

## Index

### **a**

- additive functional 18
- adjacency system 49
  - complementary 59
  - local 49
  - self-complementary 59
- Alexander's horned sphere 133
- aliasing 68
- arc 132
- auto-correlation function 35, 197

### **b**

- background 46
- ball,  $n$ -dimensional 13
- Bartlett spectrum 198, 217
- basis of a lattice 44
- binarization 120
- bisector discrete 120
- Bochner theorem 41, 216
- body 13
- body force 289
- Boolean cylinder model 251
- Boolean model 215, 243
  - structure model index 250
- boundary condition 290
- boundary of a set 13
- bounded
  - essentially 34
  - set 13
- breadth, mean 20

### **c**

- calliper diameter, mean of 20
- Campbell theorem 33
- capacity functional 28, 82
- cardinal number 12
- cardinality 12
- Cauchy formula 161
- Chamfer distance transform 113

### Choquet

- capacity 28
- theorem 28

### Choquet capacity 168

- chord length distribution 171, 192
- closed

- morphologically 13
- topologically 13

### closed foam 270

### closure

- algebraic 90
- morphological 87, 103
- topological 13

### compact set 13

- complementary adjacency systems 59
- component

- connected 131
- path-connected 130

### conductivity

- effective 282
- tensor 286

### connected

- set 129

### connected component 131

### connectedness 128–129

### connectivity 49

### constitutive relation of linear elasticity 289

### contact distribution

- function 191
- linear 191
- spherical 190–191

### convex

- hull 14
- ring 14
  - extended 14
- set 14

### convolution 94

- of a measure 35
- of functions 35

- cosine transform 180
- counting measure 30
- covariance function
  - exponential 200
  - of a random set 197
- covariance measure 199, 214
  - reduced 199
- Crofton formula 26
- Crofton formulae 152
- cross-correlation 208
  - function 209
  - measure 208
  - of functions 35
  - spatial 208
- cross-covariance function 209
- curvature 15
  - Gaussian 15
  - Germain's 15
  - mean 15
  - total 15
- curve plane 132
  
- d**
- density
  - of a random point field 31
  - of intrinsic volume 29
  - of the Euler number 29
  - of the integral of Gaussian curvature 29
  - of the integral of Germain's curvature 29
  - of the integral of mean curvature 29
  - of the integral of total curvature 29
- density measure 31
- diameter 156
  - maximum 14
- difference, of sets 12
- diffusion filter 105
- digitization
  - Gauss 45–46, 65
  - Jordan 45, 65
- dilation 84, 103
  - geodesic 110
  - reconstruction by 110
- Dirac delta function 39
- directional distribution 32
- Dirichlet boundary condition 290, 295
- discrete bisector 120
- discretization of a set 51
- displacement
  - field 289
  - virtual 290, 294
- distance transform 111
  - Chamfer 113
  - discrete 113
  - Euclidean 114
  
- e**
- edge detection filters 98
- edge of a set 13
- EDT 114
- elasticity matrix 289
- element shape function 292
- elementary neighbourhood 108
- emission 70
- empty set 11
- energy 295
- equilibrium equation of linear elasticity 289
- equivalence relation 130
- erosion 85, 103
  - geodesic 108
  - reconstruction by 108
- Euclidean distance transform 114, 169
- Euclidean norm 11
- Euclidean skeleton 117
- Euclidean space 11
- Euler number 18, 52
  - density 29
  - local 58
  - specific 61
- Euler–Poincaré characteristic 18
- Euler–Poincaré formula 18, 52
  
- f**
- Fast Fourier Transform (FFT) 80
- FEM 291
- Ferret diameter 156
  - maximum 14
  - mean 20
- fibre system 251
- Fick's second law of diffusion 105
- field features 149
- filling 128
- filter
  - diffusion 105
  - edge detection 98
  - Gauss 97
  - gradient 98
  - Hermitean 94
  - isotropic 94
  - Laplace 101
  - mean value 96
  - median 104
  - norm-preserving 94
  - quantile 104
  - rank value 105
  - Sobel 100

- filter mask 96  
finite element method 291  
fluctuation 199  
flux 286  
foam
  - ceramic 276
  - closed 270
  - Kelvin 265
  - open 61, 267, 276
  - Weaire–Phelan 265
force biased algorithm 239  
foreground 46  
Fourier–Bessel transform 37, 201, 204, 217  
Fourier co-transform 36  
Fourier–Stieltjes transform 41  
Fourier transform 36, 216
  - continuous 36
  - inverse 36
frame 29  
function
  - essentially bounded 34
  - integrable 34
  - measurable 34
functional
  - additive 18, 23
  - invariant under rigid motions 22
  - $k$ -homogeneous 23
fundamental form
  - first 16
  - second 16
Funk–Hecke theorem 181
- g**  
gauge body 82  
Gauss
  - digitization 45–46, 65
  - distribution 39
  - filters 97
Gauss–Bonnet formula 19  
Gaussian curvature 15
  - integral of 19
geodesic
  - h-minima transform 111
  - height adaptive h-minima transform 111
geodesic dilation 110  
geodesic erosion 108  
Germain’s curvature 15
  - integral of 21
global thresholding 121  
gradient 37
  - morphological 103
gradient filter 98
- granulometry 89
  - distribution 192
  - spherical 89
- h**  
h-minima transform 111
  - adaptive 111
Hadwiger’s recursive formula 20  
Hadwiger’s theorem 23  
Hankel transform 37, 217  
hard-core point field 235–236  
hard-core Voronoï tessellation 261  
Hausdorff
  - distance 13
  - measure 17
  - metric 13
height adaptive h-minima transform 111  
Hermitean filter 94  
hit-or-miss transform 82  
homogeneous
  - lattice 44
  - macroscopically 27
homotopic thinning 117  
Horvitz–Thompson procedure 164  
hyperplane
  - process 32
  - tessellation 34
- i**  
image 43, 64
  - binary 64
  - Boolean 64
  - grey-tone 64
inclusion-exclusion principle 18, 24  
independent random closed sets 27  
index function 24, 169  
indicator function 12  
inner product 11  
integrable function 34  
integral of Gaussian curvature 19
  - density 29
integral of Germain’s curvature 21
  - density 29
integral of mean curvature 21, 150, 158
  - density 29
integral of total curvature 150
  - density 29
intensity 31
  - measure 31
interior of a set 13  
intersection of sets 12  
intrinsic volume 22, 29, 149
  - density 29

- inverse lattice 65
- isotropic
  - filter 94
  - random closed set 27
- j**
- Jordan
  - arc 132
  - curve 132
  - digitization 45, 65
  - surface 133
- Jordan–Brouwer
  - complementarity 134
  - surface theorem 133
  - theorem 134
- Jordan–Veblen curve theorem 132
- k**
- $k$ -face
  - typical, of a tessellation 257
- $k$ -homogeneous functional 23
- Kelvin's foam 265
- kinematic formulae 25
- l**
- label propagation 138
- labelling 128
- labelling algorithm 135
- Laguerre
  - cell 264
  - tessellation 261
- Lamé's modulus 290
- Laplace filter 101
  - operator 37
  - smoothed 101
- lattice
  - basis 44
  - body centred cubic 45
  - cell 44
  - cubic primitive 43, 45
  - distance 43
  - face centred cubic 45
  - homogeneous 44
  - inverse 65
  - orthorhombic primitive 43
  - reciprocal 65
- Lebesgue measure 11
- linear contact distribution 191
- load tensor, global 292, 294
- locally finite measure 17
- LSE-filter 94
- m**
- macroscopically homogeneous 27
- marching cube algorithms 73
- mark space 31
- marked random point field 31
- marker image 108
- mask image 108
- Matérn hard-core model 235
- maximum diameter 14
- maximum filter 103–104
- mean breadth 20
- mean chord length 170
- mean curvature 15
  - integral of 21, 158
- mean intercept length 170
- mean value filter 96
- mean width 20
- measurable function 34
- measure
  - locally finite 17
  - signed 17
  - totally finite 17
- medial axis 117
- median filter 104
- mesh simplification 77
- Metropolis–Hastings algorithm 237
- MIL 170
- Miles' formulae 248, 251
- Miles–Lantuejoul correction 164
- minimum filter 103–104
- Minkowski
  - addition 12
  - additivity 18
  - difference 12
  - subtraction 12
  - sum 12
- Minkowski functional 24
- modulation 37
- Moiré effect 68
- morphological gradient 103
- morphologically
  - closed 13
  - open 13
  - regular 13
- multi-grid convergence 61
- multiple point of an arc 132
- n**
- nabla operator 37
- neighbourhood
  - elementary 108
  - graph 49
- non-woven 63
- norm-preserving filter 94
- normal strain 289

- normal stress 289  
 Nyquist–Shannon sampling theorem 67
- o**  
 object feature 22, 149  
 Ohm's law 283  
 opacity 70  
 open  
   – morphologically 13  
   – topologically 13  
 open ceramic foam 276  
 open foam 61, 267  
 opening  
   – algebraic 90  
   – morphological 87, 89, 103  
 Otsu's threshold 122
- p**  
 pair correlation function 224  
 pairwise interaction point field 236  
 Parseval identity 36, 216  
 path-connected component 130  
 perfect simulation 238  
 pixel 64  
   – density 44  
 pixel spacing 44  
 Plancherel identity 36  
 plane curve 132  
 point field  
   – process 31  
   – random 31  
 Poisson field of planes 215  
 Poisson–Laguerre tessellation 264  
 Poisson line field 251  
 Poisson point field 234  
 Poisson–Voronoi tessellation 260  
 Poisson's ratio 289  
 poly-convex 14  
 polytope 14  
 pores per inch 268  
 power spectrum 201  
 ppi-value 268  
 principal kinematic formula 25, 166  
 principle of virtual work 290  
 probability space 17  
 proper orthogonal group 12
- q**  
 quantile filter 104
- r**  
 radiative transfer 69  
 random closed set(s)  
   – independent 27
- isotropic 27  
 – macroscopically homogeneous 27  
 random packing 239  
   – force biased 239  
   – random sequential addition 239  
   – spatial sequential inhibition 239  
 random point field 31  
   – finite 235  
   – hard-core 236  
   – marked 31  
   – Matérn hard-core 235  
   – pairwise interaction 236  
   – Poisson 234  
   – simple 31  
   – simulation 237–238  
   – Strauss 236
- random sequential addition 239  
 random tessellation  
   – hard-core Voronoï 261  
   – Laguerre 261  
   – Poisson–Laguerre 264  
   – Poisson–Voronoi 260  
   – Voronoï 260
- rank value filter 105  
 ray casting 71  
 reciprocal lattice 65  
 reconstruction by dilation 110  
 reconstruction by erosion 108  
 reconstruction, self-dual 110  
 reflection of a set 12  
 region detection 128  
 region growing 127  
 region of interest 29, 64, 149  
 regular  
   – morphologically 13  
   – topologically 13  
 rendering  
   – direct methods 69  
   – indirect methods 72  
 representative volume element 282, 294  
 rigid motion 12  
 Robin's boundary condition 283  
 ROI 29, 149  
 rose of normal directions 180  
 Rosenfeld–Pfaltz algorithm 135–136  
 rotation group 12
- s**  
 sampling 46, 65  
   – rate 44  
   – theorem 67  
   – window 29  
 scalar product 11

- Schwartz space 35
  - Schönflies–Brouwer theorem 132
  - Schönflies problem 133
  - segment 12
  - self-dual reconstruction 110
  - separability 38, 82, 95, 114
  - separable mask 90
  - set
    - boundary 13
    - bounded 13
    - compact 13
    - connected 129
    - convex 14
    - difference 12
    - discretization 51
    - edge 13
    - empty 11
    - interior 13
    - intersection 12
    - morphologically closed 13
    - morphologically open 13
    - morphologically regular 13
    - poly-convex 14
    - random closed 27
    - reflection 12
    - simply connected 132–133
    - surface 13
    - topologically closed 13
    - topologically open 13
    - topologically regular 13
    - translation 12
    - union 12
  - shape factor 162
  - shear strain 289
  - shear stress 289
  - shift-equivariant 83, 94
  - sieving distribution 192
  - signed measure 17
  - simple
    - counting measure 30
    - random point field 31
  - sinc function 39
  - sinusoid 130
  - skeletonization 116
    - by thinning 117
    - using distance transforms 117
  - SMI 171
  - Sobel filter 100
  - spatial birth-and-death process 238
  - spatial sequential inhibition 239
  - special orthogonal group 12
  - specific surface area 29
  - spherical contact distribution 190–191
  - spherical granulometry 89
    - distribution 192
  - Steiner formula 22, 24–25, 29
    - extended 166
  - stiffness tensor 295
    - global 292, 294
  - stochastic homogenization 283
  - strain
    - effective 295
    - energy 294
    - field 289
    - normal 289
    - rate 291
    - shear 289
    - virtual 291
  - Strauss point field 236
  - stress
    - field 289
    - normal 289
    - shear 289
  - structure model index 171
  - structuring element 82
  - subspace, linear 17
  - support function 20, 153
  - support of a function 34
  - surface
    - area 18, 150
    - covariance function 214
    - density 29, 213
    - measure 17, 213
      - random 213
    - of a set 13
    - rendering 72
  - surface area 158
- t**
- tessellation 33
    - admissible 58
    - face-to-face 33, 58
    - normal 34
    - random 34
  - texture mapping 72
  - thinning, homotopic 117
  - threshold
    - isodata 122
    - Otsu's 122
  - thresholding 121
    - double 125
    - global 121
    - hysteresis 125
    - local 123
    - Niblack's 123
  - top-hat transform 89

- topological closure 13
- topologically
  - closed 13
  - open 13
  - regular 13
- total curvature 15
- totally finite measure 17
- traction 290
  - boundary condition 290
- transfer function 70, 95
- transform
  - adaptive h-minima 111
  - distance 111
  - h-minima 111
- translation of a set 12
- transmission condition 290
- transparency 70
- typical grain 243
- typical  $k$ -face 257
  
- u***
- union of sets 12
- unit cell of a lattice 44
  
- v***
- velocity, virtual 290
- virtual
  - displacement 290, 294
  
- w***
- Weaire–Phelan foam 265
- Whittaker–Shannon interpolation 66
- width 156
- width, mean 20
- Wiener–Khintchine theorem 202, 225
- window 64, 149
- window function 202
- work equation of linear elasticity 290
- wrapper algorithm 75
  
- y***
- Young's modulus 289
  
- z***
- zonotope 14