×10 ⁻⁷	1.175×10 ⁻⁶	1.390×10 ⁻⁶	1.663×10 ⁻⁶
N ₅	4.644×10 ⁻⁷	6.131×10 ⁻⁷	7.951×10 ⁻⁷	1.017×10 ⁻⁶	1.289×10 ⁻⁶	1.616×10 ⁻⁶	2.012×10 ⁻⁶	2.488×10 ⁻⁶	3.057×10 ⁻⁶	3.658×10 ⁻⁶	4.291×10 ⁻⁶	5.084×10 ⁻⁶
N ₆		1.297×10 ⁻¹⁰	1.613×10 ⁻¹⁰	1.972×10 ⁻¹⁰	2.373×10 ⁻¹⁰	2.737×10 ⁻¹⁰	2.977×10 ⁻¹⁰	3.678×10 ⁻¹⁰	4.398×10 ⁻¹⁰	5.203×10 ⁻¹⁰	6.170×10 ⁻¹⁰	8.309×10 ⁻¹⁰
N ₇							1.941×10 ⁻¹⁴	2.604×10 ⁻¹⁴	3.293×10 ⁻¹⁴	1.274×10 ⁻¹²	1.330×10 ⁻¹²	1.468×10 ⁻¹²

	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	72 Hf
K	1.025	1.113	1.208	1.309	1.420	1.539	1.668	1.808	1.961	2.122	2.300	2.490
L ₁	0.1369	0.1498	0.1638	0.1790	0.1956	0.2138	0.2335	0.2553	0.2789	0.3045	0.3329	0.3632
L ₂	0.004906	0.005579	0.006345	0.007199	0.008184	0.009281	0.01052	0.01193	0.01352	0.01530	0.01732	0.01959
L ₃	0.009527	0.01059	0.01176	0.01303	0.01446	0.01600	0.01770	0.01955	0.02158	0.02379	0.02622	0.02886
M ₁	0.03067	0.03373	0.03705	0.04066	0.04464	0.04899	0.05377	0.05901	0.06475	0.07096	0.07781	0.08525
M ₂	0.001168	0.001338	0.001532	0.001750	0.002001	0.002283	0.002605	0.002970	0.003385	0.003851	0.004382	0.004979
M ₃	0.002346	0.002630	0.002946	0.003291	0.003676	0.004098	0.004565	0.005080	0.005647	0.006268	0.006953	0.007703
M ₄	9.186×10 ⁻⁶	1.075×10 ⁻⁵	1.258×10 ⁻⁵	1.465×10 ⁻⁵	1.707×10 ⁻⁵	1.983×10 ⁻⁵	2.299×10 ⁻⁵	2.661×10 ⁻⁵	3.075×10 ⁻⁵	3.545×10 ⁻⁵	4.079×10 ⁻⁵	4.685×10 ⁻⁵
M ₅	2.813×10 ⁻⁵	3.269×10 ⁻⁵	3.789×10 ⁻⁵	4.377×10 ⁻⁵	5.051×10 ⁻⁵	5.812×10 ⁻⁵	6.676×10 ⁻⁵	7.652×10 ⁻⁵	8.756×10 ⁻⁵	9.997×10 ⁻⁵	1.140×10 ⁻⁴	1.296×10 ⁻⁴
N ₁	0.007660	0.008457	0.009331	0.01032	0.01132	0.01247	0.01373	0.01511	0.01662	0.01825	0.02012	0.02216
N ₂	2.547×10 ⁻⁴	2.938×10 ⁻⁴	3.389×10 ⁻⁴	3.916×10 ⁻⁴	4.479×10 ⁻⁴	5.138×10 ⁻⁴	5.891×10 ⁻⁴	6.748×10 ⁻⁴	7.723×10 ⁻⁴	8.822×10 ⁻⁴	0.001011	0.001158
N ₃	5.123×10 ⁻⁴	5.781×10 ⁻⁴	6.516×10 ⁻⁴	7.364×10 ⁻⁴	8.227×10 ⁻⁴	9.221×10 ⁻⁴	0.001032	0.001154	0.001288	0.001435	0.001604	0.001791
N ₄	1.982×10 ⁻⁶	2.352×10 ⁻⁶	2.783×10 ⁻⁶	3.317×10 ⁻⁶	3.855×10 ⁻⁶	4.516×10 ⁻⁶	5.277×10 ⁻⁶	6.152×10 ⁻⁶	7.157×10 ⁻⁶	8.303×10 ⁻⁶	9.685×10 ⁻⁶	1.129×10 ⁻⁵
N ₅	5.997×10 ⁻⁶	7.045×10 ⁻⁶	8.263×10 ⁻⁶	9.739×10 ⁻⁶	1.126×10 ⁻⁵	1.308×10 ⁻⁵	1.516×10 ⁻⁵	1.753×10 ⁻⁵	2.022×10 ⁻⁵	2.327×10 ⁻⁵	2.689×10 ⁻⁵	3.103×10 ⁻⁵
N ₆	1.099×10 ⁻⁹	1.433×10 ⁻⁹	1.852×10 ⁻⁹	2.553×10 ⁻⁹	2.999×10 ⁻⁹	3.769×10 ⁻⁹	4.703×10 ⁻⁹	5.833×10 ⁻⁹	7.194×10 ⁻⁹	8.824×10 ⁻⁹	1.133×10 ⁻⁸	1.442×10 ⁻⁸
N ₇	1.616×10 ⁻¹²	1.777×10 ⁻¹²	6.049×10 ⁻⁹	8.475×10 ⁻⁹	9.791×10 ⁻⁹	1.229×10 ⁻⁸	1.531×10 ⁻⁸	1.895×10 ⁻⁸	2.331×10 ⁻⁸	2.851×10 ⁻⁸	3.675×10 ⁻⁸	4.678×10 ⁻⁸
O ₁							0.001708	0.001873	0.002054	0.002249	0.002573	0.002935
O ₂											1.252×10 ⁻⁴	1.492×10 ⁻⁴

	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po
K	2.697	2.920	3.163	3.422	3.707	4.012	4.346	4.704	5.090	5.511	5.970	6.472
L ₁	0.3969	0.4335	0.4733	0.5167	0.5648	0.6165	0.6740	0.7355	0.8033	0.8781	0.9605	1.051
L ₂	0.02216	0.02504	0.02831	0.03198	0.03615	0.04083	0.04614	0.05214	0.05885	0.06647	0.07510	0.08490
L ₃	0.03175	0.03490	0.03833	0.04206	0.04615	0.05058	0.05543	0.06073	0.06644	0.07268	0.07943	0.08682
M ₁	0.09343	0.1023	0.1122	0.1228	0.1345	0.1474	0.1617	0.1770	0.1938	0.2125	0.2329	0.2557
M ₂	0.005659	0.006426	0.007298	0.008277	0.009394	0.01065	0.01209	0.01370	0.01552	0.01759	0.01995	0.02263
M ₃	0.008527	0.009431	0.01042	0.01151	0.01270	0.01400	0.01543	0.01699	0.01869	0.02055	0.02260	0.02483
M ₄	5.374×10 ⁻⁵	6.155×10 ⁻⁵	7.039×10 ⁻⁵	8.037×10 ⁻⁵	9.168×10 ⁻⁵	1.044×10 ⁻⁴	1.189×10 ⁻⁴	1.350×10 ⁻⁴	1.533×10 ⁻⁴	1.738×10 ⁻⁴	1.970×10 ⁻⁴	2.231×10 ⁻⁴
M ₅	1.472×10 ⁻⁴	1.670×10 ⁻⁴	1.891×10 ⁻⁴	2.138×10 ⁻⁴	2.414×10 ⁻⁴	2.722×10 ⁻⁴	3.067×10 ⁻⁴	3.448×10 ⁻⁴	3.873×10 ⁻⁴	4.346×10 ⁻⁴	4.871×10 ⁻⁴	5.455×10 ⁻⁴
N ₁	0.02442	0.02691	0.02964	0.03263	0.03597	0.03962	0.04371	0.04818	0.05306	0.05849	0.06448	0.07117
N ₂	0.001327	0.001519	0.001739	0.001989	0.002276	0.002601	0.002975	0.003397	0.003879	0.004431	0.005062	0.005784
N ₃	0.002000	0.002231	0.002488	0.002771	0.003086	0.003435	0.003819	0.004242	0.004707	0.005221	0.005789	0.006416
N ₄	1.314×10 ⁻⁵	1.529×10 ⁻⁵	1.775×10 ⁻⁵	2.056×10 ⁻⁵	2.380×10 ⁻⁵	2.749×10 ⁻⁵	3.172×10 ⁻⁵	3.653×10 ⁻⁵	4.202×10 ⁻⁵	4.828×10 ⁻⁵	5.541×10 ⁻⁵	6.353×10 ⁻⁵
N ₅	3.577×10 ⁻⁵	4.118×10 ⁻⁵	4.736×10 ⁻⁵	5.437×10 ⁻⁵	6.234×10 ⁻⁵	7.140×10 ⁻⁵	8.165×10 ⁻⁵	9.312×10 ⁻⁵	1.060×10 ⁻⁴	1.206×10 ⁻⁴	1.370×10 ⁻⁴	1.554×10 ⁻⁴
N ₆	1.819×10 ⁻⁸	2.279×10 ⁻⁸	2.842×10 ⁻⁸	3.520×10 ⁻⁸	4.336×10 ⁻⁸	5.314×10 ⁻⁸	6.479×10 ⁻⁸	7.862×10 ⁻⁸	9.501×10 ⁻⁸	1.144×10 ⁻⁷	1.374×10 ⁻⁷	1.644×10 ⁻⁷
N ₇	5.895×10 ⁻⁸	7.365×10 ⁻⁸	9.123×10 ⁻⁸	1.123×10 ⁻⁷	1.375×10 ⁻⁷	1.673×10 ⁻⁷	2.028×10 ⁻⁷	2.448×10 ⁻⁷	2.943×10 ⁻⁷	3.526×10 ⁻⁷	4.210×10 ⁻⁷	5.011×10 ⁻⁷
O ₁	0.003345	0.003803	0.004320	0.004893	0.005538	0.006397	0.006993	0.007935	0.008978	0.01016	0.01149	0.01299
O ₂	1.773×10 ⁻⁴	2.099×10 ⁻⁴	2.505×10 ⁻⁴	2.976×10 ⁻⁴	3.528×10 ⁻⁴	4.140×10 ⁻⁴	4.877×10 ⁻⁴	5.762×10 ⁻⁴	6.811×10 ⁻⁴	8.052×10 ⁻⁴	9.491×10 ⁻⁴	0.001117
O ₃			3.490×10 ⁻⁴	4.020×10 ⁻⁴	4.616×10 ⁻⁴	5.217×10 ⁻⁴	5.956×10 ⁻⁴	6.860×10 ⁻⁴	7.902×10 ⁻⁴	9.087×10 ⁻⁴	0.001044	0.001197
O ₄									4.614×10 ⁻⁶	5.745×10 ⁻⁶	7.128×10 ⁻⁶	8.757×10 ⁻⁶
O ₅											1.723×10 ⁻⁵	2.097×10 ⁻⁵

Table 1. Squared Amplitudes of the Bound-State Electron Radial Wavefunctions ($\beta_x^2 B_x p_x^{2(k_x-1)}$) (continued)

	⁸⁵ At	⁸⁶ Rn	⁸⁷ Fr	⁸⁸ Ra	⁸⁹ Ac	⁹⁰ Th	⁹¹ Pa	⁹² U	⁹³ Np	⁹⁴ Pu	⁹⁵ Am	⁹⁶ Cm
K	5.068	7.560	8.197	8.879	9.650	10.419	11.321	12.234	13.298	14.427	15.633	16.941
L ₁	1.152	1.253	1.372	1.499	1.646	1.796	1.971	2.153	2.365	2.591	2.838	3.109
L ₂	0.09605	0.1079	0.1220	0.1378	0.1562	0.1759	0.1994	0.2248	0.2549	0.2886	0.3263	0.3691
L ₃	0.09487	0.1033	0.1127	0.1229	0.1341	0.1459	0.1591	0.1730	0.1885	0.2051	0.2229	0.2423
M ₁	0.2807	0.3058	0.3359	0.3681	0.4053	0.4429	0.4875	0.5332	0.5872	0.6449	0.7080	0.7767
M ₂	0.02569	0.02895	0.03284	0.03721	0.04231	0.04779	0.05432	0.06141	0.06983	0.07925	0.08984	0.1019
M ₃	0.02729	0.02987	0.03278	0.03593	0.03943	0.04311	0.04726	0.05166	0.05657	0.06188	0.06762	0.07386
M ₄	2.524×10 ⁻⁴	2.844×10 ⁻⁴	3.212×10 ⁻⁴	3.622×10 ⁻⁴	4.087×10 ⁻⁴	4.595×10 ⁻⁴	5.177×10 ⁻⁴	5.813×10 ⁻⁴	6.538×10 ⁻⁴	7.342×10 ⁻⁴	8.236×10 ⁻⁴	9.231×10 ⁻⁴
M ₅	6.104×10 ⁻⁴	6.807×10 ⁻⁴	7.599×10 ⁻⁴	8.473×10 ⁻⁴	9.446×10 ⁻⁴	0.001050	0.001169	0.001298	0.001442	0.001600	0.001772	0.001962
N ₁	0.07861	0.08615	0.09510	0.1048	0.1159	0.1273	0.1407	0.1547	0.1710	0.1887	0.2079	0.2291
N ₂	0.006613	0.007504	0.008571	0.009777	0.01119	0.01272	0.01454	0.01654	0.01891	0.02157	0.02458	0.02802
N ₃	0.007109	0.007845	0.008677	0.009587	0.01060	0.01168	0.01290	0.01419	0.01565	0.01723	0.01896	0.02084
N ₄	7.277×10 ⁻⁵	8.298×10 ⁻⁵	9.481×10 ⁻⁵	1.081×10 ⁻⁴	1.234×10 ⁻⁴	1.402×10 ⁻⁴	1.596×10 ⁻⁴	1.811×10 ⁻⁴	2.057×10 ⁻⁴	2.332×10 ⁻⁴	2.640×10 ⁻⁴	2.986×10 ⁻⁴
N ₅	1.761×10 ⁻⁴	1.989×10 ⁻⁴	2.247×10 ⁻⁴	2.535×10 ⁻⁴	2.859×10 ⁻⁴	3.216×10 ⁻⁴	3.617×10 ⁻⁴	4.058×10 ⁻⁴	4.554×10 ⁻⁴	5.103×10 ⁻⁴	5.710×10 ⁻⁴	6.382×10 ⁻⁴
N ₆	1.962×10 ⁻⁷	2.329×10 ⁻⁷	2.763×10 ⁻⁷	3.269×10 ⁻⁷	3.860×10 ⁻⁷	4.539×10 ⁻⁷	5.340×10 ⁻⁷	6.255×10 ⁻⁷	7.323×10 ⁻⁷	8.558×10 ⁻⁷	9.961×10 ⁻⁷	1.157×10 ⁻⁶
N ₇	5.946×10 ⁻⁷	7.023×10 ⁻⁷	8.284×10 ⁻⁷	9.742×10 ⁻⁷	1.144×10 ⁻⁶	1.337×10 ⁻⁶	1.561×10 ⁻⁶	1.816×10 ⁻⁶	2.111×10 ⁻⁶	2.446×10 ⁻⁶	2.830×10 ⁻⁶	3.266×10 ⁻⁶
O ₁	0.01469	0.01646	0.01857	0.02090	0.02358	0.02639	0.02958	0.03300	0.03698	0.04126	0.04602	0.05136
O ₂	0.001314	0.001531	0.001794	0.002096	0.002454	0.002851	0.003303	0.003813	0.004421	0.005096	0.005886	0.006801
O ₃	0.001371	0.001561	0.001777	0.002017	0.002288	0.002584	0.002895	0.003241	0.003629	0.004040	0.004500	0.005021
O ₄	1.067×10 ⁻⁵	1.286×10 ⁻⁵	1.546×10 ⁻⁵	1.848×10 ⁻⁵	2.205×10 ⁻⁵	2.613×10 ⁻⁵	3.011×10 ⁻⁵	3.492×10 ⁻⁵	4.046×10 ⁻⁵	4.626×10 ⁻⁵	5.349×10 ⁻⁵	6.201×10 ⁻⁵
O ₅	2.530×10 ⁻⁵	3.021×10 ⁻⁵	3.601×10 ⁻⁵	4.268×10 ⁻⁵	5.040×10 ⁻⁵	5.910×10 ⁻⁵	6.787×10 ⁻⁵	7.810×10 ⁻⁵	8.967×10 ⁻⁵	1.019×10 ⁻⁴	1.160×10 ⁻⁴	1.327×10 ⁻⁴
O ₆			1.939×10 ⁻⁹	2.367×10 ⁻⁹	2.831×10 ⁻⁹	3.324×10 ⁻⁹	4.439×10 ⁻⁸	5.772×10 ⁻⁸	7.344×10 ⁻⁸	8.496×10 ⁻⁸	1.062×10 ⁻⁷	1.385×10 ⁻⁷
O ₇					6.591×10 ⁻¹³	8.052×10 ⁻¹³	9.564×10 ⁻¹²	1.076×10 ⁻¹¹	1.214×10 ⁻¹¹	1.326×10 ⁻¹¹	2.825×10 ⁻¹¹	3.795×10 ⁻¹¹
O ₈							9.295×10 ⁻¹³	1.090×10 ⁻¹²	1.279×10 ⁻¹²	1.456×10 ⁻¹²	1.489×10 ⁻¹¹	1.709×10 ⁻¹¹
O ₉												8.657×10 ⁻¹⁵

	⁹⁷ Bk	⁹⁸ Cf	⁹⁹ Es	¹⁰⁰ Fm	¹⁰¹ Md	¹⁰² No
K	18.391	19.970	21.673	23.601	25.626	27.879
L ₁	3.413	3.747	4.112	4.529	4.974	5.469
L ₂	0.4183	0.4742	0.5374	0.6112	0.6933	0.7882
L ₃	0.2634	0.2862	0.3109	0.3380	0.3669	0.3984
M ₁	0.8544	0.9392	1.033	1.140	1.253	1.382
M ₂	0.1157	0.1315	0.1494	0.1702	0.1935	0.2204
M ₃	0.08071	0.08816	0.09624	0.1052	0.1147	0.1252
M ₄	0.001035	0.001159	0.001298	0.001453	0.001625	0.001817
M ₅	0.002172	0.002402	0.002654	0.002933	0.003237	0.003571
N ₁	0.2529	0.2792	0.3080	0.3409	0.3762	0.4160
N ₂	0.03198	0.03650	0.04165	0.04767	0.05441	0.06223
N ₃	0.02292	0.02519	0.02766	0.03040	0.03335	0.03660
N ₄	3.375×10 ⁻⁴	3.813×10 ⁻⁴	4.302×10 ⁻⁴	4.858×10 ⁻⁴	5.473×10 ⁻⁴	6.166×10 ⁻⁴
N ₅	7.128×10 ⁻⁴	7.955×10 ⁻⁴	8.867×10 ⁻⁴	9.883×10 ⁻⁴	0.001100	0.001223
N ₆	1.342×10 ⁻⁶	1.553×10 ⁻⁶	1.795×10 ⁻⁶	2.072×10 ⁻⁶	2.386×10 ⁻⁶	2.745×10 ⁻⁶
N ₇	3.768×10 ⁻⁶	4.336×10 ⁻⁶	4.980×10 ⁻⁶	5.714×10 ⁻⁶	6.542×10 ⁻⁶	7.484×10 ⁻⁶
O ₁	0.05724	0.06384	0.07113	0.07948	0.08852	0.09872
O ₂	0.007850	0.009065	0.01046	0.01209	0.01394	0.01609
O ₃	0.005563	0.006176	0.006847	0.007593	0.008401	0.009293
O ₄	7.100×10 ⁻⁵	8.155×10 ⁻⁵	9.345×10 ⁻⁵	1.070×10 ⁻⁴	1.222×10 ⁻⁴	1.395×10 ⁻⁴
O ₅	1.494×10 ⁻⁴	1.690×10 ⁻⁴	1.908×10 ⁻⁴	2.152×10 ⁻⁴	2.422×10 ⁻⁴	2.723×10 ⁻⁴
O ₆	1.602×10 ⁻⁷	1.942×10 ⁻⁷	2.340×10 ⁻⁷	2.804×10 ⁻⁷	3.342×10 ⁻⁷	3.967×10 ⁻⁷
O ₇	4.272×10 ⁻⁷	5.171×10 ⁻⁷	6.207×10 ⁻⁷	7.408×10 ⁻⁷	8.782×10 ⁻⁷	1.033×10 ⁻⁶
O ₈	1.868×10 ⁻¹¹	2.090×10 ⁻¹¹	2.336×10 ⁻¹¹	2.617×10 ⁻¹¹	2.922×10 ⁻¹¹	3.267×10 ⁻¹¹