

Contents

Notations	XI
1 Finite Elements Overview	1
1.1 Modeling Basics	1
1.2 Discretization Outline	3
1.3 Elements	7
1.4 Material Behavior	12
1.5 Weak Equilibrium and Spatial Discretization	13
1.6 Numerical Integration and Solution Methods for Algebraic Systems	17
1.7 Convergence	23
2 Uniaxial Structural Concrete Behavior	27
2.1 Scales and Short-Term Stress–Strain Behavior of Homogenized Concrete	27
2.2 Long-Term Behavior – Creep and Imposed Strains	34
2.3 Reinforcing Steel Stress–Strain Behavior	40
2.4 Bond between Concrete and Reinforcing Steel	42
2.5 The Smeared Crack Model	45
2.6 The Reinforced Tension Bar	47
2.7 Tension Stiffening of Reinforced Tension Bar	52
3 Structural Beams and Frames	55
3.1 Cross-Sectional Behavior	55
3.1.1 Kinematics	55
3.1.2 Linear Elastic Behavior	57
3.1.3 Cracked Reinforced Concrete Behavior	59
3.1.3.1 Compressive Zone and Internal Forces	59
3.1.3.2 Linear Concrete Compressive Behavior with Reinforcement .	61
3.1.3.3 Nonlinear Behavior of Concrete and Reinforcement .	65
3.2 Equilibrium of Beams	68
3.3 Finite Element Types for Plane Beams	71
3.3.1 Basics	71
3.3.2 Finite Elements for the Bernoulli Beam	72
3.3.3 Finite Elements for the Timoshenko Beam	75

3.4	System Building and Solution Methods	77
3.4.1	Elementwise Integration	77
3.4.2	Transformation and Assemblage	78
3.4.3	Kinematic Boundary Conditions and Solution	80
3.5	Further Aspects of Reinforced Concrete	83
3.5.1	Creep	83
3.5.2	Temperature and Shrinkage	86
3.5.3	Tension Stiffening	90
3.5.4	Shear Stiffness for Reinforced Cracked Concrete Sections	92
3.6	Prestressing	95
3.7	Large Deformations and Second-Order Analysis	101
3.8	Dynamics of Beams	108
4	Strut-and-Tie Models	115
4.1	Elastic Plate Solutions	115
4.2	Modeling	117
4.3	Solution Methods for Trusses	119
4.4	Rigid-Plastic Truss Models	125
4.5	More Application Aspects	131
5	Multiaxial Concrete Material Behavior	135
5.1	Basics	135
5.1.1	Continua and Scales	135
5.1.2	Characteristics of Concrete Behavior	136
5.2	Continuum Mechanics	138
5.2.1	Displacements and Strains	138
5.2.2	Stresses and Material Laws	139
5.2.3	Coordinate Transformations and Principal States	141
5.3	Isotropy, Linearity, and Orthotropy	143
5.3.1	Isotropy and Linear Elasticity	143
5.3.2	Orthotropy	144
5.3.3	Plane Stress and Strain	145
5.4	Nonlinear Material Behavior	147
5.4.1	Tangential Stiffness	147
5.4.2	Principal Stress Space and Isotropic Strength	148
5.4.3	Strength of Concrete	151
5.4.4	Phenomenological Approach for the Biaxial Anisotropic Stress–Strain Behavior	154
5.5	Isotropic Plasticity	157
5.5.1	A Framework for Multiaxial Elastoplasticity	157
5.5.2	Pressure-Dependent Yield Functions	161
5.6	Isotropic Damage	165
5.7	Multiaxial Crack Modeling	171
5.7.1	Basic Concepts of Crack Modeling	171
5.7.2	Multiaxial Smeared Crack Model	174
5.8	The Microplane Model	177

5.9	Localization and Regularization	180
5.9.1	Mesh Dependency	180
5.9.2	Regularization	182
5.9.3	Gradient Damage	186
5.10	General Requirements for Material Laws	190
6	Plates	193
6.1	Lower Bound Limit Analysis	193
6.1.1	The General Approach	193
6.1.2	Reinforced Concrete Contributions	195
6.1.3	A Design Approach	200
6.2	Crack Modeling	205
6.3	Linear Stress–Strain Relations with Cracking	209
6.4	2D Modeling of Reinforcement and Bond	213
6.5	Embedded Reinforcement	219
7	Slabs	221
7.1	A Placement	221
7.2	Cross-Sectional Behavior	222
7.2.1	Kinematic and Kinetic Basics	222
7.2.2	Linear Elastic Behavior	225
7.2.3	Reinforced Cracked Sections	226
7.3	Equilibrium of Slabs	228
7.3.1	Strong Equilibrium	228
7.3.2	Weak Equilibrium	230
7.3.3	Decoupling	232
7.4	Structural Slab Elements	234
7.4.1	Area Coordinates	234
7.4.2	A Triangular Kirchhoff Slab Element	235
7.5	System Building and Solution Methods	237
7.6	Lower Bound Limit Analysis	240
7.6.1	General Approach and Principal Moments	240
7.6.2	Design Approach for Bending	242
7.6.3	Design Approach for Shear	247
7.7	Kirchhoff Slabs with Nonlinear Material Behavior	250
8	Shells	255
8.1	Approximation of Geometry and Displacements	255
8.2	Approximation of Deformations	258
8.3	Shell Stresses and Material Laws	260
8.4	System Building	263
8.5	Slabs and Beams as a Special Case	264
8.6	Locking	266
8.7	Reinforced Concrete Shells	270
8.7.1	The Layer Model	270
8.7.2	Slabs as Special Case	272
8.7.3	The Plastic Approach	276

9 Randomness and Reliability	281
9.1 Basics of Uncertainty and Randomness	281
9.2 Failure Probability	283
9.3 Design and Safety Factors	291
A Solution of Nonlinear Algebraic Equation Systems	297
B Crack Width Estimation	303
C Transformations of Coordinate Systems	309
D Regression Analysis	313
E Reliability with Multivariate Random Variables	317
F Programs and Example Data	321
Bibliography	325
Index	333