4. Positron Annihilation-in-Flight

The absolute intensity of 511-keV photons per 100 disintegrations ($\gamma^{\pm}(\%)$) from positrons annihilating at thermal energies in an absorber is:

$$\gamma^{\pm}(\%) = 2[\beta^{+}(\%) - \beta_f^{+}(\%)] \,, \tag{1}$$

where $\beta^{+}(\%)$ and $\beta_f^{+}(\%)$ are the emitted and annihilated-in-flight absolute positron intensities, respectively.

There is a significant probability for annihilation-in-flight to result in *two quantum annihilation* (TQA) or *one quantum annihilation* (OQA) with a continuous photon energy distribution. The maximum photon energy is E_0+1, where E_0 is the maximum positron kinetic energy (endpoint) in units of the electron rest mass $m_e c^2$. The OQA probability for annihilation-in-flight of a positron of energy E by collision with an atomic electron is given by Bethe[1] as

$$^{OQA}\Phi(E,Z) = \frac{2\pi\alpha^4 Z^5 r_0^2 \left(E^2 + \frac{2}{3}E + \frac{4}{3} - (E+2)\ln[E + (E^2-1)^{1/2}]\,(E^2-1)^{-1/2} \right)}{(E+1)^2 (E^2-1)^{1/2}} \,, \tag{2}$$

and the TQA probability, as

$$^{TQA}\Phi(E) = \frac{\pi r_0^2 \left((E^2 + 4E + 1)\ln[E + (E^2-1)^{1/2}] - (E+3)(E^2-1)^{1/2} \right)}{(E^2-1)(E+1)} \,, \tag{3}$$

where $r_0 = 2.82\times10^{-13} cm$ is the classical electron radius, $\alpha \approx 1/137$ is the fine structure constant, and Z is the atomic number of the absorber. Positron energies are given in units of the electron rest mass ($m_e c^2$).

The OQA probability given in equation (2) is generally small, except for high-Z absorbing materials where it is ≈16% that of the TQA probability. Equation (2) includes collisions with electrons from the K atomic shell only. The TQA probability given in equation (3) includes collisions with electrons from all atomic shells. The total probability for annihilation-in-flight by positrons of energy E is given by[2]

$$P(E,Z) = \frac{N\rho}{A}\int_0^E [Z\,^{TQA}\Phi(E) + 2\,^{OQA}\Phi(E,Z)](-dE/dx)^{-1} dE \,, \tag{4}$$

where N is Avogadro's number, A, ρ and dE/dx, are the atomic weight, the density, and the stopping power of the absorber, respectively.

Figure 7 shows the total probability for annihilation-in-flight of fully absorbed positrons in Be, Al, Cu, Ag, Pb and U, calculated with equation (4). The integration was done numerically and the probability corrected for the fact that a positron that has already annihilated can not reappear at a lower energy. The stopping power, (dE/dx), was calculated for both collision- and bremsstrahlung-energy losses; for collisions, as described by Nelms,[3] with mean excitation energies and density-effect corrections of Sternheimer[4]; for bremsstrahlung, as described by Koch and Motz.[5]

[1]H.A. Bethe, *Proc. Roy. Soc. Lond.* **A150**, 129 (1935).

[2]G. Azuelos and J.E. Kitching, *At. Data and Nucl. Data Tables* **17**, 103 (1976).

[3]A.T. Nelms, *Energy Loss and Range of Electrons and Positrons, NBS Circular* 577 (1956).

[4]R.M. Sternheimer, *Phys. Rev.* **103**, 511 (1956).

[5]H.W. Koch and J.W. Motz, *Rev. Mod. Phys.* **31**, 920 (1959).

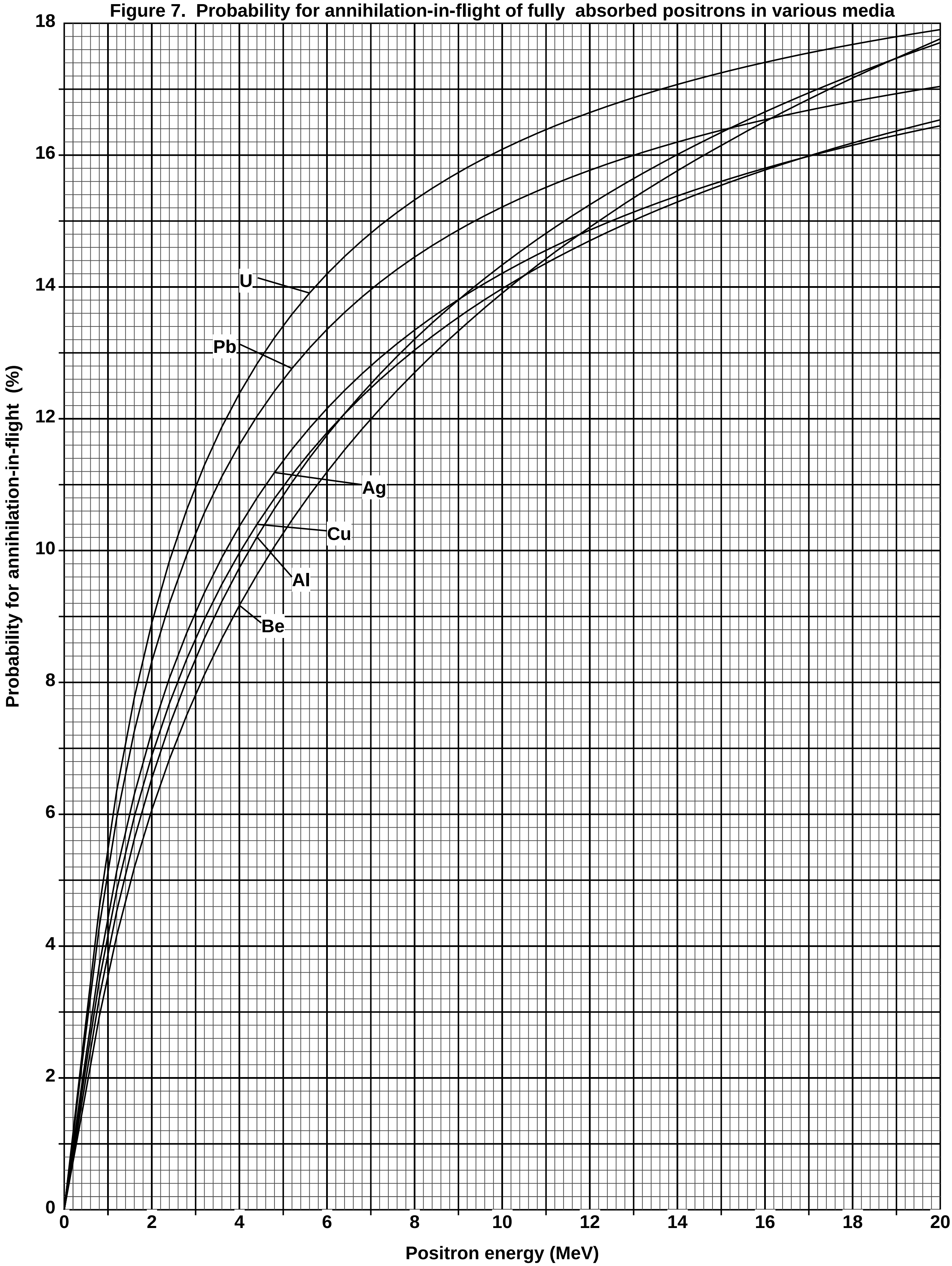

Figure 7. Probability for annihilation-in-flight of fully absorbed positrons in various media
Probability for annihilation-in-flight (%)
0
2
4
6
8
10
12
14
16
18
0
2
4
6
8
10
12
14
16
18
20
Positron energy (MeV)
U
Pb
Ag
Cu
Al
Be