

Content

1	Introduction	1
1.1	Verification Methods	1
1.2	Methods to Determine the Internal Forces and Moments	2
1.3	Element Types and Fields of Application	4
1.4	Linear and Nonlinear Calculations	6
1.5	Designations and Assumptions	7
1.6	Fundamental Relationships	13
1.7	Limit States and Load Combinations	16
1.8	Introductory Example	19
1.9	Content and Outline	23
1.10	Computer Programs	24
2	Cross Section Properties	25
2.1	Overview	25
2.2	Utilisation of Symmetry Properties	29
2.3	Standardisation Part I: Centre of Gravity, Principal Axes and Moments of Inertia	31
2.4	Calculation of Standardised Cross Section Properties Part I	40
2.4.1	Separation of the Cross Section into Partial Areas	40
2.4.2	Partial Areas of Thin-Walled Rectangles	43
2.4.3	Basic Cross Sections and Elementary Compound Cross Section Shapes	46
2.4.4	Tabular Calculation of Cross Section Properties	51
2.4.5	Numeric Integration / Fibre and Stripe Model	53
2.5	Standardisation Part II: Shear Centre, Warping Ordinate and Warping Constant	58
2.6	Warping Ordinate	63
2.7	Shear Centre M	67
3	Principles of FEM	72
3.1	General Information	72
3.2	Basic Concepts and Methodology	72
3.3	Progress of the Calculations	78
3.4	Equilibrium	80
3.4.1	Preliminary Remarks	80
3.4.2	Virtual Work Principle	81
3.4.3	Principle of Minimum of Potential Energy	83

3.4.4	Differential Equations	84
3.5	Basis Functions for the Deformations	87
3.5.1	General	87
3.5.2	Polynomial Functions for Beam Elements	87
3.5.3	Trigonometric and Hyperbolic Functions for Beam Elements	91
3.5.4	Basis Functions for Plate Buckling	95
3.5.5	One-Dimensional Functions for Cross Sections	99
3.5.6	Two-Dimensional Functions for Cross Sections	103
4	FEM for Linear Calculations of Beam Structures	108
4.1	Introduction	108
4.2	Beam Elements for Linear Calculations	108
4.2.1	Linking Deformations to Internal Forces and Moments	108
4.2.2	Axial Force	110
4.2.3	Bending	113
4.2.4	Torsion	116
4.2.5	Arbitrary Stresses	120
4.3	Nodal Equilibrium in the Global Coordinate System	123
4.4	Reference Systems and Transformations	126
4.4.1	Problem	126
4.4.2	Beam Elements in the X-Z Plane	131
4.4.3	Beam Elements in a Three-Dimensional X-Y-Z COS	134
4.4.4	Loads	138
4.4.5	Warping Moment and Derivative of the Angle of Twist	139
4.4.6	Finite Elements for Arbitrary Reference Points	146
4.5	Systems of Equations	147
4.5.1	Aim	147
4.5.2	Total Stiffness Matrix	147
4.5.3	Total Load Vector	149
4.5.4	Geometric Boundary Conditions	151
4.6	Calculation of the Deformations	153
4.7	Determination of the Internal Forces and Moments	154
4.8	Determination of Support Reactions	156
4.9	Loadings	157
4.9.1	Concentrated Loads	157
4.9.2	Distributed Loads	157
4.9.3	Settlements	158
4.9.4	Influences of Temperature	159
4.10	Springs and Shear Diaphragms	159
4.11	Hinges	164

5	FEM for Nonlinear Calculations of Beam Structures	168
5.1	General	168
5.2	Equilibrium at the Deformed System	168
5.3	Extension of the Virtual Work	171
5.4	Nodal Equilibrium with Consideration of the Deformations	178
5.5	Geometric Stiffness Matrix	180
5.6	Special Case: Bending with Compression or Tension Force	185
5.7	Initial Deformations and Equivalent Geometric Imperfections	189
5.8	Second Order Theory Calculations and Verification Internal Forces	193
5.9	Stability Analysis / Critical Loads	201
5.10	Eigenmodes / Buckling Shapes	203
5.11	Plastic Hinge Theory	206
5.12	Plastic Zone Theory	210
5.12.1	Application Areas	210
5.12.2	Realistic Calculation Assumptions	210
5.12.3	Influence of Imperfections	213
5.12.4	Calculation Example	214
6	Solution of Equation Systems and Eigenvalue Problems	217
6.1	Equation Systems	217
6.1.1	Problem	217
6.1.2	Solution Methods	218
6.1.3	<i>Gaussian</i> Algorithm	219
6.1.4	<i>Cholesky</i> Method	220
6.1.5	<i>Gaucho</i> Method	220
6.1.6	Calculation Example	222
6.1.7	Additional Notes	224
6.2	Eigenvalue Problems	224
6.2.1	Problem	224
6.2.2	Explanations for Understanding	225
6.2.3	Matrix Decomposition Method	230
6.2.4	Inverse Vector Iteration	236
6.2.5	Combination of the Solution Methods	241
7	Stresses According to the Theory of Elasticity	245
7.1	Preliminary Remarks	245
7.2	Axial Stresses due to Biaxial Bending and Axial Force	247
7.3	Shear Stresses due to Shear Forces	250
7.3.1	Basics	250
7.3.2	Calculation Formula for τ	254

7.3.3	Open Cross Sections	255
7.3.4	Closed Cross Sections	260
7.4	Stresses due to Torsion	261
7.4.1	General	261
7.4.2	Arbitrary Open Cross Sections	264
7.4.3	Closed Sections	270
7.5	Interaction of All Internal Forces and Verifications	270
7.6	Limit Internal Forces and Moments on the Basis of the Theory of Elasticity	272
8	Plastic Cross Section Bearing Capacity	273
8.1	Effect of Single Internal Forces	273
8.2	Limit Load-Bearing Capacity of Cross Sections	275
8.2.1	Preliminary Remarks	275
8.2.2	Plastic Cross Section Reserves	277
8.2.3	Calculation Methods and Overview	281
8.3	Limit Load-Bearing Capacity of Doubly-Symmetric I-Cross Sections	288
8.3.1	Description of the Cross Section	288
8.3.2	Perfectly Plastic Internal Forces S_{pl}	289
8.3.3	Equilibrium between Internal Forces and Partial Internal Forces	291
8.3.4	Combined Internal Forces N , M_y , M_z , V_y and V_z	293
8.3.5	Interaction Conditions of DIN 18800 and Comparison with the PIF-Method	296
8.4	Computer-Oriented Methods	303
8.4.1	Problem Definition	303
8.4.2	Strain Iteration for a Simple Example	304
8.4.3	Strain Iteration for σ Internal Forces	307
8.4.4	Consideration of the τ Internal Forces	314
8.4.5	Examples / Benchmarks	317
9	Verifications for Stability and according to Second Order Theory	319
9.1	Introduction	319
9.2	Definition of Stability Cases	321
9.3	Verification according to Second Order Theory	323
9.4	Verifications for Flexural Buckling with Reduction Factors	329
9.4.1	Preliminary Remarks	329
9.4.2	Axial Compression	330
9.4.3	Uniaxial Bending with Compression Force	338
9.4.4	Modified Reduction Factors κ	340

9.5	Calculation of Critical Forces	342
9.5.1	Details for the Determination	342
9.5.2	Replacement of Structural Parts by Springs	348
9.5.3	Compression Members with Springs	352
9.6	Verifications for Lateral Torsional Buckling with Reduction Factors	360
9.6.1	Preliminary Remarks	360
9.6.2	Beams Not Susceptible to Lateral Torsional Buckling	360
9.6.3	Scheduled Centric Compression	362
9.6.4	Uniaxial Bending without Compression Force	364
9.6.5	Uniaxial Bending with Axial Compression Force	368
9.6.6	Reduction Factors according to Eurocode 3	369
9.6.7	Accuracy of Reduction Factors	373
9.7	Calculation of Critical Moments	375
9.8	Verifications with Equivalent Imperfections	381
9.8.1	Verification Guidance	381
9.8.2	Equivalent Geometric Imperfections	381
9.9	Calculation Examples	393
9.9.1	Single-Span Beam with Cantilever	393
9.9.2	Beam with Scheduled Torsion	396
9.9.3	Two Hinged Frame – Calculation in the Frame Plane	399
9.9.4	Two Hinged Frame – Stability Perpendicular to the Frame Plane	404
9.9.5	Frame Considering Joint Stiffness	413
10	FEM for Plate Buckling	420
10.1	Plates with Lateral and In-Plane Loading	420
10.2	Stresses and Internal Forces	420
10.3	Displacements	422
10.4	Constitutive Relationships	423
10.5	Principle of Virtual Work	425
10.6	Plates in Steel Structures	428
10.7	Stiffness Matrix for a Plate Element	429
10.8	Geometric Stiffness Matrix for Plate Buckling	432
10.9	Plates with Longitudinal and Transverse Stiffeners	434
10.10	Verifications for Plate Buckling	438
10.11	Determination of Buckling Values and Eigenmodes with FEM	448
10.12	Calculation Examples	451
10.12.1	Single Panel with Constant σ_x and $\alpha \approx 1.5$	451
10.12.2	Beam Web with Longitudinal Stiffeners	454
10.12.3	Web Plate of a Composite Bridge with Shear Stresses	457
10.12.4	Web Plate with High Bending Stresses	459

11	FEM for Cross Sections	461
11.1	Tasks	461
11.2	Principle of Virtual Work	464
11.3	One-Dimensional Elements for Thin-Walled Cross Sections	469
11.3.1	Virtual work	469
11.3.2	Element Stiffness Relationships	472
11.3.3	Equation Systems	474
11.3.4	Calculation of Cross Section Properties and Stresses	476
11.3.5	Compilation	479
11.4	Two-Dimensional Elements for Thick-Walled Cross Sections	480
11.4.1	Preliminary Remarks	480
11.4.2	Virtual Work for Thick-Walled Cross Section Elements	482
11.4.3	Element Geometry	484
11.4.4	Transformation Relationships	486
11.4.5	Stiffness Relationships	488
11.4.6	Numerical Integration	490
11.4.7	Cross Section Properties and Stresses	493
11.4.8	Performance of the Approximate Solutions	495
11.4.9	Special Case: Rectangular Elements	497
11.5	Calculation Procedure	501
11.6	Calculation Examples	503
11.6.1	Preliminary Remarks	503
11.6.2	Single-Celled Box Girder Cross Section	503
11.6.3	Bridge Cross Section with Trapezium Stiffeners	508
11.6.4	Rectangular Solid Cross Section	511
11.6.5	Doubly Symmetric I-Profile	518
11.6.6	Crane Rail	525
	References	528
	Index	534