3. Beta Transition Probabilities

For an n^{th}-forbidden beta decay (n=0 for allowed), the transition probability can be written as[1]

$$\frac{\ln 2}{t_{1/2}} = \frac{g^2}{2\pi^3}\int_1^{W_0} pW(W_0 - W)^2 S_n(Z,W)\,dW \,, \tag{1}$$

where g is the weak interaction coupling constant, Z is the atomic number of the daughter nucleus, W is the total energy of the β-particle in units of electron mass, $p=\sqrt{W^2-1}$ is the momentum of the β-particle, and W_0 is the maximum β-particle energy. For β^- decay $W_0=Q+1$, and for β^+ decay $W_0=Q-1$, where Q is the mass difference between initial and final states in neutral atoms. For an n^{th}-forbidden β-branch, $\Delta J=n$ (nonunique) or $\Delta J=n+1$ (unique), and $\Delta\pi=(-1)^n$. ΔJ=0 is allowed for n=1.

$S_n(Z,W)$ is the shape factor defined as

$$S_n(Z,W) = \sum_{k_e,k_\nu=1}^{\infty}[\lambda_k M_L^2 + \lambda_k' m_L^2 - 2\lambda_k'' M_L m_L] \tag{2}$$

where λ_k, λ_k', and λ_k'' are bilinear combinations of the radial components of the continuum electron or positron wavefunctions. M_L and m_L contain the nuclear matrix elements. L is the orbital angular momentum carried off by the electron and the neutrino. For k_e and k_ν, $k=|\kappa|$, where $\kappa=l$ for $j=l-1/2$, and $\kappa=-l-1$ for $j=l+1/2$. For allowed and unique forbidden transitions, a single nuclear matrix element is dominant, while a combination of nuclear matrix elements contribute to non-unique forbidden transitions.

The partial half-life $t_{1/2}^{\beta}$ for an n^{th}-forbidden decay can be obtained from equation (1). A reduced half-life $f_n t$ is defined as

$$f_n t = \frac{2\pi^3 \ln 2}{g^2\eta^2} \,, \tag{3}$$

where

$$f_n = \int_1^{W_0} pW(W_0 - W)^2 \frac{S_n(Z,W)}{\eta^2}\,dW \,, \tag{4}$$

and η^2 contains the nuclear matrix elements. For allowed decays,

$$\eta^2 = \frac{S_0(Z,W)}{\lambda_1(Z,W)} \tag{5}$$

and

$$f_0 = \int_1^{W_0} pW(W_0 - W)^2 \lambda_1(Z,W)\,dW \,. \tag{6}$$

More generally, for unique forbidden decays,

$$f_n = (2n+1)!\int_1^{W_0} pW(W_0 - W)^2 \sum_{k=1}^{n+1}\frac{\lambda_k(Z,W)p^{2(k-1)}(W_0-W)^{2(n-k+1)}}{(2k-1)![2(n-k-1)+1]!}\,dW \tag{7}$$

For first-forbidden unique decay, equation (7) reduces to

$$f_1 = \int_1^{W_0} pW(W_0 - W)^2[\lambda_1(Z,W)(W_0-W)^2 + \lambda_2(Z,W)p^2]\,dW \,. \tag{8}$$

The f function can be calculated with corrections for finite nuclear size and screening by atomic electrons. Values of f_0 and f_1 for β^- decay with Z=10-100 are plotted in Figure 6. Electron-capture decay always competes with β^+ decay. Equation (4) applies to electron capture, except that the function f is written as a combination of the radial components of the bound-state electron wavefunctions $g_i(Z)$, $f_i(Z)$ corresponding to electron capture from ith atomic state.

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[1] N.B. Gove and M.J. Martin, *Nucl. Data Tables* **A10**, 205 (1971)

For allowed decay,

$$f_0^{EC} = \frac{\pi}{2}\left[q_K^2 g_K^2 B_K + q_{L1}^2 g_{L1}^2 B_{L1} + q_{L2}^2 f_{L2}^2 B_{L2} + \sum_i q_i^2 g_i^2 B_i + \sum_j q_j^2 f_j^2 B_j\right]. \tag{9}$$

B_i are the atomic exchange and overlap corrections, and $q_i = Q_{EC} - E_B$ are the neutrino energies where Q_{EC} is the mass difference between the initial and final state and E_B is the electron binding energy in the daughter nucleus. For first-forbidden unique transitions,

$$f_1^{EC} = \frac{\pi}{2}\left[q_K^4 g_K^2 B_K + q_{L1}^4 g_{L1}^2 B_{L1} + q_{L2}^4 g_{L2}^2 B_{L2} + \sum_i q_i^4 g_i^2 B_i + \sum_j q_j^4 f_j^2 B_j + \frac{9}{R^2}\left\{q_{L3}^2 g_{L3}^2 B_{L3} + \sum_m q_m^2 g_m^2 B_m\right\}\right] \tag{10}$$

where i, j, m refer to the s, p, d electrons, respectively. The functions f_0^{EC} and f_1^{EC} are plotted for Z=10-100 in Figure 7.

If the available electron-capture decay energy q is greater that $2m_e c^2$ (1022 keV), both positron emission and electron-capture decay will compete. The nuclear matrix elements are identical for both modes and cancel in the ratio $\frac{EC}{\beta^+} = \frac{f_n^{EC}}{f_n^{\beta+}}$ for unique transitions. The electron-capture to positron decay ratios for allowed and first forbidden transitions are plotted in Figure 8.

Because *ft* values vary over several orders of magnitude, it is customary to consider the quantity log*ft* rather than *ft*. The log*ft* values for non-unique forbidden transitions cannot be calculated directly, so it is conventional to use the allowed log*ft* value for systematic comparisons. Table 5 outlines the systematics of beta transitions[2,3] as a function of log*ft*.

Table 5. Systematics of Log*ft* Values

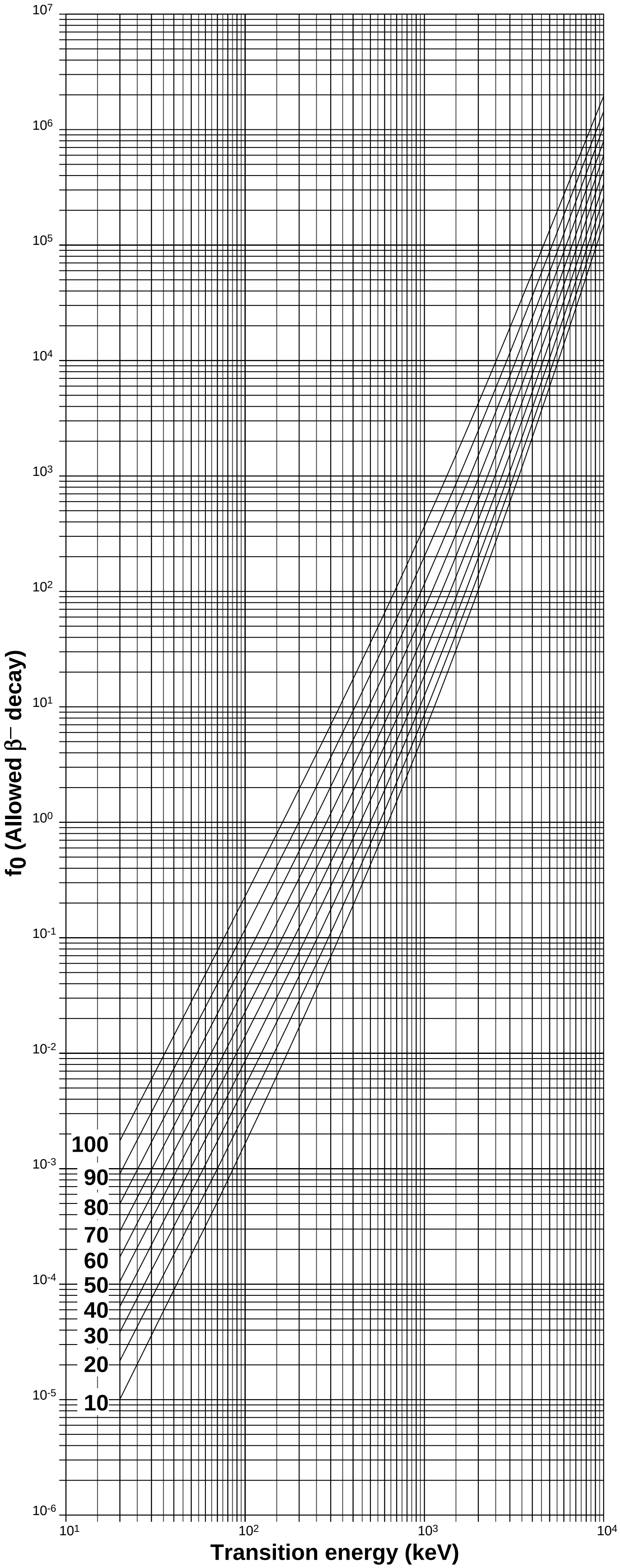
Log*ft*	Transitions
<3.5	$0^+ \rightarrow 0^+$ Superallowed (ΔT=0) and n, ^{3}H, ^{6}He, ^{18}Ne decays
3.6-5.9†	Allowed (ΔJ=0,1; no parity change)
≤6.4	Not $0^+ \rightarrow 0^+$ (ΔT>0)
≤8.5[1u]	ΔJ=0,1
<11.0	ΔJ=0,1; ΔJ=2, parity change
<12.8	ΔJ=0,1,2

†For heavier nuclei, higher-order nuclear matrix elements, especially those proportional to αZ^2, become important. This leads to anomalously low log*ft* values for forbidden transitions. For the *Z*~82 mass region, the upper limit for allowed transitions should be lowered to ~5.1.

[2] S. Raman and N.B. Gove, *Phys. Rev.* **C7**, 1995 (1973).

[3] M.J. Martin, *Nucl. Data Sheets* **74**, ix (1995).

Figure 6. Allowed and first-forbidden unique $f^{\beta-}$ function for Z=10-100



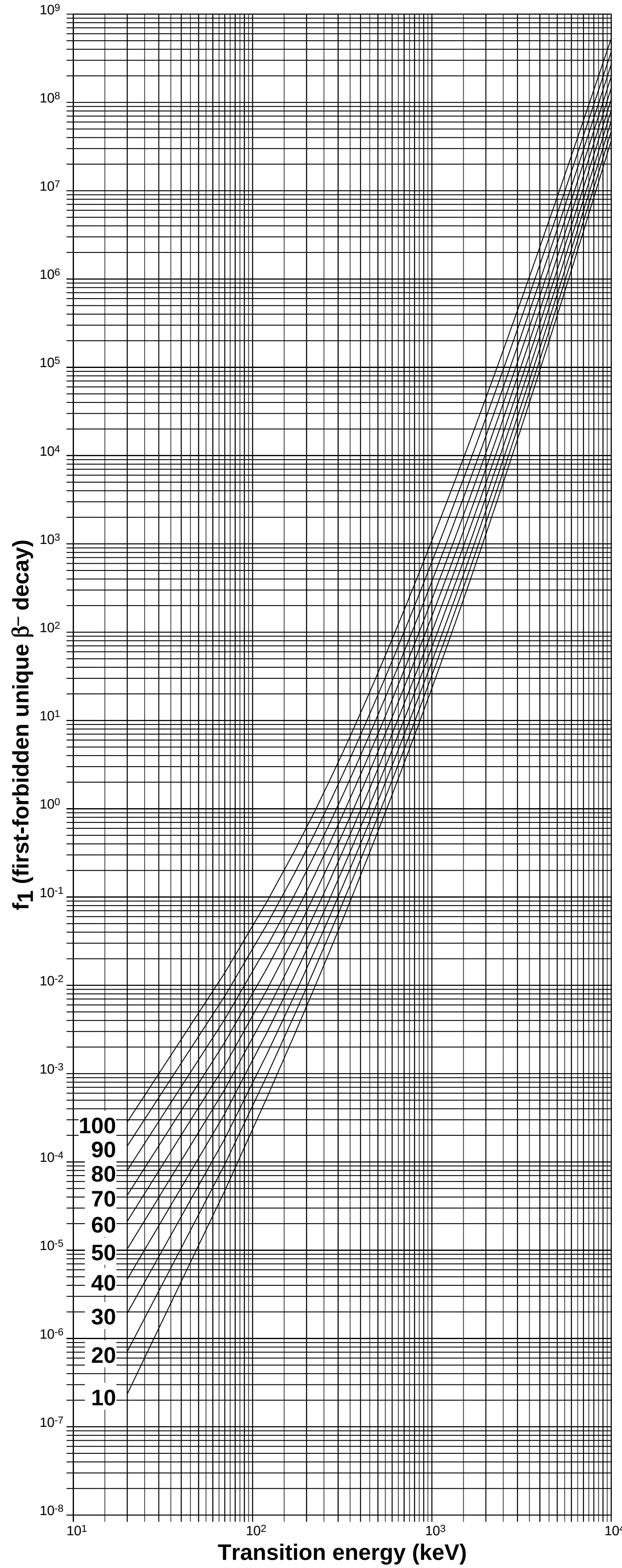


Figure 7. Allowed and first-forbidden unique f^{EC} function for Z=10-100

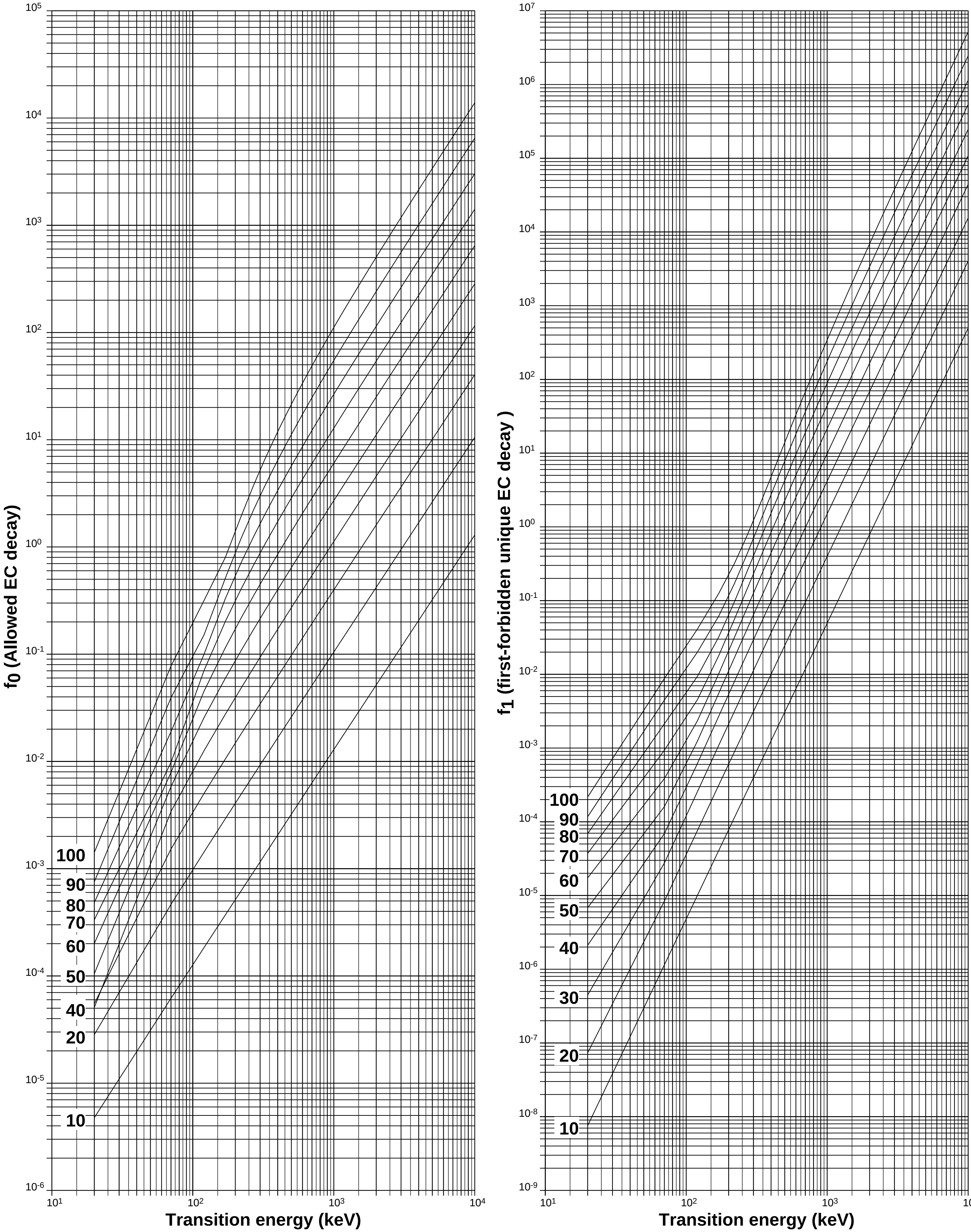


Figure 8. EC/β⁺ for allowed and first-forbidden unique decays with Z=10-100