1 Introduction to shells

\[ Z = p \cdot R \]
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Shells are naturally beautiful and efficient structures. The reason for this is that their flowing double-curved form is able to transfer loads without bending, by transmitting tension and compression forces solely within the surface. They therefore require significantly less material than flat structures under bending stress, as for instance beam or slab structures. There is however a discrepancy between favourable load-bearing behaviour and difficult double-curved construction. Solving this discrepancy is an important step towards successful shell design.

To design a transparent or glazed shell, it is necessary to fragment it into bars, thus creating a structure that offers maximum transparency. Double-curved surface structures with a triangular mesh net provide a favourable basis for optimal transparency. An essential condition for single-layer membrane shells is to transfer forces solely within the surface, and to do so without a substantial bar deflection. It is the triangular grid alone that is fit for this purpose.

The economic efficiency of transparent shells depends largely on the way the grid members are joined in the nodes, and on the shape of the cladding.

1.1 Designing shells

Today, high performance software facilitates the calculation of shells. These programmes offer easy to use geometry and load input functions, as well as a clear visualisation of results.

Nonetheless, a structural engineer needs sound theoretical knowledge of the load bearing behaviour of shells to make sure that the right decisions towards an aesthetic and efficient structure are made right from the early design stage. Although an incorrect structural concept can be made feasible by means of computing and determining the corresponding dimensions of load-bearing members, the result will be neither effective, nor innovative.

It is therefore vital to be in possession of adequate knowledge about the load bearing behaviour of shell structures.

The design should always strive towards a membrane; which is to say, a moment-free state. The basis therefore a continuous double-curved shape. Contrary to an arch, where a moment-free load transfer is only possible, if the geometrical form matches the type of load (thrust line, Fig. 1.1), a single shell form is able to transfer diverse loads moment-free. The support of the shells should be in accordance with the membrane theory. This means the thrust line is tangent to the shell surface and point loads on the shell surface must either be avoided or applied as a distributed load. The bending stresses arising from the compatibility conditions within the shell can be reliably determined by computer. In a membrane state the shell forces can easily be estimated. These simple estimation methods are of great importance for the design process, and to double-check computer results.

The membrane forces can easily be estimated for various rotation-symmetrically stressed shells, once the resultant load \( P_1 \) above the horizontal cut is known (Fig. 1.2).
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Thrust line for arches under dead load

\[ y = a \cdot \cosh \frac{x}{a} - a \] (catenary)

Thrust line for arches under uniformly distributed load

\[ x^2 = 2p \cdot y \] (parabola)

support forces:

\[ N_b = \frac{q \cdot L^2}{8 \cdot f}, \quad N_v = q \cdot \frac{L}{2} \] (1)

Meridian force \( n_\phi \) can easily be determined by double application of the ring formula \( Z = p \cdot R \).

The ring force \( n_\theta \) produces a deviation force \( u \) in outward direction of \( u = \frac{n_\phi}{R_\phi} \) and the ring formula \( Z = p \cdot R \) produces a ring force \( n_\theta = \frac{n_\phi}{R_\phi} \cdot R_\theta \).

The external load \( p \) induces a radial load component \( p_\theta \). The ring formula \( Z = p \cdot R \) gives us the ring force \( n_\theta = p_\theta \cdot R_\theta \) and therefore ring force

\[ n_\theta = -R_\phi \left( p_\theta + \frac{n_\phi}{R_\phi} \right) \] (3)
In the special case of a spherical shell under uniformly distributed load the radial load component is \( p_r = p \cdot \cos^2 \varphi \) and the resultant load is \( P_1 = \pi \cdot p \cdot R_y \cdot R_z \cdot \sin^2 \varphi \). Under dead load \( g \) the radial load component is \( p_r = g \cdot \cos \varphi \) and the resultant load is \( P_1 = 2\pi \cdot g \cdot R_y^2 \cdot (1 - \cos \varphi) \).

To obtain the membrane forces of the spherical shell \( p_r \) and \( P_1 \) are to be substituted in the equation above. These are compiled in Fig. 1.3. Regarding other cases, we refer to the corresponding literature.

If the support of a shell does not comprise optimum membrane supports, if only vertical support forces can be absorbed for example; a stiff edge beam can create a membrane-like state that permits much lower bending moments within the shell that diminish quickly. Hoop forces can easily be determined in manual calculations, as long as the edge beam is formed as a ring (Fig. 1.4).
A distinctive characteristic of a ring beam under carding moment \( m = q \cdot e \) is that the stress resultant mobilised within the ring is not a torsion, but a bending moment \( M \), which is distributed into tensile and compressive hoop forces [13]. This characteristic has been implemented several times in curved footbridges [14].

Due to the equilibrium of forces, a resulting transverse force \( Q \) applied to the ring beam via radial forces \( n = \pi \cdot \cos \varphi \) induces the shear forces \( t = \pi \cdot \sin \varphi \) (Fig. 1.5).

\[
\sum Q = 0 \text{ results in: } \\
\int_0^{2\pi} \pi \cdot \cos^2 \varphi \cdot R \cdot d\varphi = \int_0^{2\pi} \pi \cdot \sin^2 \varphi \cdot R \cdot d\varphi \\
\pi \cdot R \cdot \pi = \pi \cdot R \cdot \pi = Q \\
\max t = \bar{t} = \frac{Q}{\pi \cdot R} \\
\text{maximum shear force } \bar{t} \text{ from } Q. \tag{8}
\]