2. E0 Transition Probabilities for $0^+ \rightarrow 0^+$ Transitions

An E0 transition results from a penetration effect caused by the Coulomb interaction between a nucleus and its surrounding atomic electrons. It is highly forbidden and can occur only between levels with identical quantum numbers $J^{\pi}K$. For $0^+ \rightarrow 0^+$ E0 transitions, there are no competing γ rays emitted and only internal conversion or pair production is possible. E0 transitions may also compete with very retarded M1 and E2 transitions. The treatment of mixed E0 transitions is complex and has been discussed by Aldushchenkov and Voinova[1]. Experimental data are seldom extensive enough to allow full analysis of mixed E0 transitions. The following discussion is limited to $0^+ \rightarrow 0^+$ transitions.

The theoretical transition probability for E0 decay by the emission of internal conversion electrons has been derived by Church and Weneser[2]. This probability may be presented in Wilkinson single-particle units (*Wi*) [3] defined for internal conversion as

$$\Gamma_{Wi}(E0)_e = 2.901\times10^{19} k\ [A(E0)_K + A(E0)_{L1} + A(E0)_{L2} + \ldots]\ sec^{-1}. \qquad (1)$$

Here k is the transition energy in units of $m_o c^2$ (energy(*keV*)/510.9991) and *A(E0)* is a coefficient tabulated by Hager and Seltzer[4] for the *K, L1,* and *L2* atomic shells, assuming the atomic mass A=atomic weight. $\Gamma(E0)_e$ is also tabulated in single-particle units (where $\Gamma_{Wi}(E0)_e = \Gamma(E0)_e/4.91$)[5] by Passoja and Salonen[6] for Z≤40 (K shell only, neglecting screening) and by Bell *et al*,[7] for Z≥40. Internal conversion in the *L3* shell is very small and can be neglected. Analytic expressions for *A(E0)* (neglecting finite size correction) are given below.

For $E>2m_o c^2$, E0 decay may also proceed by pair emission. The corresponding transition probability may be given in Wilkinson single-particle[3] units defined for pair production as

$$\Gamma_{Wi}(E0)_{\pi} = 7.41\times10^4 A^{4/3}\left(\frac{k}{2}-1\right)^3\left(\frac{k}{2}+1\right)^2 B(s)C(Z,k)\ sec^{-1}. \qquad (2)$$

Here A is the mass number and k is the transition energy. The function *B(s)* with $s = \frac{(k-2)}{(k+2)}$ is

$$B(s) = \frac{3}{8}\pi\left(1 - \frac{s}{4} - \frac{s^2}{8} + \frac{s^3}{16} - \frac{s^4}{64} + \frac{5s^5}{512} + \cdots\right),$$

and $C(Z,k)$ is the Coulomb correction factor. Functions *B(s)* and *C(Z,k)* are tabulated in Tables 3 and 4, respectively.

The total Wilkinson single-particle transition probability for internal conversion plus pair production is

$$\Gamma_{Wi}(E0) = \Gamma_{Wi}(E0)_e + \Gamma_{Wi}(E0)_{\pi}. \qquad (3)$$

The experimental E0 transition probability is

$$\Gamma_{\exp}(E0) = \frac{\ln 2}{t_{1/2}} BR, \qquad (4)$$

where $t_{1/2}$ is the half-life of the initial state and BR is the branching fraction for the E0 transition (internal conversion plus pair production). The reduced E0 transition probability can be presented in Wilkinson units, analogous to the Weisskopf photon transition probabilities for higher multipoles, as

$$B_{Wi}(E0) = \frac{\Gamma_{\exp}(E0)}{\Gamma_{Wi}(E0)}\ . \qquad (5)$$

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[1] A.V. Aldushchenkov and N.A. Voinova, *Nucl. Data Tables* **11**, 299 (1972).

[2] E.L. Church and J. Weneser, *Phys. Rev.* **103**, 1035 (1956).

[3] D.H. Wilkinson, *Nucl. Phys.* **A133**, 1 (1969).

[4] R.S. Hager and E.C. Seltzer, *Nucl. Data Tables* **A6**, 1 (1969).

[5] P.M. Endt, *At. Data and Nucl. Data Tables* **26**, 47 (1981).

[6] A. Passoja and T. Salonen, report JYFL RR-2/86 (1986).

[7] D.A. Bell, C.E. Aveledo, M.G. Davidson, and J.P. Davidson, *Can. J. Phys.* **48**, 2542 (1970).

[8] P.M. Endt, *At. Data and Nucl. Data Tables* **55**, 171 (1993).

Systematics of E0 transition probabilities are given for A≤150 by Endt[5,8,9]. The following examples illustrate the calculation of reduced E0 transition probabilities.

Example 1. In ^{150}Sm, the 740-*keV* 0^+ level deexcites by a 1.32% E0 branch to the 0^+ ground state. The level half-life is 19.7 *ps*. *A(E0)* values, calculated as shown below, are

$$A(E0)_K = 1.02\times10^{-10} ,$$

$$A(E0)_{L1} = 1.45\times10^{-11} ,$$

$$A(E0)_{L2} = 2.41\times10^{-13} .$$

The transition energy is k=1.45 in $m_o c^2$ units. From equation (1), the Wilkinson single-particle E0 transition probability becomes $\Gamma_{Wi}(E0)_e = 4.9\times10^9$ sec^{-1}. The experimental E0 transition probability, defined in equation (4), is $\Gamma_{\exp}(E0) = 4.64\times10^8$ sec^{-1}. Since pair production is energetically forbidden in this decay, the reduced transition probability from equation (5) is simply

$$B_{Wi}(E0) = \frac{\Gamma_{\exp}(E0)_e}{\Gamma_{Wi}(E0)_e} = 0.095.$$

Example 2. In ^{96}Zr the 1594-keV 0+ level deexcites by a 100% E0 branch to the 0+ ground state. The level half-life is 38 *ns*. *A(E0)* values, calculated as shown below, are

$$A(E0)_K = 4.18\times10^{-12} ,$$

$$A(E0)_{L1} = 4.42\times10^{-13} ,$$

$$A(E0)_{L2} = 3.52\times10^{-15} .$$

The transition energy is k=3.12 in $m_o c^2$ units. From equation (1) the Wilkinson single-particle internal conversion probability is $\Gamma_{Wi}(E0)_e = 4.19\times10^8$ sec^{-1}. Pair production also contributes to this E0 transition. *B(s)*=1.107 from Table 3 for *s*=0.22, and *C(Z,k)*=1.91 from Table 4 for *Z*=40 and *k*=3.12. From equation (2), the Wilkinson single-particle pair production probability becomes $\Gamma_{Wi}(E0)_\pi = 7.35\times10^7$ sec^{-1}. The experimental transition probability from equation (4) is $\Gamma_{\exp}(E0) = 1.82\times10^7$ sec^{-1}. The reduced transition probability as defined in equation (5) is then

$$B_{Wi}(E0) = \frac{\Gamma_{\exp}(E0)}{\Gamma_{Wi}(E0)_e + \Gamma_{Wi}(E0)_\pi} = 0.037.$$

Analytic expressions for *A(E0)*

The derivation of the single-particle E0 internal conversion probability is described by Church and Weneser.[2] A point nucleus is assumed and only the higher-order Coulomb and momentum terms are considered. Using the formalism adopted by Hager and Seltzer[4], the analytic expression for $A(E0)_K$ for the K atomic shell is

$$A(E0)_K = \frac{1}{8\pi\alpha k}\frac{\alpha^2}{36}\frac{1+\gamma}{\Gamma(2\gamma+1)}\frac{P_K(W_K+\gamma)}{(\alpha Z)}(2\alpha ZR)^{2\gamma+2}F(Z,P_K)S_K^2. \qquad (6)$$

Here k is the transition energy in units of $m_o c^2$, α=1/137, $\gamma=[1-(\alpha Z)^2]^{1/2}$, P_K is the K-electron linear momentum, $R = 0.426\alpha A^{1/3} \equiv 1.2A^{1/3}$ fm, S_K^2 is the atomic screening correction for the K shell, and $F(Z,P_K)$ is the Fermi function given by

$$F(Z,P_K) = 2(1+\gamma)(2P_K R)^{2\gamma-2}e^{\pi\alpha ZW_K/P_K}\left|\frac{\Gamma(\gamma+i\alpha ZW_K/P_K)}{\Gamma(2\gamma+1)}\right|^2 . \qquad (7)$$

[9] P.M. Endt, *At. Data and Nucl. Data Tables* **23**, 547 (1979).

In equation (7), $W_K=[P_K^2+1]^{1/2}$ is the total energy of the emitted electron. The Gamma functions in equation (7) can be calculated by using equations 6.1.3, 6.1.15, 6.1.18, and 6.1.25 of Abramowitz and Stegun[10]. These yield

$$\left|\frac{\Gamma(\gamma+i\alpha ZW_K/P_K)}{\Gamma(2\gamma+1)}\right|^2=\frac{\pi(\gamma+1/2)^2e^{2C(\gamma+1/2)}\left|\prod_{n=1}^{\infty}\left(1+\frac{(\gamma+1/2)}{n}\right)e^{-(\gamma+1/2)/n}\right|^2\prod_{m=0}^{\infty}\left[1+\left(\frac{\alpha ZW_K}{P_K(\gamma+m)}\right)^2\right]^{-1}}{2^{4\gamma}\gamma^2}, \quad (8)$$

where C=0.5772156649 is Euler's constant. Expressions for $A(E0)_{L1}$ and $A(E0)_{L2}$ may be derived from

$$\frac{A(E0)_K}{A(E0)_{L1}}=2\frac{P_K(W_K+\gamma)F(Z,P_K)S_K^2}{P_L(W_L+\gamma)F(Z,P_L)S_{L1}^2}\frac{X+1}{X+2}\frac{X^{2\gamma+2}}{2\gamma+1} \quad (9)$$

and

$$\frac{A(E0)_{L1}}{A(E0)_{L2}}=\frac{2+X}{2-X}\frac{X-1}{X+1}\frac{W_L+\gamma}{W_L-\gamma}\frac{S_{L1}^2}{S_{L2}^2}, \quad (10)$$

where X=$[2(1+\gamma)]^{1/2}$ and quantities with L subscripts are analogous to those above with K subscripts. Neglecting screening, $W_K=k+\gamma$ and $W_L=k+\frac{X}{2}$. The screening corrections S_K, S_{L1}, and S_{L2} were calculated by Brysk and Rose[11] and are shown in Figures 3, 4 and 5.

Table 3. The function *B(s)*

s	*B(s)*	s	*B(s)*
0.00	1.1781	0.52	0.9945
0.02	1.1722	0.54	0.9866
0.04	1.1661	0.56	0.9786
0.06	1.1599	0.58	0.9706
0.08	1.1536	0.60	0.9626
0.10	1.1472	0.62	0.9545
0.12	1.1408	0.64	0.9465
0.14	1.1342	0.66	0.9384
0.16	1.1275	0.68	0.9302
0.18	1.1207	0.70	0.9221
0.20	1.1139	0.72	0.9139
0.22	1.1069	0.74	0.9058
0.24	1.0999	0.76	0.8976
0.26	1.0928	0.78	0.8894
0.28	1.0856	0.80	0.8812
0.30	1.0784	0.82	0.8731
0.32	1.0710	0.84	0.8649
0.34	1.0636	0.86	0.8567
0.36	1.0562	0.88	0.8485
0.38	1.0487	0.90	0.8404
0.40	1.0411	0.92	0.8323
0.42	1.0334	0.94	0.8242
0.44	1.0257	0.96	0.8161
0.46	1.0180	0.98	0.8080
0.48	1.0102	1.00	0.8000
0.50	1.0024		

[10] M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards Applied Mathematics Series-**55**, (1964).

[11] N. Brysk and M.E. Rose, Oak Ridge National Laboratory, Report USAEC ORNL-1830 (1955).

Table 4. The Coulomb function *C(Z,k)*

	k										
Z	2.3	2.5	2.8	3.3	4	5	7	10	15	20	25
5	1.0172	1.0151	1.0137	1.0126	1.0118	1.0111	1.0102	1.0094	1.0084	1.0076	1.0071
10	1.0619	1.0569	1.0526	1.0487	1.0456	1.0428	1.0392	1.0356	1.0314	1.0284	1.0260
15	1.116	1.114	1.116	1.108	1.101	1.0950	1.0866	1.0781	1.0684	1.0613	1.0558
20	1.190	1.201	1.204	1.192	1.181	1.169	1.154	1.138	1.119	1.106	1.0961
30	1.393	1.458	1.466	1.448	1.431	1.402	1.362	1.320	1.272	1.239	1.213
40	1.732	1.877	1.921	1.902	1.852	1.799	1.709	1.616	1.514	1.443	1.390
60	3.153	3.819	4.066	4.054	3.849	3.594	3.213	2.825	2.427	2.173	1.991
80	7.861	10.80	12.07	11.93	11.02	9.671	7.831	6.181	4.656	3.788	3.219
100	31.39	48.71	56.18	54.29	48.63	37.94	26.49	17.70	11.00	7.774	5.916

Figure 3

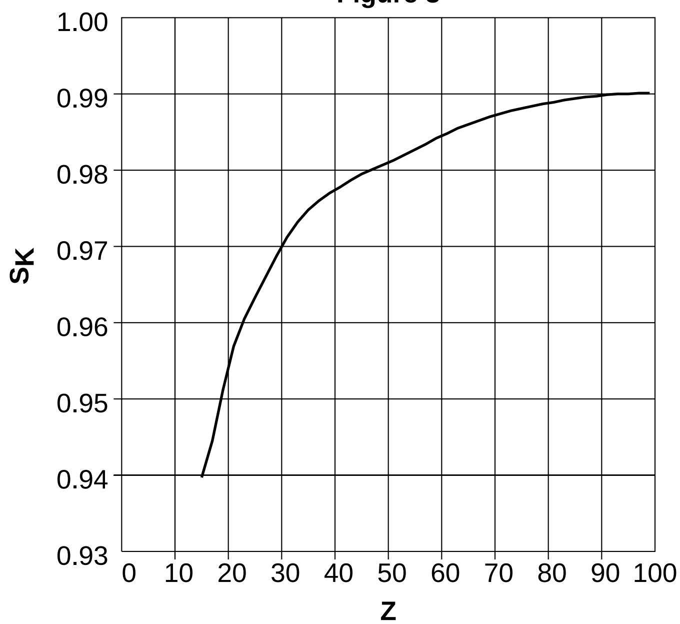

Figure 4

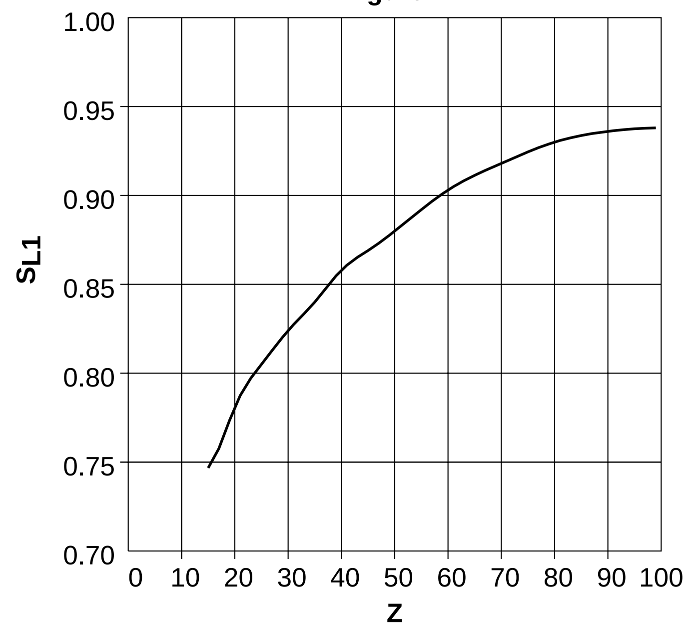

Figure 5