1
Gas Physics and Vacuum Technology

1.1
The term "vacuum"

In standard specification list DIN 28400, Part 1, the term "vacuum" is defined as follows:

Vacuum is the state of a gas, the particle density of which is lower than the one of the atmosphere on the earth's surface. As within certain limits the particle density depends on place and time, a general upper limit of vacuum cannot be determined.

In practice, the state of a gas can mostly be defined as vacuum in cases in which the pressure of the gas is lower than atmosphere pressure, i.e. lower than the air pressure in the respective place.

The correlation between pressure (p) and particle density (n) is

$$p = n \cdot K \cdot T \tag{1-1}$$

k Boltzmann constant
T thermodynamic temperature

Strictly speaking, this formula is valid only for ideal gases.

The legal pressure unit is Pascal (Pa) as SI unit. The usual pressure unit in vacuum technology is millibar (mbar). This pressure unit is valid for the whole vacuum range from coarse vacuum to ultrahigh vacuum.

1.2
Application of vacuum technology

Vacuum is often used in chemical reactions. It serves to influence the affinity and therefore the reaction rate of the phase equilibrium gaseous – solid, gaseous – liquid and liquid – solid. The lowering of the pressure causes a decrease in the reaction density of a gas. This effect is used e. g. in the metallurgy for the bright-annealing of metals. There are several kilograms of metal for 1 liter annealing space, whereas less than 1/3 of the total volume is filled with gas; as a result, the oxygen content of

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the residual air is less than 1 mg in the pressure range of 10 mbar. Compared to the metal mass, the oxygen content decreases to 10^{-5}. This leads to a retardation of the oxidation process, thus allowing higher process temperatures. It also causes an increase in the ductility of the products. When teeming melted materials, such as metals, apart from a retarded oxidation also degassing (desorption) takes place at the same time. The result is metal of particular purity. In the metal and sinter ceramics industry, sintering processes are based on the same principle. The impediment of fermentation caused by aerobe micro-organisms with the help of vacuum can also be called reactive retardation, an example of which is vacuum packaging. On the other hand, a reactive acceleration is reached, e. g. when after the evacuation of the materials to be treated, gases or liquids are discharged in order to increase the reaction density. The reaction density can also be controlled as required by means of a decrease in pressure, e.g. when chlorinating. In this case, diluting gases are not required.

The selection of the adequate technology for a chemical-physical process depends on pressure-related parameters and the specific characteristics of the material to be treated. This requires e.g.

- the determination of the optimal ranges of vacuum and temperature,
- the determination of the required equipment,
- the determination of all necessary auxiliary means, which vacuum pumps or vacuum devices belong to.

The dimensions of a vacuum plant are not only determined by the performance data of the process devices, but also by the operating range of the vacuum. In the range of high vacuum the sizes of the individual devices are not as important for the dimensions of the total plant as the required suction capacity and the sizes and dimensions of the vacuum pumps, i.e. the vacuum pump stations.

Generally, in vacuum process engineering of the chemical industry or related branches vacuum plants usually consist of the following main components:

- Vacuum devices for the execution of the process
- Condensation devices for the compression of the arising vapor
- vacuum pump or combination of pumps
- accessories, such as separators, heat exchangers, vacuum vessels, metering and control devices.

1.2.1
Basic operations in process engineering

In the industrial process engineering, basic operations are usually carried out in coarse vacuum, seldom in fine vacuum. The application of high vacuum is considered only in particular cases.

The machines used here are vacuum pumps and compressors. With lower vacuums and higher flow rates mostly extractor fans are used.

Regarding the use of waste heat and the careful heating of thermally sensitive material it is advantageous for the performing of the vacuum process to work at low temperatures. The most different processes are carried out through vaporizing, drying, condensing, degassing, filtering etc. under vacuum. Generally, it can be said that the total operating costs of vacuum plants increase with higher vacuum.

Mechanical vacuum pumps can be designed as dry or wet running pumps with pistons or rotating elements.

Dry running vacuum pumps are used for pumping dry and non-condensable gases. In case of existing condensable vapors, condensers have to be installed on the suction side in which the condensable particles are condensed through cooling. In the field of coarse vacuum, usually surface condensers or mixing condensers are used, while low-temperature condensers or absorption condensers are used in the fine vacuum range.

Wet running vacuum pumps are particularly suitable for the suction of condensable vapors or gases, as well as for mixtures of gases and liquids. In wet running vacuum pumps driven by an operating liquid (e.g. water or another liquid chosen according to the process), the process gas can be condensed. Owing to this fact, condensers installed on the suction side of the pumps are not required. The diagram of the basic layout of a vacuum device is shown in fig. 1-1.

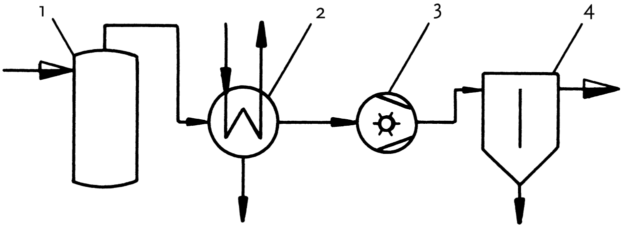


Figure 1-1. Basic scheme of a vacuum plant
1 vacuum vessel
2 condenser
3 liquid ring vacuum pump
4 liquid separator

1.2.2
Basic fields and worked-out examples for the application of vacuum technology

Vacuum technology is dominant in many fields of research and industry (Table 1-1) and is applied by using the most different process technologies (Table 1-2).

Table 1-1. Fields of application of vacuum technology [1.1]

Field of knowledge	Branch of industry/technology
Physics (mechanics, continuum mechanics, thermodynamics, electrodynamics, optics, nuclear physics, surface physics- and chemistry)	Scientific instrument production (precision mechanics) Mechanical engineering and heavy engineering industry Electronics (for measuring and control problems) Automation and controlling Cryogenic engineering
Biophysics	Chemical process engineering (oils, greases, waxes, resins etc.)
Physical chemistry	Metallurgy
Chemistry	
Material engineering	State-of-the-art technologies (glass, ceramics and metallic compounds)
Pharmacy	
Medicine	

Field of application	Examples
Nuclear technology	Crystal growing (scintillation detectors) Evaporation (solid-state detectors) Working with closed systems (hot laboratories, plutonium, etc.) Filtration Sintering under vacuum (nuclear metals, ceramics, carbide)
Optical industry	Vaporization technologies (interference layers, laser, maser, glass fiber optics, optoelectronics)
Electrical engineering/electronics	Drying (insulation oils, coolants) Impregnation (insulation material) Hermetic sealing (boosters) Evacuation and degassing (e.g. tubes, lamps) Evaporation and sputtering (e.g. condenser production, thin-film technology) Encapsulation (tubes, semiconductor elements) Welding and surface treatment (micro-circuits) Crystal growing (epitaxial growth) Surface reactions (transistors, circuits)
Scientific instrument production	Physical and chemical analyses Analyzing appliances (surface analysis, UV examination, electron and ion microscopes, X-ray analyzers, microwave devices) Lowest temperature analyses Particle accelerators, storage rings Fusion plants
Chemical industry	Distillation (fatty acids, oils, alcohol, etc.) Filtration Drying, dehydration vaporization, sublimation

Field of application	Examples
Food industry	Freeze-drying (fresh and cooked food) Preservation and conservation Dehydration and concentration (milk, coffee ...) Crystallization (e.g. sugar)
Pharmaceutical industry	Distillation (vitamin A, E, ...) Freeze-drying (blood, ...) Drying (antibiotics, hormones, ...) Sterilization (dressing materials, ...)
Metallurgy and semiconductor manufacturing	Distillation (Mg, Ca, Li, Se, Na, K, ...) Reduction (Ti, Mg, Zr, Fe,Cr, ...) Sintering (high-melting and reactive metals, carbides, ...) Melting and casting (Pb, Sn, Mn, Ge, alloys, high-melting and reactive metals) Drying (powder) Heat treatment
Production engineering	Impregnation (molds for casting) Injection molding (Mg alloy components without voids) Fastening (chucks) Welding and soldering (precision devices) Surface finish (hard material or anticorrosion coating)
Space engineering	Biological processes and developments Material development and control Development and control of devices (motors, gauges, ...)
Office machines industry	Welding and treatment Registration Heat insulation
Transportation in various industry branches	Lifting and transporting (paper, metal sheets, pavement plates, cathode ray tubes, ...)
Miscellaneous applications	Evaporation (paper, plastics, fabrics, ...) Thermal insulation (Dewar flasks, ...) Forming (plastics, vacuum casting) Concrete hardening

1.2.3
Overview of the most important vacuum processes

Tab. 1-2 contains processes preferably carried out under vacuum.

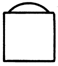
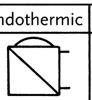
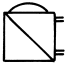
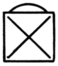
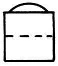
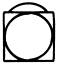
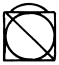
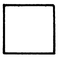
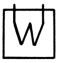
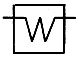
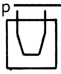
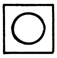
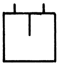
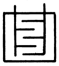
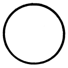
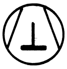
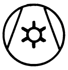
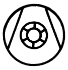
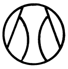
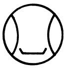
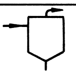
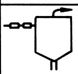
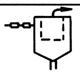
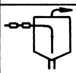
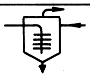
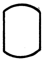
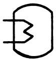
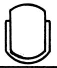
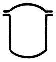
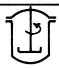
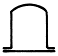
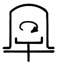
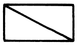
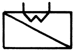
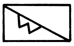
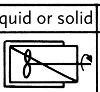
Table 1-2. Vacuum processes in process engineering [1.2]

Process	Important advantages through vacuum
Endothermic processes	
Vacuum vaporization	Low temperature of material and heating agent Increased heat efficiency
Vacuum distillation	Better separation effect, molecular distillation; Oxide-free and gas-free metal distillation
Vacuum sublimation	Under triple point (freeze-drying)
Vacuum drying	Quick and careful drying without shrinking; increased dissolving speed
Vacuum calcination	Shifting of phase equilibrium, decomposing temperature drops
Vacuum annealing and sintering	Bright annealed products are free from oxides, gases and scale
Vacuum melting	Gas-free melted product, high-purity metals, chemicals, plastics, sealing compounds
Vacuum casting	Non-porous cast products with high density
Vacuum soldering	Furnace soldering without flux, oxide-free hard soldering
Vacuum evaporation	Surface finish through vapor deposition of thin films of metals and non-metals
Vacuum reaction	Thermal conversion at low temperatures and decreased reaction density
Vacuum steam generation	Water vapor heating below 100 °C, rapid control
Processes without catalytic oxidation	
Vacuum degassing	Gas-free liquid, viscose, plastic masses
Vacuum gas injection	Fumigation, disinfection, sterilisation, sorption
Vacuum mixing	Modified sorption, improved wettability
Vacuum extraction	Higher dissolver speed, dissolver recovery
Vacuum filtration	Continuous residue decreasing
Vacuum impregnation	Complete impregnation of porous bodies, agglutination
Vacuum transport	Fluidized bed transport of bulk materials by means of induced draught
Vacuum insulation	Thermo-barochamber
Vacuum packaging	Improved shelf life, no aroma losses
Exothermic processes	
Vacuum condensation	Distillate recovery, higher energy yield
Vacuum cooling	Ice generation without coolants
Vacuum crystallization	Higher crystal yield through flash distillation of solvents
Vacuum reaction	Higher distribution rate, low reaction density
Vacuum presses	Non-porous agglomeration or agglutination of powders and laminates

1.2.4
Basic designs of apparatus for mass transfer and mass combination

The most important vacuum processes applied in process engineering are given in Table 1-3. They are subdivided according to thermal processes and grouped together according to the apparatus equipment.

Table 1-3. Basic symbols, apparatus and process technique in vacuum engineering [1.2]

Component	Basic symbol	Symbol range					
Vacuum apparatus							
		Mass transfer				Mass combination	
	Heat tone	endothermic	endothermic	exothermic	without	without	endothermic
							
	Process	evaporating distilling deodorising	drying subliming calcining	degassing cooling crystallising	filtering extracting	agglomerating shaping impregnating	sintering, soldering, melting, casting, coating
Condensers							
	Condenser	liquid	liquid and solid		solid	liquid	dissolved
	Cooling fluid	through pipes	around pipes	Condensation trap	rotat. cooler	Injection	Absorption
							
	Process	Surface condensation with condensate separation				Co-condensation	
Pumps							
		Piston	Rotor	Rotor	Rotor	Motive fluid	Motive fluid
							
	Process	Positive displacement vacuum pump oscillating	rotating	rotating	Kinetic vacuum pump rotating		
Separators							
	Fluid	liquid	solid	solid	solid	gaseous	
							
	Process	Droplet separation	Dust separation by Gravity	Filter	Submerged washing	Circulatory washing	
Vacuum vessels							
	Fluid	liquid	liquid	liquid or solid	liquid or solid	solid	solid
							
		Heating, cooling coil	Heating, cooling jacket	with cover opening	with stirrer	Bottom opening	Bottom opening with stirrer
Heat exchangers							
	Fluid	liquid	liquid	liquid or solid			
							
		Tube system	Tube coil	Double shell with stirrer			

1.2.5

Limits to the application of vacuum in process engineering

In the field of vacuum, the kind of gas flow depends on the respective prevailing vacuum.

According to the Hagen-Poisseuille law, laminar gas flow exists in coarse vacuum. In the range of high vacuum, the internal friction is no longer decisive, as the collision of the molecules and the tube wall occurs more often than the collision among the molecules themselves. This kind of flow is called Knudsen molecular flow, i.e. the average molecular speed and the mean particle path of the gas molecules determine the flow process. The range between coarse and high vacuum is called fine vacuum. The fine vacuum range is the transition zone between the Hagen-Poisseuille flow and the Knudsen flow. The range of vacuums higher than in high vacuum is called ultrahigh vacuum.

According to the Knudson equation

$$K = \frac{\bar{l}}{d} \tag{1-2}$$

the different types of flow are subdivided as shown in table 1-4.

Table 1-4. Flow types in vacuum [1.3]

K	< 0.5	0.5 – 3.0	> 3.0
Type of flow	Hagen-Poiseuille flow	Transition zone	Knudsen molecular flow

K	Knudsen number
$\bar{l}$	mean free path [m]
d	diameter of the flow channel [m]

Therefore, for the type of flow arising in tubes, the ratio of the mean free path (a gas molecule does on average until its collision with another molecule), which increases with decreasing pressure and the diameter of the flow channel is decisive.

Material transport. With the increasing vacuum, the transport of gases and vapors gets more and more difficult. This is a result of the fact that with decreasing pressures the available forces diminish and the volumes increase. With pressures lower than 0.1 kPa (= 1.0 mbar), in practice only insignificant quantities of gas and vapor are transported in pipes.

Heat transport. Only in the range of atmospheric pressure heat transfer through convection is technically applicable, whereas high vacuum is a good heat insulator. In vacuum processes, the heating-up of the material occurs practically only in direct contact with heating elements through radiation, rarely through dielectric heating or inductive heating.

1.3
Operating ranges and measuring ranges of vacuum

Vacuum ranges are ranges of pressures or particle densities according to which it is agreed to classify vacuum.

The rounded down limits of these ranges are listed as pressure values or equivalent particle density values in Tab. 1-5.

The particle density values given in the table apply to a temperature of $\delta = 20\,°C$.

1.3.1
Vacuum pressure ranges

Table 1-5. Vacuum ranges (acc. to DIN 28400, Part 1, July 1979)

Formula character	Unit	Coarse vacuum, CV	Fine vacuum, FV	High vacuum, HV	Ultrahigh vacuum, UHV
P	[Pa]	$1 \cdot 10^{15}$ to $1 \cdot 10^{2}$	$1 \cdot 10^{2}$ to $1 \cdot 10^{-1}$	$1 \cdot 10^{-1}$ to $1 \cdot 10^{-5}$	$< 1 \cdot 10^{-5}$
P	[mbar]	$1 \cdot 10^{3}$ to 1	1 to $1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$ to $1 \cdot 10^{-7}$	$< 1 \cdot 10^{-7}$
n	[m^{-3}]	$2.5 \cdot 10^{25}$ to $2.5 \cdot 10^{22}$	$2.5 \cdot 10^{22}$ to $2.5 \cdot 10^{19}$	$2.5 \cdot 10^{19}$ to $2.5 \cdot 10^{15}$	$< 2.5 \cdot 10^{15}$

It seems to suggest itself to divide measuring ranges in decimal powers, as follows:

Millibar range	from 1000 to 1 mbar, essentially the normal and coarse vacuum range
Microbar range	from 1 to 10^{-3} mbar, the fine vacuum range
Nanobar range	the high vacuum range
Picobar range	and below, the ultrahigh vacuum range

1.3.2
Vapor pressure curve of water in vacuum

For vacuum process engineering with a prevailing thermal mass transfer, it is practical and clearer to divide into vacuum operating ranges following the thermometric fixed points of water as so-called fundamental material which the chemists, process engineers and technicians have to deal with every day (Fig 1-2). According to this, in the boiling range of pure water between 0 and 100 °C corresponding to 6.11 mbar to 1013mbar, the normal or basic vacuum range results, in which the boiling process always occurs as pure vaporization.

Processes with lower pressures at which vaporization takes place through sublimation from the solid phase (ice) below 0 °C are to be allocated to the fine or high vacuum range.

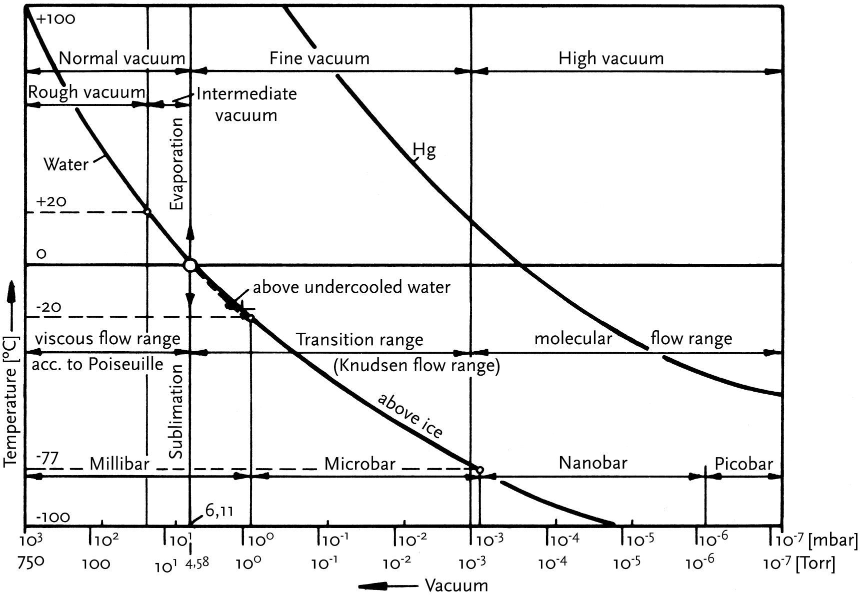


Figure 1-2. Vapor pressure curve of water in different vacuum ranges [1.2]

1.3.3
Vacuum operation ranges, temperature pressure table

In Tab. 1-6, vacuum operating and measuring ranges in millibar and Torr are compared to the specific boiling points of pure water (H_2O), mercury (Hg), methanol (CH_3OH) and ethyl alcohol (C_2H_5OH). From this, the difference between the vapor pressures of the individual fluids and the dependence on the pressure temperature are clearly deriving. In fine and high vacuum, the operating ranges coincide with the measuring ranges.

Table 1-6. Operating ranges and measuring ranges of vacuum [1.4]

			Measuring ranges			Boiling ranges in °C				
			mbar 1)		Torr 2)		H_2O	Hg	$C_2H_3 \cdot OH$	CH_3OH
Normal vacuum	Rough vacuum		1013		760		100	357	78,3	64,7
			1000		750		99,6	356,2	78	64,2
			800		600		93,5	344	71	56,5
			600		450		85,5	329	61,5	48
			400		300		75	309	54	40,5
			200		150		60,1	278	45	32
		↑ Millibar range ↓	100	↑ Torr range ↓	75	Evaporation	45,7	251	29,5	16,5
			80		60		41	242	24	11
			60		45		36,4	232	17	5,5
			50		37,5		32,5	225	14	3
			40		30		29	218	11,5	0
			30		22,5		24	208	8	−3,5
	Intermediate vacuum		23,4		17,53		20	201	5	−6
			20		15		17,2	196	3,5	8
			15		12		14	190	−1	−11,5
			12		9		9,7	181	−4	−15,5
			10		7,5	↑	7	176	−9	−19,6
			8		6		3,8	170	−12,5	−24
			6,11		4,58		0	162,5	−15,6	−25
Fine vacuum range			6		4,5	↓	−0,2 3)	162	−17	−20,5
			4		3		−5	152	−21,5	−32
			2		1,5		−13	135,5	−28	−38
			1		0,75		−20,3	119	−38,5	−47,5
		↑ Microbar ↓	1000	↑ Millitorr ↓	750	Sublimation				
			800		600		−22,7	115,5	−42	−50,5
			600		450		−25,4	110	−45,5	−53
			400		300		−29,3	102	−48,5	−56,5
			200		150		−36	89	−53	−61
			100		75		−42	77,5	−60	−67,5
			80		60		−43,7	73,5	−63	−70
			40		30		−50	63	−68	−74,5
			10		7,5		−60,5	42,5	−75	−83
			8		6		−62	39,8	−77	−85
			4		3		−67	31	−81	−88,5
			1		0,75					
High vacuum range		↑ Nanobar ↓	1000	↑ Microtorr ↓	750		−76,3	14,4	−87	−94
			800		600		−77,8	12	−88,5	−95
			400		300		−82	5	−91,5	−98
			100		75		−90	−9	−94	−100
			80		60		−91,5	−11		
			40		30		−96	−17		
			10		7,5		−101,5	−29		
			8		6		−103	−31		
			4		3		−106	−36		
			1		0,75		−112	−42		
Ultra vacuum range		↑ Picobar ↓	1000	↑ Nanotorr ↓	750					
			800		600		−113			
			400		300		−116			
			100		75		−121,5			
			80		60		−122			
			40		30		−124,5			
			10		7,5		−129,5			
			8		6		−130			
			4		3		−132,5			
			1	Picotorr	0,75		−137	−100		
			1000		750					
			800		600		−137,5			
			400		300		−139,5			
			100		75		−143,5			

1) 10^3 N/m^2

2) mm Hg

3) above ice

1.3.4
Total pressure measuring

Pressure units. In vacuum technology, the Torr unit is used as the practical unit for pressure, which is tantamount to millimeter mercury column (mm Hg), the same applies to the unit millibar (mbar). In the international unit system (SI), Pascal (Pa) or Newton/square meter (N/m^2) are used as pressure units. There are still other customary pressure units which should no longer be used, however. When pressure values are quoted, usually negative decimal powers (e.g. 2×10^{-1} Torr) are employed. Table 1-7 may help in converting old units, no longer permitted in business and official communication since 31/12/1977, into new units.

Table 1-7. Conversion table for pressure units (acc. to DIN 28400, part 1 July 1979)

	Pa [N · m^{-2}]	**1 bar = 1000 mbar**	**Atm**	**Torr**
1 Pa = 1 N · m^{-2} =	1	10^5	$0.986923 \cdot 10^{-5}$	$0.750062 \cdot 10^{-2}$
1 bar = 1000 mbar =	10^5	1	0.986923	$0.750062 \cdot 10^3$
1 atm =	$1.01325 \cdot 10^5$	1.01325	1	$0.760000 \cdot 10^3$
1 Torr =	$1.33322 \cdot 10^2$	$1.33322 \cdot 10^3$	$1.315789 \cdot 10^3$	1

Depression and vacuum. In technology, the terms overpressure (p_e), depression (p_u), and vacuum are used. The lowest pressure theoretically possible is 0 Pa = 0 N · m^{-2} = 0 mbar.

This would correspond to a 100 per cent vacuum (Fig. 1-3). Vacuum is a rarefied air or gas space. With 100 percent vacuum there is a total absence of air and gas.

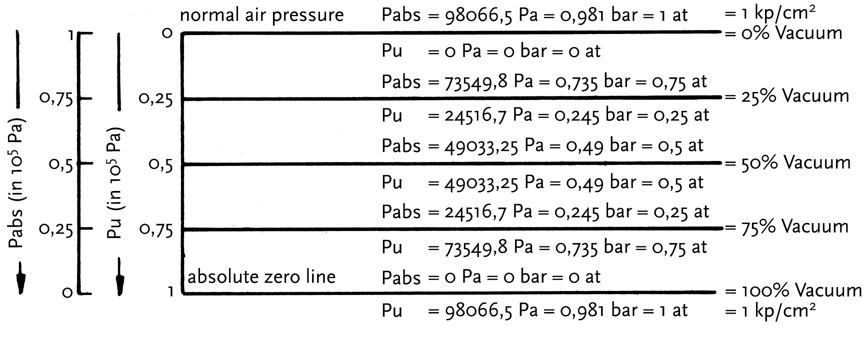


Figure 1-3. Underpressure and vacuum [1.5]

According to the following formula, vacuum can be calculated in percent:

$$\text{Vacuum} = \frac{P_u}{P_{air}} \cdot 100\,[\%] \tag{1-3}$$

As this is a pressure ratio, any pressure unit can be used for calculation. However, for p_u and p_{air} the same pressure units have to be used in the formula..

Vacuum in percent can also be determined from barometer readings B in Pa and the readings of a vacuum gauge H in Pa using the following formula:

$$\text{Vacuum} = \frac{1.01325 \cdot 10^5\ \text{Pa} - (B - H)}{1.01325 \cdot 10^5\ \text{Pa}} \cdot 100\ [\%] \tag{1-4}$$

Example 1.1:
How much underpressure is in a vessel with a vacuum of 80% and an air pressure of $0.98 \cdot 10^{-5}$ Pa?

Solution:
According to equation (1-3), the result is:

$$p_u = \text{Vacuum in } \% \cdot p_{air}$$
$$p_u = 0.80 \cdot 98000\ \text{Pa} = 78\,400\ \text{Pa} = 784.0\ [\text{mbar}]$$

Absolute pressure, overpressure and underpressure. A pressure higher than the outside air pressure is called overpressure p_e. A pressure below the outside air pressure is called underpressure p_u. (Fig. 1-4)

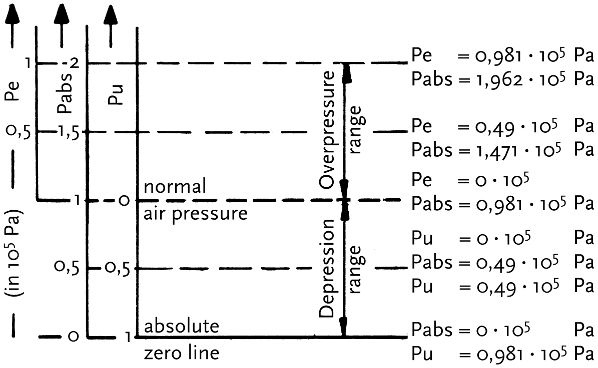


Figure 1-4. Absolute pressure, overpressure and underpressure [1.5]

In technology, the absolute pressure p_{abs} is used for calculations taking the currently prevailing air pressure into consideration.

The absolute pressure p_{abs} is the pressure calculated from the absolute zero line.

$$p_{abs} = p_e + p_{air} \quad \left[\text{Pa};\ \frac{\text{N}}{\text{m}^2};\ \text{mbar}\right] \tag{1-5}$$

$$p_{abs} = p_{air} - p_u \quad \left[\text{Pa};\ \frac{\text{N}}{\text{m}^2};\ \text{mbar}\right] \tag{1-6}$$

Example 1.2

In a condenser, an underpressure of 650.00 mbar prevails, with the barometer showing 980.00 mbar.

How much is the absolute pressure in mbar, in Pa and the vacuum in percent?

Solution:

Acc. to equation (1-6), the result is:

$$p_{abs} = p_{air} - p_u$$

$$p_{abs} = 980 \text{ mbar} - 650 \text{ mbar} = 330\,[\text{mbar}] = 3{,}30 \cdot 10^4\,[\text{Pa}]$$

and acc. to equation (1-3) the result is:

$$\text{Vacuum} = \frac{P_u}{P_{air}} \cdot 100\ [\%]$$

$$\text{Vacuum} = \frac{650 \text{ mbar}}{980 \text{ mbar}} \cdot 100\ [\%]$$

$$\text{Vacuum} = 66,32\ [\%]$$

1.3.5

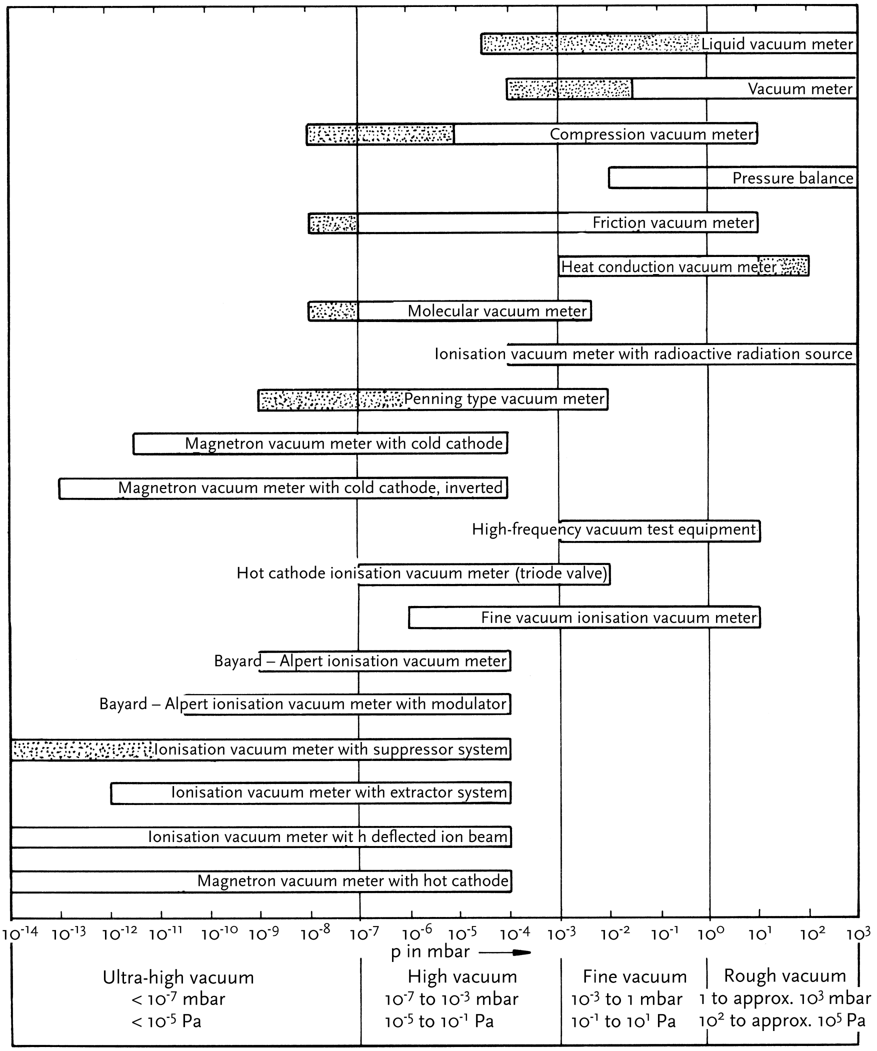
Pressure meters

In order to be able to record the pressure range from 10^5 to 10^{-11} Pa metrologically, various physical basic principles are employed according to which the pressure meters are subdivided.

There are

- mechanical gauges
- heat conduction vacuum manometers
- friction manometers
- radiometer manometers
- ionization manometers.

Up to pressures of 1 Torr, the direct measuring and display of the mechanical force acting upon a surface is carried out by means of mechanical manometers. Pressures below 1 Torr are determined by other measuring methods. Here, physical quantities such as friction, thermal conduction and ionization are measured. The measured quantities can be converted into electric currents or voltages, thus enabling the application of most modern electric and electronic methods of analysis.



The limits of the measuring ranges are without engagement. In the diagram are shown the usual limits.

▨ Measuring range for special execution or for special operating data.

Figure 1-5. Measuring ranges of common vacuum gauges (acc. to DIN 28400, Part 3, October 1980)

Mechanical gauges

In the following, only the group of mechanic gauges will be described [1.1]. These devices utilize the action of forces of the pressure for pressure display. To this group includes:

- Bourdon pressure gauge
- Diaphragm gauge
- Modulation meter
- Liquid manometer

Bourdon pressure gauge. This gauge (Fig. 1-6a) is a bent tube made of elastic material with an oval cross section. One tube end is closed while the other one is connected to a vacuum vessel. Owing to different actions of forces onto the upper and lower broadside of the tube (Fig. 1-6b) the tube bends during evacuation. By means of a lever mechanism, the tube deformation is transferred to a pointer.

Features

- Pressure readings independent from kind of gas
- Measuring range: atmospheric pressure up to about 1000 Pa
- special versions for laboratories up to about 1 Pa, e.g. versions made of corrosion-resistant materials (glass or quartz) are known
- Accuracy of readings depends on design, usually relatively low

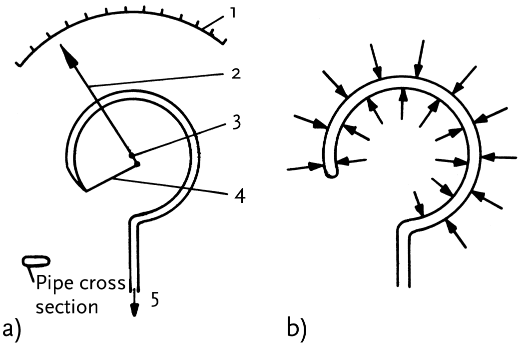


Figure 1-6. Bourdon tube
a) basic design b) force balance
1 scale, 2 pointer, 3 pointer's centre of motion, 4 lever, 5 vacuum vessel

Diaphragm gauge. This type of gauge uses the deformation of a diaphragm (or the changing of the length of a bellows) dividing two ranges of different pressures for pressure display. The deformation is displayed mechanically, optically or electrically (fig. 1-7). As the diaphragm deformation does not occur proportionally to the pressure, gauging devices with linearization appliances are partly used. The reference pressure is mostly selected in the lower pressure range, i.e. in fine and high vacuum, to be independent from altitude, fluctuating air pressure and weather. The readings are independent from temperature and outside air pressure, as the sensing element and the display are not in contact with the measured gas. In these devices, the clearance between the solid wall and the nesting ripple diaphragm, or the so-called pressure capsule in versions with two nesting ripple diaphragms, (acc. to [1.6]) is evacuated (fig. 1-8).

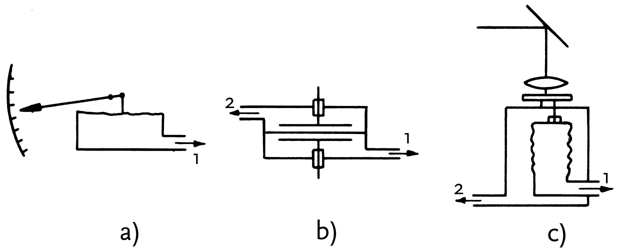


Figure 1-7. Diaphragm vacuum gauge with different kinds of display
a) mechanical display (pointer) b) electrical display c) optical display
1 to vacuum vessel, 2 to reference vacuum

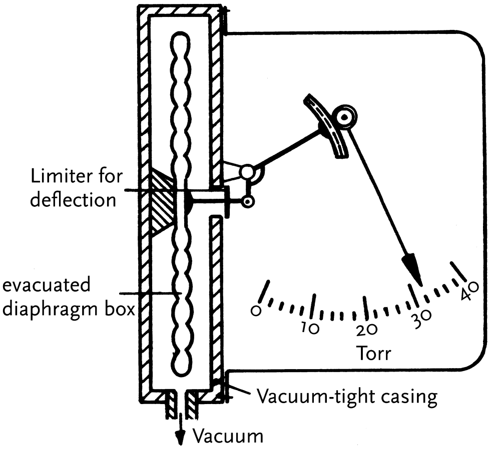


Figure 1-8. Diaphragm vacuum gauge with diaphragm box

Features

- Pressure readings independent from kind of gas
- Measuring range: depending on the versions, from 10^5 Pa to 10^{-2} Pa or for pressure differences of some hundreds of Pa up to 10^{-2} Pa, in laboratory versions up to some 10^{-3} Pa
- Accuracy: depending on the version, in some ranges up to several percent. Owing to the combination with switches, suitable for controlling; with electric display, remote readings and registration are possible.

Modulation manometer. The change of pressure according to the state equation for ideal gases arising from the periodical change of volume is used for the pressure measurement.

$$\Delta p = -p_o \frac{\Delta V}{V} \tag{1-7}$$

p_o average pressure
V_o average volume
$\Delta V = \Delta V_o \cdot \cos \omega \cdot t$
(constant gas mass being presupposed)

The periodic volume change can be produced e.g. with a piezo-ceramic measuring transmitter, the periodic pressure change can be detected by means of a sensitive microphone (fig. 1-9).

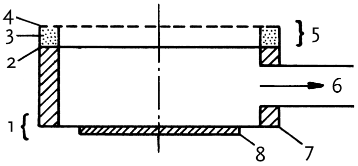


Figure 1-9. Modulation gauge according to Hartung and Jurgeit
1 – modulator, 2 – diaphragm, 3 – insulator, 4 – electrode, 5 – receiver, 6 – vacuum vessel, 7 – metal diaphragm, 8 – piezo-oscillator

Features

- In case of suitable design, pressure display independent from the kind of gas – at least in certain pressure ranges
- Measuring ranges: from atmospheric pressure up to about 10^{-4} Pa
- Accuracy: several percent
- Due to electronics, process control in vacuum, remote indication and registration of pressure are possible

U-tube gauge. The hydrostatic pressure of a liquid column compensating the pressure of the gas serves the pressure measurement.

$$p = \rho \cdot g \cdot h \tag{1-8}$$

ρ liquid density
h liquid column height
g acceleration of free fall
p hydrostatic pressure

The open, simplest type of U-tube gauge (fig. 1-10) is not really common in vacuum engineering, as for the measuring required for the determination of the gas pressure the air pressure has to be read as well. For technical purposes, a shortened closed U-tube gauge has become established (Fig.1-10b). In order to record the measuring range up to air pressure, a U-tube with a branch length of more than 760 mm would be required in case of mercury filling. Mercury is mostly used as filling medium. Over the mercury in the closed branch, the Torriccelli vacuum exists.

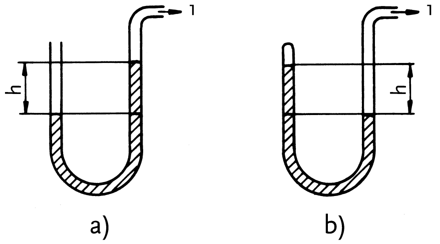


Figure 1-10. U-tube gauge
a) open type b) closed type (1 to vacuum vessel)

Features:

- Pressure readings independent from gas type
- Liquid mostly mercury (ρ =13.55 g/cm^3 at T = 281 K), in some cases dibutylphtalate (ρ =1.05 g/cm^3) is used
- Measuring range depending on design, from 10^5 Pa to about 150 Pa with mercury as liquid, and from 10^3 Pa to about 15 Pa with dibutylphtalate
- In case of mercury filling in laboratory versions, switching operations, remote display and registration of pressure can be realized (fig. 1-11)

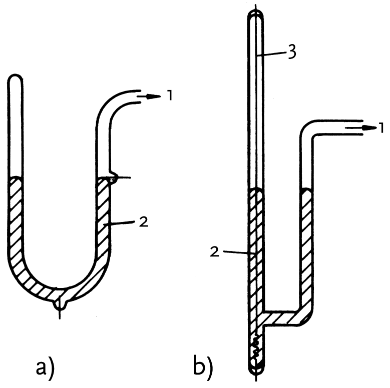


Figure 1-11. U-tube gauge in laboratory version
a) with a switch
b) for pressure registration
1) vacuum vessel
2) mercury
3) resistance wire

Compression gauge acc. to McLeod. With the compression gauge invented by McLeod in 1879, even pressure values below 150 Pa can be measured. Through isothermal compression of a measuring volume V_M onto the compression volume V'_M, the gas pressure is increased to such an extent that it can still be compensated by a mercury column of the readable height h. The McLeod compression gauge shown in Fig 1-12 disposes of a pressure-proportional display.

In the state (a), the measuring volume is connected with the measuring point. If h is the height difference between the mercury meniscuses in the closed and in the reference capillary after compression, the following formula is applicable:

$$p \approx \rho \cdot g \cdot h \cdot \frac{V'_M}{V_M} \qquad (1\text{-}9)$$

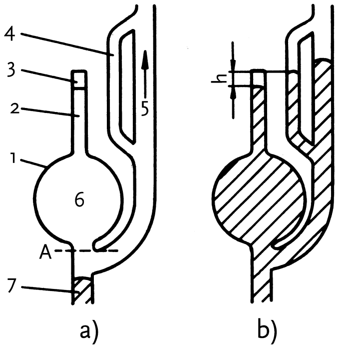


Figure 1-12. McLeod compression gauge with a linear scale
a) before compression, b) after compression
1) ball, 2) measuring capillary
3) compression volume V'_M
4) corresponding capillary
5) to vacuum vessel, 6) measuring volume V_M
7) mercury column

Fig. 1-13 shows a measuring layout in which the mercury in the measuring capillary does not rise up to a fixed mark (so that the known and constant compression volume V'_M is generated), but to the same (fixed) height in the reference capillary as the end of the measuring capillary. Then the mercury column in the measuring capillary shows different heights. For the pressure goes (r = capillary radius):

$$p \approx \rho \cdot g \cdot \pi \cdot r^2 \cdot \frac{h^2}{V_M} \qquad (1\text{-}10)$$

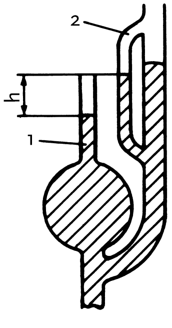


Figure 1-13. McLeod compression gauge with square display
1 measuring capillary, 2 corresponding capillary tube

Features:

- Pressure readings independent from kind of gas
- Measuring range depends on design (V_M and V'_M sizes)
- Upper measuring limit is up to some hundreds of Pascal, in exceptional cases up to 10^4 Pa; the lowest limit is at about 10^{-2} Pa to 10^{-3} Pa, in special cases up to 10^{-4} Pa

- The measuring is discontinuous
- Filling liquid: mercury of purest quality
- Measuring accuracy decreases with decreasing pressure and achieves values of 50–100% at the lowest measurable pressures (height difference of mercury columns about 1 mm). The compression gauge is not suitable for the measuring of condensable vapors, as its function is based on the application of the equation for ideal gases in thermodynamic balance ($p \cdot V = m \cdot R \cdot T$). If such vapors are contained in the gas to be measured, extremely high measuring errors occur.

1.3.6
Definition of terms for vacuum measuring devices

(extract from DIN 28400, Part 3, October 1980)

Absolute vacuum gauge. An absolute vacuum gauge is an absolute pressure gauge for vacuum.

Absolute pressure gauge, absolute gauge. An absolute pressure gauge is a gauge which is used for the determination of pressure as quotient from the pressure force applied onto a surface and the surface area. An absolute pressure gauge is independent from the kind of gas.

Operation and display device of a vacuum gauge. An operation and display device of the vacuum gauge is the part of a vacuum gauge which contains power supply and appliances for the functioning of the vacuum gauge as well as for pressure readings.

Differential pressure gauge, differential pressure measuring device. A differential pressure gauge is a device measuring the difference between pressures acting on one of both sides of a pressure-sensitive partition surface at the same time, e. g. on an elastic diaphragm or a movable separation liquid. Differential pressure gauges are independent from the kind of gas.

Differential pressure vacuum gauge. A differential pressure vacuum gauge is a differential pressure gauge for vacuum.

Pressure gauge. Manometer. According to the corresponding norm, a pressure gauge is a device for measuring gas and vapor pressures, independent from the pressure range.

Piston manometer. A piston manometer is an absolute vacuum gauge in which the pressure to be measured acts onto a piston-cylinder-combination with a very small clearance in between and a known cross section. The force acting onto the piston is determined by weighing.

Vacuum gauge sensitiveness. The sensitiveness of a vacuum gauge is a quotient of the change of value displayed on a vacuum gauge and the corresponding change of pressure within a small pressure interval. With certain kinds of vacuum gauges, the sensitiveness depends on the kind of gas. In such cases, the kind of gas has to be specified. In case of a lacking specification, the sensitiveness refers to nitrogen. Together with the specification of sensitiveness the operating conditions and pressure ranges must be indicated.

Integrated measuring systems. An integrated measuring system is a vacuum measuring system without a special casing. The pressure-sensitive elements are situated directly in the vacuum vessel.

Elastic spring vacuum gauge. This vacuum gauge is a differential pressure vacuum gauge in which the pressure-sensitive partition is an elastic element. The pressure difference can be determined either by changing of the position of the elastic element (direct method) or through the force needed for maintaining the original position (zero method). (Examples: diaphragm vacuum gauge, Bourdon tube).

Note: Such devices are also used in pressure ranges above vacuum. Then they are called elastic spring manometers or elastic spring barometers.

Liquid vacuum gauge. A liquid vacuum gauge is a differential vacuum gauge in which the pressure-sensitive element is a movable separation liquid (e.g. mercury in a U-tube vacuum gauge). The pressure difference is determined by measuring the differences of the liquid level.

Note: Such devices are even used in pressure ranges above vacuum. Then they are called liquid manometer. A form frequently used is the U-tube manometer.

Compression vacuum gauge. A compression vacuum gauge is a vacuum gauge in which a known volume of a gas is compressed by a known compression ratio at the pressure to be measured (e.g. by moving of a liquid column, usually mercury) and with which the higher pressure resulting from this is measured.

Measuring with a compression vacuum gauge is independent from the kind of gas. In case of existing vapors, condensation processes must be taken into consideration. A well-known example is the McLeod vacuum gauge.

Measuring range of a vacuum gauge. The measuring range of a vacuum gauge is the pressure range in which the inaccuracy of a single pressure reading under defined conditions does not exceed the maximum permissible inaccuracy. For certain kinds of vacuum gauges, this range depends on the type of gas. In such cases, the type of gas has to be specified. In case of lacking specification, the measuring range refers to nitrogen.

Partial pressure vacuum gauge. A partial pressure vacuum gauge is a vacuum gauge for the measuring of partial pressures of the individual components of a gas mixture.

Relative sensitiveness of vacuum gauge. The relative sensitiveness of a vacuum gauge for a certain gas is the quotient resulting from the sensitiveness of this gas and the nitrogen sensitiveness at the same pressure and under the same operating conditions.

Nitrogen equivalent pressure. Nitrogen equivalent pressure is the pressure of pure nitrogen which would cause the same vacuum gauge reading as the pressure of the gas to be measured.

Total pressure vacuum gauge. A total pressure vacuum gauge is a vacuum gauge for measuring the total pressure of a gas or gas mixture.

Vacuum measuring system. Vacuum measuring system is the part of a vacuum gauge consisting of pressure-sensitive elements and connected to the area in which the gas pressure is to be measured.

Some kinds of vacuum measuring systems are also called vacuum measuring tubes, vacuum measuring cells or vacuum measuring heads.

Vacuum gauge; vacuum measuring device. A vacuum gauge is a pressure measuring instrument for vacuum. It can consist of a vacuum measuring system, as well as of an operation and display device.

Note: Some common kinds of vacuum gauges do not measure pressure directly (i.e. acc. to quotients from force and surface), but another physical quantity depending on pressure or particle number.

1.4
Gas flow and vacuum ranges

1.4.1
Vacuum ranges and types of flow

For the designing of pipeworks and vacuum plants, exact knowledge about flow processes is significant, i.e. in vacuum technology gas flows play an important role. In the following, steady flows will be discussed. Non-steady flow processes, as for example in ducts of rotary piston pumps and rotary plunger pumps or gas mass vibration in pipework cannot be described within the framework of this treatise.

Gas flows are subdivided into very low gas velocities, and flow speeds in the range of speed of sound and supersonic speed. In case of low gas-flow velocities, the gas temperature can be regarded as constant, if the temperature is not affected by external influences. For flows in the sound and supersonic range, the laws of gas dynamics are applicable.

In any flow, whether liquid or gas – jointly called fluids – forces that produce, accelerate or delay flows are active. There is a distinction between pressure forces and friction forces; gravity forces may generally be ignored in gas flow.

Flow processes, in which friction forces are small or do not exist at all, are called inviscid flows. In this case, the acceleration or delay of the mass elements of the gas is determined by the pressure forces; here the Bernoulli- equation is applied. Otherwise, the flow is influenced by friction forces. Usually, pressure forces and friction forces are equally high and counteract each other. Friction forces are determined by the inside friction or the viscosity. In these cases, this type of flow is called viscous flow.

1.4.2
Mean free path

For the definition of flow processes in different vacuum ranges, the ratio between the mean free path $\bar{l}$ (Fig. 1-14) of the gas molecules and the flow channel width d (e.g. pipe diameter) is particularly important.

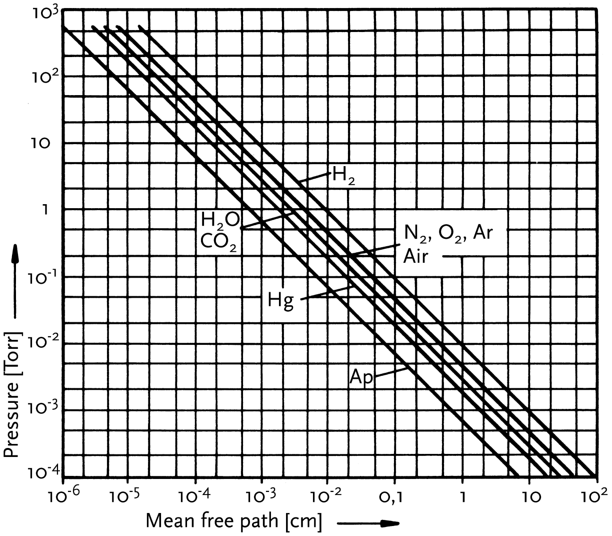


Figure 1-14. Mean free path of some substances at 20 °C [1.6]

Ar	argon	CO_2	carbon dioxide
H_2	hydrogen	H_2O	water vapor
N_2	nitrogen	O_2	oxygen
Hg	mercury	Ap	apiezon oil vapor

This ratio

$$K = \frac{\bar{l}}{d} \tag{1-2}$$

$\bar{l}$	mean free path (m)
d	flow channel width

is called Knudsen number.

As shown in table 1-8, the kind of gas flow is determined by the Knudsen number (K) acc. to the pressure ranges.

Table 1-8. Vacuum ranges and kinds of flow [1.7]

Coarse vacuum	**Fine vacuum**	**High-/ultrahigh vacuum**
Viscous flow	Knudsen flow	molecular flow

p = gas pressure, d = pipe diameter, $\bar{l}$ = mean free path,
K = $\bar{l}$/d = Knudsen number, Re = Reynolds number

In coarse vacuum, gas molecules frequently bang together with neighboring molecules, i.e. the molecular structure can be ignored. As the dynamic viscosity is independent from pressure, the flow is viscous. Viscous flow distinguishes between laminar flow and turbulent flow. The turbulent flow occurs when in the range of higher speeds random flow instead of laminar flow exists.

1.4.3
Reynolds number

Reynolds number Re determines the boundary between laminar and turbulent flow.

$$\mathrm{Re} = \frac{\rho \cdot \bar{\mathrm{v}} \cdot \mathrm{d}}{\eta} \tag{1-11}$$

Re	Reynolds number
ρ	fluid density [kg/m^3]
$\bar{\mathrm{v}}$	average flow speed [m/s]
d	characteristic length, e.g. pipe diameter [m]
η	dynamic viscosity [kg/(m · s)]

Viscous flow exists whenever $K < 10^{-2}$.

The critical Reynolds number for round pipes has the value of $\mathrm{Re}_{\mathrm{crit}} = 2300$. With this limit value for viscous flow, the laminar flow changes to turbulent flow.

Laminar flow exists with Re <2300

Turbulent flow exists with Re > 4000

While in viscous flow, i. e. in the coarse vacuum range, the molecules close to each other continuously touch each other, and the gas flows like a viscous liquid with adhesion to the pipe wall (fig. 1-15) in high and in ultra-high vacuum, i.e. with thinner gas, the gases behave differently. In this flow range, the mean free path $\bar{l}$ of the gas molecules is almost equal to the width of the flow channel, or larger. The number of molecule collisions decreases to such an extent that at a certain lowest pressure collisions of the molecules do not take place any more. Thus the flow is influenced only by the collisions of the gas particles with the pipe wall (fig. 1-16). This type of flow in high and ultra-high vacuum is called molecular flow.

Molecular flow takes place with K > 0.5.

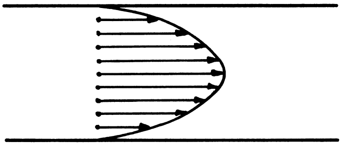


Figure 1-15. Scheme of laminar flow [1.6]

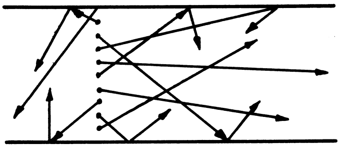


Figure 1-16. Scheme of molecular flow [1.6]

Turbulent flow is characterized by a strongly flattened speed profile and strongly decreasing boundary layers near the pipe wall.

Laminar flow shows a parabolic speed course formed across the cross section with a maximum in the middle and speed decreasing to zero near the pipe wall.

In the *molecular flow*, an abrupt velocity jump occurs near the pipe wall as soon as the mean free path increases compared to pipe diameter at very small pressures.

In practice, turbulent-laminar and laminar-molecular types of flow occur between the three typical types of flow (Fig. 1-17).

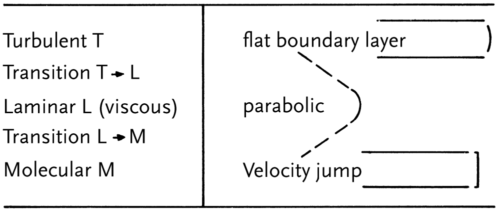


Figure 1-17. The three typical types of flow and their limits [1.2]

The range between viscous flow and molecular flow is called Knudsen flow, as defined acc. to DIN 28400. This type of flow occurring in fine vacuum is also called Knudsen number.

Knudsen flow occurs when $0.5 > K > 0.01$.

1.4.4
Gas flow, suction power, suction capacity [1.7]

The gas quantity, gas mass, the quantity of volume, particle numbers or substances ΔM flowing through a definite cross section within a definite period of time Δt is called flow rate q.

$$q = \frac{\Delta M}{\Delta t} \tag{1-12}$$

Therefore, we get

volume flow rate $$q_V = \frac{\Delta V}{\Delta t} = \dot{V}\left[\frac{m^3}{s}\right] \tag{1-13}$$

mass flow rate $$q_m = \frac{\Delta m}{\Delta t} = \dot{m}\left[\frac{kg}{s}\right] \tag{1-14}$$

amount of substance flow rate $$q_v = \frac{\Delta V}{\Delta t} = \dot{v}\left[\frac{mol}{s}\right] \tag{1-15}$$

particle number flow rate $$q_N = \frac{\Delta N}{\Delta t} = \dot{N}\left[\frac{1}{s}\right] \tag{1-16}$$

Instead of flow rate, the terms volume flow, mass flow, molecular flow and particle number flow can be used.

For ideal gases in a steady condition where pressure and temperature remain constant acc. to the general state equation of ideal gas, the following is applicable:

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

for time period Δt

$$p \cdot \frac{\Delta V}{\Delta t} = p \cdot \dot{V} = \frac{\Delta m}{\Delta t} \cdot R \cdot T$$

or

$$p \cdot \frac{\Delta V}{\Delta t} = \dot{m} \cdot R \cdot T = q_m \cdot R \cdot T \tag{1-18}$$

Substance mass flow rate $\dot{m}$ is proportional to the product $p \cdot \dot{V}$.

The $p \cdot \dot{V}$-flow rate or the $p \cdot \dot{V}$-flow

$$q_{pV} = p \cdot \dot{V} \left[\frac{P_a \cdot m^3}{s}; \frac{mbar \cdot 1}{s}\right] \tag{1-19}$$

is proportional to the substance mass flow rate $\dot{m}$.

The gas volume (= volume flow rate) flowing into a suction nozzle of a vacuum pump and related to the time and surface of the suction nozzle is called suction volume flow rate $\dot{S}$, which is the suction capacity of a vacuum pump.

$$\dot{S} = \dot{V}_{pump} = q_{Vpump} \left[\frac{m^3}{h}; \frac{m^3}{s}; \frac{1}{s}\right] \tag{1-20}$$

On the other hand, the $p \cdot \dot{V}$ flow rate at the pump inlet is called suction power $\dot{Q}$ of the vacuum pump.

$$\dot{Q} = p \cdot \dot{V}_{pump} = q_{pVpump} \left[\frac{P_a \cdot m^3}{s}; \frac{mbar \cdot 1}{s}\right] \tag{1-21}$$

From equations (1-20) and (1-21), follows:

$$\dot{Q} = p \cdot \dot{S} \left[\frac{mbar \cdot 1}{s}\right] \tag{1-22}$$

In vacuum pumps with a suction capacity independent from the intake pressure – this is the case in many vacuum pumps within a certain pressure range – the suction power $\dot{Q}$ decreases proportionally to the intake pressure. The suction power $\dot{Q}$ decreasing with lower pressure and having the same pump suction capacity can be explained with the fact that with smaller intake pressures fewer molecules per volume unit exist.

The characteristic curve of a vacuum pump with a constant suction capacity within a determined pressure range is shown in the fig. 1-18. Fig. 19 shows the suction power curve of a vacuum pump with constant suction capacity.

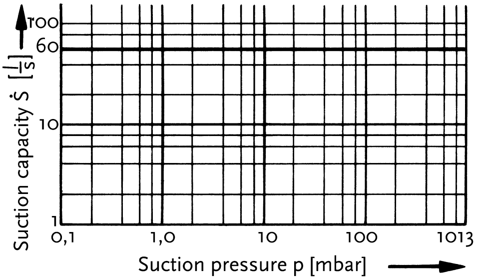


Figure 1-18. Characteristic curve, vacuum pump with constant suction capacity of 60 l/s within the pressure range of 1013 mbar to 0.10 mbar

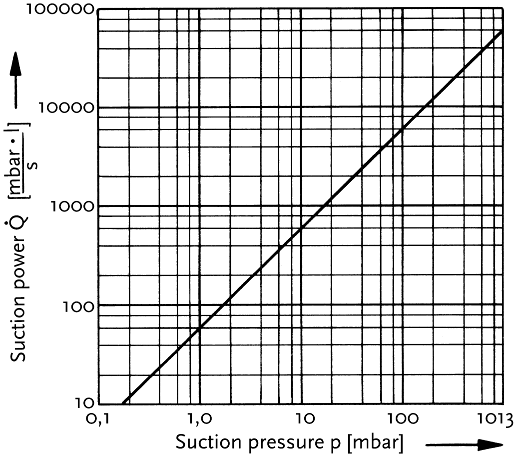


Figure 1-19. Characteristic curve with suction power of a vacuum pump with a constant suction capacity of 60 l/s within the pressure range from 1013 mbar to 0.10 mbar

1.4.5
Flow losses in pipework [1.7]

Vacuum pumps are usually connected with the vacuum vessels via pipes (fig.1-20).

With gas flow q flowing through a pipe, a flow resistance occurs in the pipe resulting in a pressure difference

$$\Delta p = p_K - p_A > 0 \qquad (1\text{-}23)$$

between the pipe ends.

As in electrical engineering, even in vacuum technology the ratio $\Delta p/q$ is called resistance or more precisely, flow resistance W.

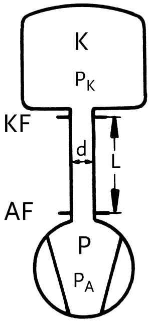


Figure 1-20. Vacuum pump with suction pipe and vacuum vessel Vessel K (pressure p_K) and pump P (pressure p_A). Pressure on suction flange AF equal to p_A. Vacuum vessel flange KF.

$$W = \frac{\text{difference in pressure}}{\text{flow rate}} = \frac{\Delta p}{q} \tag{1-24}$$

The flow resistance in vacuum pipes depends on the pressure difference or gas flow rate. The reciprocal value of the resistance is called conductivity L, just like in electrical engineering. In laminar ranges for different gases, conductivities behave like their viscosities. In the molecular range, conductivities depend only on the temperature and the molecular mass.

$$L = \frac{1}{W} = \frac{q}{\Delta p} \tag{1-25}$$

In vacuum technology, resistance or conductivity can be different quantities depending on the flow rate applied.

If the $p \cdot \dot{V}$ flow rate is used, the following results:

$$W = \frac{\Delta p}{q_{pV}} = \frac{\mathrm{Pa}}{\frac{\mathrm{Pa \cdot m^3}}{\mathrm{s}}} \left[\frac{\mathrm{s}}{\mathrm{m^3}}\right] \tag{1-26}$$

and

$$L = \frac{q_{pV}}{\Delta p} = \frac{\frac{\mathrm{Pa \cdot m^3}}{\mathrm{s}}}{\mathrm{Pa}} \left[\frac{\mathrm{m^3}}{\mathrm{s}}\right] \tag{1-27}$$

With volume flow rate $\dot{S}$, the equation is:

$$W = \frac{\Delta p}{q_V} = \frac{\mathrm{Pa}}{\frac{\mathrm{m^3}}{\mathrm{s}}} \left[\frac{\mathrm{Pa \cdot s}}{\mathrm{m^3}}\right] \tag{1-28}$$

and

$$L = \frac{q_V}{\Delta p} = \frac{\frac{\mathrm{m^3}}{\mathrm{s}}}{\mathrm{Pa}} \left[\frac{\mathrm{m^3}}{\mathrm{Pa \cdot s}}\right] \tag{1-29}$$

With a series connection of electrical resistances, the single resistances are added up, whereas with a parallel connection, the reciprocals of the electrical resistances, i.e. of the conductivities are added up.

These rules were applied analogously to the gas flow.

Series connection

$$W = W_1 + W_2 + W_3 + \ldots \tag{1-30}$$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots \tag{1-31}$$

Parallel connection

$$L = L_1 + L_2 + L_3 + \ldots \tag{1-32}$$

$$\frac{1}{W} = \frac{1}{W_1} + \frac{1}{W_2} + \frac{1}{W_3} + \ldots \tag{1-33}$$

In contrast to the electrical calculation of resistances, the calculation of the gas flow resistance shows slight deviations. This differences result from the fact that the gas flow in a pipe is not constant throughout the whole length. Therefore, in case of very short pipes the total resistance is not proportional to the pipe length due to the inlet flow. Strictly speaking, for the calculation e.g. in case of series connections, respective transition resistances have to be considered.

For the calculation of several pipework systems connected to each other, for example, a subdivision in individual sections is recommendable.

The calculation method described above is applicable for steady flow processes. In calculations of non-steady flows, complex resistances are used. The pipe volumes can be compared to the capacity of condensers and the inertia of the flowing gas masses to the inductivity of coils, according to the electric calculations.

1.4.6
Effective suction capacity of vacuum pumps

In a connecting pipe between vacuum chamber and vacuum pump, the gas flow, or at a constant temperature, the $p \cdot \dot{V}$ flow through each cross-section of the pipe is the same, as according to the continuity equation gas particles cannot get lost or be created anywhere. Considering the gas flow from a chamber with a higher gas density to another chamber with lower gas density, flow practically occurs only in direction of the pressure drop, as due to the higher density at the beginning of the pipe there are more moving molecules than at the end of the pipe on the gas outlet side. This is the explanation for an increasing flow rate owing to an increased gas temperature; even here the molecular velocity is higher than at low temperatures.

If, for example, in front of a vacuum pump with flow rate $\dot{S}$ and intake pressure p, a valve is installed that increases the line resistance (fig. 1-21), then, according to the rules described above, a so-called effective suction rate on the suction side of the installed valve is an inevitable result. [1.6]

Referring to the vessel flange KF and pump flange AF in fig. 1-20, the following correlation results acc. to [1.7]:

$$q_{pV} = p_K \cdot \dot{V}_K = p_A \cdot \dot{V}_A \tag{1-34}$$

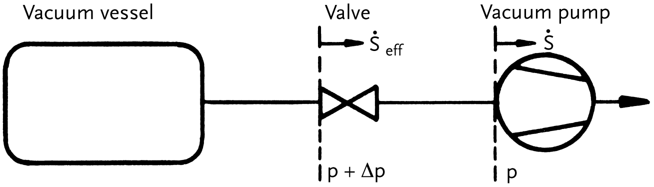


Figure 1-21. Scheme vacuum pump with valve installed on the suction side

$$\dot{V}_A = \dot{S} = \text{suction capacity of the pump} \tag{1-35}$$

$$p \cdot \dot{V} = \dot{Q} = \text{suction power of the pump} \tag{1-36}$$

If

$$\dot{V}_K = \dot{S}_K \tag{1-37}$$

$\dot{S}_K$ = effective suction capacity on vessel flange then the following will result from equation (1-34):

$$\dot{S}_K = \frac{p_A}{p_K} \cdot \dot{S} \tag{1-38}$$

Therefore $\dot{S}_K$ is smaller than $\dot{S}$, as already stated above, because in order to maintain the flow through the pipe, $p_K > p_A$.

With regard to the continuity equation, however, the suction power $\dot{Q} = \dot{Q}_K$ remains constant.

According to the equations

$$L = \frac{1}{W} = \frac{q}{\Delta p} \tag{1-25}$$

$$q_{pV} = p_K \cdot \dot{V}_K = p_A \cdot \dot{V}_A \tag{1-34}$$

$$\dot{V}_K = \dot{S}_K \tag{1-37}$$

the following correlation between pressure ratio p_K / p_A, suction capacity $\dot{S}$ and conductivity L results:

$$\frac{p_K}{p_A} = 1 + \frac{\dot{S}}{L} \tag{1-39}$$

and from the equation (1-38), the effective suction capacity

$$\frac{1}{\dot{S}_K} = \frac{1}{\dot{S}} + \frac{1}{L} \tag{1-40}$$

or

$$\dot{S}_K = \frac{\dot{S}}{1 + \frac{\dot{S}}{L}} \tag{1-41}$$

In fig. 1-22, the relation between the effective suction capacity $\dot{S}_K$, the actual suction capacity $\dot{S}$ of the vacuum pump, and conductivity L is shown acc. to equation (1-40).

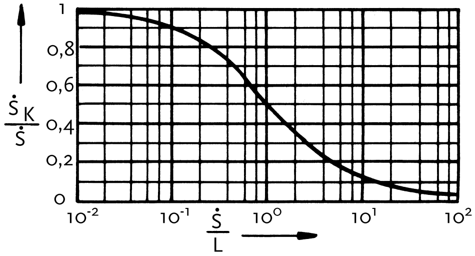


Figure 1-22. Pump efficiency $\dot{S}_K$ / $\dot{S}$ in relation to the ratio $\dot{S}$/L [1.7]

$\dot{S}_K$ effective suction capacity on the vacuum vessel flange
$\dot{S}$ suction capacity of the pump
L flow conductivity

It is apparent that a pump efficiency $\dot{S}_K / \dot{S} \approx 0.9 \approx 90\%$ can only be achieved if the conductivity is 10 times as high as the suction capacity of the vacuum pump. If conductivity $L \ll \dot{S}$, then acc. to the equation

$$\dot{S}_K = \frac{\dot{S}}{1+\frac{\dot{S}}{L}} \tag{1-41}$$

it results $\dot{S}_K \approx L$, i.e. the effective suction capacity is influenced only by conductivity and no longer by the suction capacity of the pump. From this derives that with flow losses in the vacuum pipe being too high even vacuum pumps with any form of suction capacity are not able to increase the effective suction capacity. This principle should always be taken into consideration when planning vacuum pipes and systems

The rule says that a vacuum pump can only be operated at a capacity of up to 90% if the conductivity of the respective pipe is 10 times as high as the suction capacity of the pump.

A pump utilization of 50% can be achieved acc. to [1.7], if the conductivity is at least as high as the suction capacity of the pump.

1.4.7
Gas-inflow and outflow on a vacuum chamber

According to fig. 1-23, gas inflow and gas outflow at a vacuum vessel is distinguished [1.1]

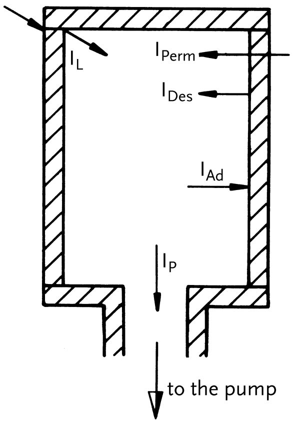


Figure 1-23. Gas inlet and outlet flow of the vacuum vessel

Inflow I_E

- Leakage gas flow I_L
- Permeation gas flow I_{Perm}
- Desorption gas flow I_{Des}

Outflow I_A

- Adsorption flow I_{Ad}
- Gas flow to the pump I_P

Adsorption and desorption are of particular importance in vacuum processes (comp. fig. 1-24).

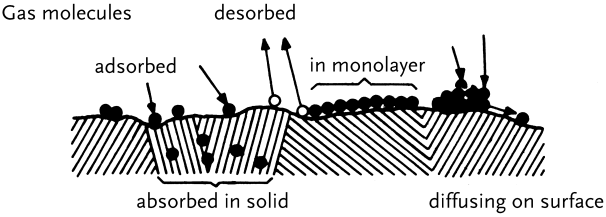


Figure 1-24. Sorption on interfaces of vacuum vessels [1.6]

The higher the vacuum in a chamber is, the greater is the influence of foreign substances on inner surfaces on evacuation. Coarse contaminations can be removed by mechanical cleaning and rinsing. The deeper penetration of adsorbed molecules in a material is called absorption. Adsorption is the intake of molecules, desorption is the discharge.

From a molecular point of view, sorption is an exchange process of different molecules on solid and liquid boundary surfaces of the vacuum chamber. The exchange on the surfaces is accelerated by increasing the temperature. By means of cooling, for example in working processes in vacuum, gas discharge from surfaces can be diminished or even totally prevented.

Examination of the pumping process in time units

For a constant volume of the vacuum chamber, a differential equation for pumping process can be elaborated according to [1.1]

$$\tau = \frac{V_0}{\dot{S}_{peff}} \tag{1-42}$$

$$pe = \frac{I_E}{\dot{S}_{peff}} \tag{1-43}$$

The differential equation is:

$$V_0 \cdot \left(\frac{d_p}{d_t}\right) = -p \cdot \dot{S}_{peff} + I_E \tag{1-44}$$

For constant $\dot{S}_{peff}$, the following solution results:

$$p - p_e = (p_0 - p_e) \cdot e^{-\frac{t}{\tau}} \tag{1-45}$$

or

$$p = (p_0 - p_e) \cdot e^{-\frac{t}{\tau}} + p_e \tag{1-46}$$

V_0	Volume of vacuum vessel
$\dot{S}_{peff}$	suction capacity of the pump acting effectively at the vacuum vessel
I_E	Inflow into the vacuum vessel
p	pressure in the vacuum vessel
p_0	Starting pressure at the time $t = 0$ (initial state)
p_e	Stationary end pressure of the system for $t \to \infty$
t	pumping-out time
τ	time constant of the system

With a suction capacity $\dot{S}_{peff}$ supposed to be constant at the vacuum vessel, the pressure p prevailing in the vacuum vessel can be determined for each period of evacuation.

If the suction capacity $\dot{S}_{peff}$ is not constant throughout the whole pressure range, this calculation has to be carried out gradually by means of iteration with the suction capacity corresponding to the pressure ranges.

1.4.8
Practice oriented application of the gas flow calculation [1.6]

For the calculation of flow resistances, it is necessary to determine the existing type of flow. In order to ascertain whether laminar or turbulent flow exists in the coarse and fine vacuum range, the so-called Reynolds number can be calculated. The Reynolds number Re is a dimensionless number calculated with the help of the known formula:

$$\mathrm{Re} = \frac{\rho \cdot \bar{v} \cdot d}{\eta} \qquad (1\text{-}11)$$

For dry air at 20 °C temperature, the following formula is applicable in practice:

$$\mathrm{Re} = 11.2 \cdot \frac{\dot{Q}}{d} \qquad (1\text{-}47)$$

$\dot{Q}$ flow power [(Torr · l)/s]
d pipe diameter [cm]

Laminar flow takes place if

$$\frac{\dot{Q}}{d} > 200$$

Turbulent flow is to be expected if

$$\frac{\dot{Q}}{d} > 360$$

For values between 200 and 360, i.e. in the transition range between laminar and turbulent flow, the flow type is significantly influenced by the existing marginal conditions.

The calculation of the resistance of vacuum pipes is based on the following known consideration.

The flow power in a pipe is directly proportional to the pressure difference $\Delta p = p_1 - p_2$ between the beginning and the end of the pipe and inversely proportional to the pipe resistance W.

For dry air at 20 °C, the resistances of round pipes can be determined by the following formulas:

$$\bar{p} = \frac{p_1 + p_2}{2} \ [\mathrm{Torr}] \qquad (1\text{-}48)$$

$\bar{p}$ mean pressure [Torr]
p_1 pressure at the beginning of the pipe [Torr]
p_2 pressure at the pipe end [Torr]

$$W_{\mathrm{laminar}} = \frac{a}{182 \cdot d^4 \cdot \bar{p}} \quad \left[\frac{s}{l}\right] \qquad (1\text{-}49)$$

$$W_{\mathrm{molecular}} = \frac{a}{12.1 \cdot d^3} \quad \left[\frac{s}{l}\right] \qquad (1\text{-}50)$$

W resistance [s/l]
a pipe length [cm]
d pipe diameter [cm]

For components without normal pipe cross-sections, the calculation in case of molecular flow can be carried out by replacing these components (e. g. valves or control devices) by pipe sections of the respective rated width and of a length approximately corresponding to the gas flow path inside the component.

The calculation of the resistance can be also carried out according to the diagram in the fig. 1-25.

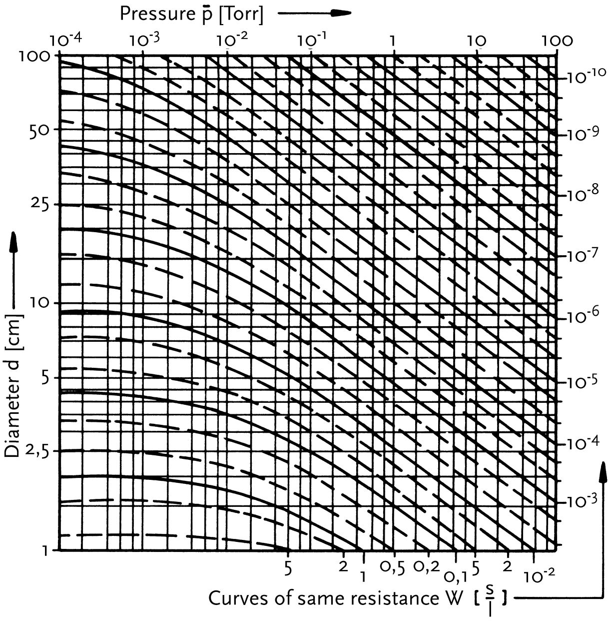


Figure 1-25. Flow resistance of a pipe of 100 cm length, for dry air at room temperature. [1.6]

The diagram in fig. 1-25 shows resistance curves of a pipe with a length of 100 cm and a clear width diameter of 1 to 100 cm, as a function of the mean pressure in the middle of the pipe section.

$$\bar{p} = \frac{p_1 + p_2}{2} \text{ [Torr]} \tag{1-48}$$

From fig. 1-25 follows that in the range of laminar flow the line resistance continuously increases with falling pressure. During the transition to the molecular flow, a constant maximum value is achieved at smaller pressures in the left half of the diagram. Furthermore, it is apparent that the maximum line resistances at small pipe diameters already occur at pressures that are not so low.

Example 1.3

For dry air at room temperature, the flow resistance in a pipe with a clear width of 10 cm and a length of 100 cm can be determined for pressures of 10^{-1} Torr and $3 \cdot 10^{-3}$ Torr with the help of the diagram shown in fig. 1-25.

Pressure $\bar{p}$ in the pipe (mean pressure in the pipe)	**Flow resistance W in the pipe**
$\bar{p} = 10^{-1}$ [Torr]	$W = 6.8 \cdot 10^{-4}$ [s/l]
$\bar{p} = 3 \cdot 10^{-3}$ [Torr]	$W = 7.00 \cdot 10^{-3}$ [s/l]

Calculation of the resistance of short pipes

For short pipes with a length of less than 1m, the resistance cannot be calculated proportionally, because even with very small pipe lengths or orifices in a thin wall (where the maximum diameter of the orifice may be only 1/10 of the space before it), finite resistance remains.

With the diameter d at the orifice, the resistance for the medium dry air at 20 °C can be calculated according to the following formulas:

$$W_{\text{laminar}} = \frac{1}{15.7 \cdot d^2} \quad \left[\frac{s}{1}\right] \tag{1-51}$$

$$W_{\text{molecular}} = \frac{1}{9 \cdot d^2} \quad \left[\frac{s}{1}\right] \tag{1-52}$$

d diameter (cm)

The resistances ascertained by the formulas and the diagram above are valid for dry air of 293 K. These values determined for air can be converted for other gases and temperatures with the help of the formulas given below, with the air-specific values being multiplied by the factor C.

For molecular flow, the following formula is applicable:

$$C_{\text{mol}} = 3.18 \cdot \sqrt{\frac{M}{T}} \tag{1-53}$$

M molecular mass [kg/kmol]
T thermodynamic temperature [K]

For laminar flow, the following formula is applicable:

$$C_{\text{lam}} = \frac{\eta_{\text{gas}}}{\eta_{\text{air}}} \tag{1-54}$$

η dynamic viscosity [micropoise]

Table 1-9. Dynamic viscosity of different gases at 293 K

Gas	H_2	He	H_2O	Air	Ar	CO_2	Hg
η_{293} [micropoise]	88	196	97	181	221	146	280

At temperatures T deviating from 293 K, the viscosity values (comp. Table 1-9) referring to 293 K have to be multiplied by the root of the temperature ratio.

$$\eta_T = \eta_{293} \cdot \sqrt{\frac{293}{T}} \qquad (1\text{-}55)$$

With increasing temperature the line resistance usually decreases.

Example 1.4:

The following is to be determined:

Resistances of a pipe with a length of 2 m and a diameter of 5 cm for

1) air and
2) water vapor

at pressures of 10^{-4} Torr and 1 Torr at 20 °C and 150 °C.

1) Solution for air

a) $p = 10^{-4}$ Torr
$t = 20\,°C$

$$\bar{l}_{air} = 66\,\text{cm} \qquad \text{(acc. to figure 1-14)}$$

$$K = \frac{\bar{l}}{d} \qquad \text{(equation 1-2)}$$

$$K = \frac{66\,\text{cm}}{5\,\text{cm}} = 13.2$$

$K > 0.5$, i.e. molecular flow

$$W_{mol} = \frac{a}{12.1 \cdot d^3} \qquad \text{(equation 1-50)}$$

$$W_{mol} = \frac{200}{12.1 \cdot 125} = 1.32 \cdot 10^{-1} \left[\frac{s}{l}\right]$$

b) $p = 10^{-4}$ Torr
$t = 150\,°C$

$$C_{mol} = 3.18 \cdot \sqrt{\frac{M}{T}} \qquad \text{(equation 1-53)}$$

$$M_{air} = 29 \cdot \frac{kg}{kmol}$$

$$T = 423\,K$$

$$C_{mol} = 3.18 \cdot \sqrt{\frac{29}{423}} = 0.832$$

$$W = 1.32 \cdot 10^{-1} \left[\frac{s}{l}\right] \cdot 0.832$$

$$W_{mol} = 1.10 \cdot 10^{-1} \left[\frac{s}{l}\right]$$

c) p = 1 Torr
t = 20 °C

$$\bar{l}_{air} = 6.4 \cdot 10^{-3}$$ (acc. to figure 1-14)

$$K = \frac{\bar{l}}{d}$$ (equation 1-2)

$$K = \frac{0.0064\,\text{cm}}{5\,\text{cm}} = 0.00128$$

$K < 10^{-2}$, i.e. viscous respectively laminar flow

$$W_{lam} = \frac{a}{182 \cdot d^4 \cdot \bar{p}}$$ (equation 1-49)

$$W_{lam} = \frac{200}{182 \cdot 625 \cdot 1} = 1.76 \cdot 10^{-3} \left[\frac{s}{l}\right]$$

d) p = 1 Torr
t = 150 °C

$$\eta_T = \eta_{293} \cdot \sqrt{\frac{293}{T}}$$ (equation 1-55)

T = 423 K

η_{air} = 181 micropoise

$$\eta_T = 181 \cdot \sqrt{\frac{293}{423}} = 150.56$$

$$C_{lam} = \frac{\eta_{gas}}{\eta_{air}}$$ (equation 1-54)

$$C_{lam} = \frac{150.56}{181} = 0.832$$

$$W = 1.76 \cdot 10^{-3} \left[\frac{s}{l}\right] \cdot 0.832$$

$$W_{lam} = 1.47 \cdot 10^{-3} \left[\frac{s}{l}\right]$$

2) Solution for water vapor

a) $p = 10^{-4}$ Torr
$t = 20\,°C$

$\bar{l}_{H_2O} = 50$ cm (acc. to figure 1-14)

$$K = \frac{\bar{l}}{d}$$ (equation 1-2)

$$K = \frac{50\,\text{cm}}{5\,\text{cm}} = 10.0$$

$K > 0.5$, i.e. molecular flow

$$C_{mol} = 3.18 \cdot \sqrt{\frac{M}{T}}$$ (equation 1-53)

$$M_{H_2O} = 18\,\frac{kg}{kmol}$$

$T = 293$ K

$$C_{mol} = 3.18 \cdot \sqrt{\frac{18}{293}} = 0.7876$$

$$W = 1.32 \cdot 10^{-1}\,\frac{s}{l} \cdot 0.7876$$

$$W_{mol} = 1.04 \cdot 10^{-1} \left[\frac{s}{l}\right]$$

b) $p = 10^{-4}$ Torr
$t = 150\,°C$

$$C_{mol} = 3.18 \cdot \sqrt{\frac{M}{T}}$$ (equation 1-53)

$$M_{H_2O} = 18\,\frac{kg}{kmol}$$

$T = 423$ K

$$C_{mol} = 3.18 \cdot \sqrt{\frac{18}{423}} = 0.656$$

$$W = 1.32 \cdot 10^{-1}\,\frac{s}{l} \cdot 0.656$$

$$W_{mol} = 8.7 \cdot 10^{-2} \left[\frac{s}{l}\right]$$

c) $p = 1$ Torr
$t = 20\,°C$

$\bar{l}_{H_2O} = 4.5 \cdot 10^{-3}$ cm (acc. to figure 1-14)

$$K = \frac{\bar{l}}{d} \quad \text{(equation 1-2)}$$

$$K = \frac{0.0045\ \text{cm}}{5\ \text{cm}} = 0.0009$$

$K < 10^{-2}$, i.e. viscous respectively laminar flow

$$C_{lam} = \frac{\eta_{gas}}{\eta_{air}} \quad \text{(equation 1-54)}$$

$\eta_{H_2O} = 97$ micropoise

$\eta_{air} = 181$ micropoise

$$C_{lam} = \frac{97}{181} = 0.535$$

$$W = 1.76 \cdot 10^{-3}\ \frac{s}{l} \cdot 0.535$$

$$W_{lam} = 9.4 \cdot 10^{-4} \left[\frac{s}{l}\right]$$

d) $p = 1$ Torr
$t = 150\,°C$

$$\eta_T = \eta_{293} \cdot \sqrt{\frac{293}{T}} \quad \text{(equation 1-55)}$$

$T = 423$ K

$\eta_{H_2O} = 97$ micropoise

$$\eta_T = 97 \cdot \sqrt{\frac{293}{423}} = 80.69$$

$$C_{lam} = \frac{\eta_{gas}}{\eta_{air}} \quad \text{(equation 1-54)}$$

$$C_{lam} = \frac{80.69}{181} = 0.445$$

$$W = 1.76 \cdot 10^{-3}\,\frac{s}{l} \cdot 0.445$$

$$W_{\mathrm{lam}} = 7.836 \cdot 10^{-4}\,\frac{s}{l}$$

Summary of results

20 °C (293 K)			**150 °C (423 K)**		
1) air [Torr]	resistance [s/l]	C_{mol}	η T	C_{lam}	resistance [s/l]
10^{-4}	a) $1.32 \cdot 10^{-1}$	0.832	–	–	b) $1.1 \cdot 10^{-1}$
1	c) $1.76 \cdot 10^{-3}$	–	150.56	0.832	d) $1.47 \cdot 10^{-3}$
2) H_2O					
10^{-4}	a) $1.04 \cdot 10^{-2}$	0.656	–	–	b) $8.7 \cdot 10^{-2}$
1	c) $9.4 \cdot 10^{-4}$	–	80.69	0.445	d) $7.83 \cdot 10^{-4}$

Example 1.5

A vacuum pump shall suck off from a vacuum chamber in the high vacuum range. The suction line is to be regarded as an orifice in a thin wall.

1) Which is the maximum possible suction capacity of the pump to enable the suction of all molecules flowing through the orifice?
2) Which is the specific suction capacity of the pump with reference to the suction cross-section?

Solution:

As the pump is operating in the high vacuum range, the calculation can be based on the molecular type of flow.

1) Flow resistance can be determined by following formula

$$W = \frac{1}{9 \cdot d^2} \quad \left[\frac{s}{l}\right] \tag{1-52}$$

For the flow capacity, the following relation exists:

$$W = \frac{\Delta p}{q_{pV}} \left[\frac{s}{l}\right] \tag{1-26}$$

Consequently,

$$q_{pv} = \Delta p \cdot 9 \cdot d^2 \quad \left[\frac{\mathrm{Torr} \cdot l}{s}\right] \tag{1-56}$$

From the equation

$$\dot{Q} = p \cdot \dot{S} = q_{pv} \left[\frac{\mathrm{Torr} \cdot l}{s}\right] \tag{1-22}$$

the result is

$$\dot{S} = \frac{q_{pV}}{p} \left[\frac{l}{s}\right] \tag{1-57}$$

and as $p = \Delta p$:

$$\dot{S} = \frac{\Delta p}{\Delta p} \cdot 9 \cdot d^2 \left[\frac{l}{s}\right]$$

and therefore

$$\dot{S} = 9 \cdot d^2 \left[\frac{l}{s}\right] \tag{1-58}$$

2) The suction capacity referred to the circular cross-section is calculated according to the equation

$$\dot{S} = 9 \cdot d^2 \left[\frac{l}{s}\right] \tag{1-58}$$

Through multiplication by the factor $\pi/4$ the result is

$$\dot{S} \cdot \frac{\pi}{4} = 9 \cdot d^2 \frac{\pi}{4}$$

with the relation

$$F = \frac{d^2 \cdot \pi}{4} \left[cm^2\right] \tag{1-59}$$

d diameter [cm]

The calculation is:

$$\dot{S} \cdot \pi / 4 = 9 \cdot F$$

and the specific suction capacity is:

$$\frac{\dot{S}}{F} = 9 \cdot \frac{4}{\pi} \left[\frac{l}{s \cdot cm^2}\right] \tag{1-60}$$

$$\frac{\dot{S}}{F} = 11.46 \left[\frac{l}{s \cdot cm^2}\right]$$

$\frac{\dot{S}}{F} = S_{spec} =$ specific suction capacity referring to the inlet cross-section of the vacuum pump.

1.5
Physical states of matter

1.5.1
The terms gases, vapors, vacuum

The three physical states of matter of a substance - solid, liquid and gaseous – differ as follows:

In the solid state of matter, strong forces of attraction exist between the particles, i.e. the distance between these particles is relatively small.

In the liquid state of matter, there are larger distances between these particles which are able to move relatively to each other. The forces of attraction are smaller compared to the solid state of matter.

In the gaseous state of matter, there aren't any forces of attraction between the molecules or atoms in the ideal state. In this case, the gas is called "ideal gas".

In practice, in vacuum technology there are pure gases or pure vapors, or mixtures of gases and vapors at pressures which are lower than the atmospheric pressure.

Gas is the state of an element which cannot be transferred into another state of matter in the existing pressure and temperature range.

Vapor is a substance in a gaseous state of matter which however can be transferred into a liquid or solid state within the temperature or pressure range relevant for the process flow.

Vacuum is defined as the thermodynamic state in a chamber filled with gas or vapor at pressures below the atmospheric pressure.

1.5.2
Physical basic principles of ideal gases

Boyle's/Mariotte's law. For calculations in which the gas temperature can be considered as constant, the Boyle Mariott law is applicable (equation 1-61).

$$p \cdot V = \text{constant} \tag{1-61}$$

T = constant
p pressure [N/m^2; Pa; bar]
V volume [m^3]

Gay-Lussac's law. With changing volume at varying temperatures and constant pressure the Gay-Lussac's law according equation 1-62 is applicable.

$$\frac{V_{T1}}{V_{T2}} = 1 + a(T_1 - T_2) \tag{1-62}$$

p constant
T thermodynamic temperature [K]

0 °C = 273.15 [K]

$$\alpha = \frac{1}{273} \tag{1-63}$$

General state equation. With the general state equation (1-17), the pressure or volume of the gas mass can be determined for any temperature. R is the specific gas constant dependent on the substance. With equation (1-64), even the apparent molecular mass of gas mixtures can be determined.

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

$$\frac{N}{m^2} \cdot m^3 = kg \cdot \frac{Nm}{kg \cdot K} \cdot K\ [Nm]$$

$$p \cdot V_m = M \cdot R \cdot T \tag{1-64}$$

$$\frac{N}{m^2} \cdot \frac{m^3}{kmol} = \frac{kg}{kmol} \cdot \frac{Nm}{kg \cdot K} \cdot K \left[\frac{Nm}{kmol}\right]$$

$$p \cdot V = m \cdot \frac{R_o}{T} \cdot T \tag{1-65}$$

$$\frac{N}{m^2} \cdot m^3 = kg \cdot \frac{\frac{Nm}{kmol \cdot K}}{\frac{kg}{kmol}} \cdot K\ [Nm]$$

m	gas mass [kg]
R	specific gas constant [Nm/(kg · K)]
R_0	general molar gas constant [Nm/(kmol · K)]
M	relative molecular mass [kg/kmol]
V_m	molar volume = 22.41 [m^3/kmol] at 0 °C and 1013 mbar

Volume. The volume is determined by the product of gas mass and the specific volume according to equation (1-66).

$$V = m \cdot v \tag{1-66}$$

v	specific volume [m^3/kg]

General gas constant. According to equation (1-67), the general gas constant is connected to the specific gas constant by the relative molecular mass ratio.

$$R_o = R \cdot M \tag{1-67}$$

$$R_o = \frac{\mathrm{Nm}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 8314 \left[\frac{\mathrm{Nm}}{\mathrm{kmol{\cdot}K}}\right]$$

or

$$R_o = \frac{\mathrm{Pa{\cdot}m^3}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 8314 \left[\frac{\mathrm{Pa{\cdot}m^3}}{\mathrm{kmol{\cdot}K}}\right]$$

or

$$R_o = \frac{\mathrm{mbar{\cdot}1}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 83140 \left[\frac{\mathrm{mbar{\cdot}1}}{\mathrm{kmol{\cdot}K}}\right]$$

or

$$R_o = \frac{\mathrm{mbar{\cdot}1}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 83,14 \left[\frac{\mathrm{mbar{\cdot}1}}{\mathrm{mol{\cdot}K}}\right]$$

or

$$R_o = \frac{\mathrm{J}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 8314 \left[\frac{\mathrm{J}}{\mathrm{kmol{\cdot}K}}\right]$$

or

$$R_o = \frac{\mathrm{kpm}}{\mathrm{kg{\cdot}K}} \cdot \frac{\mathrm{kg}}{\mathrm{kmol}} = 848 \left[\frac{\mathrm{kpm}}{\mathrm{kmol{\cdot}K}}\right]$$

1 Nm = 1 J
1 Pa = 1 N/m^2

Molar volume. The molar volume is calculated according to equation (1-68) with a value of

22.41 m^3/kmol at 0 °C and 1013 mbar. It can also be calculated from the quotient of the Avogadro Number and the Loschmidt's Number acc. to equation (1-69).

$$V_m = \frac{R_o{\cdot}T}{p} \tag{1-68}$$

$$V_m = \frac{8314\,\frac{\mathrm{Nm}}{\mathrm{kmol{\cdot}K}}{\cdot}273.15\mathrm{K}}{1.01325{\cdot}10^5\frac{\mathrm{N}}{\mathrm{m^2}}} = 22.41 \left[\frac{\mathrm{m^3}}{\mathrm{kmol}}\right]$$

or

$$V_m = \frac{N_A}{N_L} \tag{1-69}$$

$$V_m = \frac{6.022{\cdot}10^{26}\,\frac{1}{\mathrm{kmol}}}{2.686{\cdot}10^{25}\,\frac{1}{\mathrm{m^3}}} = 22.41 \left[\frac{\mathrm{m^3}}{\mathrm{kmol}}\right]$$

N_A Avogadro Constant $= 6.022 \cdot 10^{26}$ [molecules/kmol]
N_L Loschmidt's Number $= 2.686 \cdot 10^{25}$ [molecules/m^3]

Density. Acc. to equation (1-70), the quotient of relative molecular mass and molar volume results in the gas density.

$$\rho = \frac{M}{V_m} \tag{1-70}$$

$$\rho = \frac{M\,\frac{kg}{kmol}}{22.41\,\frac{m^3}{kmol}} = 4.462 \cdot 10^{-2} \cdot M \left[\frac{kg}{m^3}\right]$$

ρ density

Absolute molar mass. Absolute molar mass can be calculated from the relative molar mass and the Avogadro Constant acc. to equation (1-71).

$$m_u = \frac{M}{N_A} \tag{1-71}$$

$$m_u = \frac{1\,\frac{kg}{kmol}}{6.022 \cdot 10^{26}\,\frac{1}{kmol}} = 1.6605 \cdot 10^{-27}\,[kg]$$

m_u absolute molar mass

Bolzmann Constant. The Bolzmann Constant is calculated from the general gas constant and the Avogadro Number acc. to equation (1-72).

$$k = \frac{R_o}{N_A} \tag{1-72}$$

$$k = \frac{8314\,\frac{Nm}{kmol \cdot K}}{6.022 \cdot 10^{26}\,\frac{1}{kmol}} = 1.380 \cdot 10^{-23} \left[\frac{Nm}{K}\right]$$

or

$$k = \frac{8314\,\frac{J}{kmol \cdot K}}{6.022 \cdot 10^{26}\,\frac{1}{kmol}} = 1.380 \cdot 10^{-23} \left[\frac{J}{K}\right]$$

k Boltzmann constant

Loschmidt's and Avogadro constants. In case of equal volume, equal pressure and equal temperature, each ideal gas has the same number of molecules or atoms. The Loschmidt's Number N_L gives the number of molecules or atoms contained in 1 m^3 of gases or gas mixtures.

The Avogadro Constant N_A gives the number of molecules or atoms contained in 1 mol or kilomol.

One mol in grams weighs as much as the relative molecular mass of the gas states. The ratio of the mass of the respective molecules to one twelfth of the mass of a carbon isotope (^{12}C) is called relative molecular mass.

Loschmidt's number. From the division of the Avogadro Constant by the molar volume, the Loschmidt's Number results acc. to equation (1-73).

$$N_L = \frac{N_A}{V_m} \tag{1-73}$$

$$N_L = \frac{6.022 \cdot 10^{26} \frac{1}{\text{kmol}}}{22.41 \frac{m^3}{\text{kmol}}} = 2.686 \cdot 10^{25} \left[\frac{1}{m^3}\right]$$

N_L Loschmidt's number

Avogadro Constant. By the division of the relative molecular weight by the absolute molar mass, the Avogadro Constant is determined according to equation (1-74).

$$N_A = \frac{M}{m_u} \tag{1-74}$$

$$N_A = \frac{1 \frac{\text{kg}}{\text{kmol}}}{1.6605 \cdot 10^{-27} \text{kg}} = 6.022 \cdot 10^{26} \left[\frac{1}{\text{kmol}}\right]$$

N_A Avogadro's constant

Particle density. For ideal gases, the number of particles (molecules, atoms) in a system is connected to pressure and temperature. This derives from the general state equation for ideal gases.

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

$$p \cdot V = \frac{m}{M} \cdot R_o \cdot T \tag{1-65}$$

$$m = N \cdot m_u \ [\text{kg}] \tag{1-75}$$

m mass [kg]
N particle number
m_u particle mass, absolute molar mass [kg]

$$p \cdot V = \frac{N \cdot m_u}{M} \cdot R_o \cdot T \tag{1-76}$$

$$N = n \cdot V \tag{1-77}$$

N particle number density [1/m^3]
V volume [m^3]

$$\frac{m_u}{M} = \frac{1}{N_A} \tag{1-78}$$

$$p \cdot V = \frac{n \cdot V}{N_A} \cdot R_o \cdot T \tag{1-79}$$

$$\frac{R_o}{N_A} = k \tag{1-72}$$

and consequently

$$p = n \cdot k \cdot T \tag{1-1}$$

$$n = \frac{p}{k \cdot T} = \frac{\frac{N}{m^2}}{1.38 \cdot 10^{-23}\, \frac{Nm}{K} \cdot K} \left[\frac{1}{m^3}\right] \tag{1-80}$$

With the equation (1-80) the particle density for any temperature and pressure can be calculated.

Example 1.6
Determination of the standard particle density at 1013 mbar and 0 °C.

Solution:

$$n = \frac{p}{k \cdot T} \tag{1-80}$$

$p = 101325$ [N/m^2]
$T = 273.15$ [K]
$K = 1.380 \cdot 10^{-23}$ [Nm/K]

$$n = \frac{101325 \frac{N}{m^2}}{1.38 \cdot 10^{-23} \frac{Nm}{K} \cdot 273.15 K} = 2.686 \cdot 10^{25} \left[\frac{1}{m^3}\right]$$

Consequently, the particle density *n* determined for the standard temperature and pressure corresponds to the Loschmidt's Number N_L.

Dalton's law, gas mixtures. According to the Dalton's law, the total pressure of a gas mixture is the sum of the partial pressures of the gas particles, as the individual gas particles fill the whole chamber as if other particles did not exist.

The calculation of a gas mixture can occur according to equation (1-81).

According to the general state equation

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

the following is applicable for the gas mixture:

$$p \cdot V = (m_1 \cdot R_1 + m_2 \cdot R_2 + \ldots\ldots + m_k \cdot R_k) \cdot T \tag{1-81}$$

V total volume [m^3]

For the calculation of the total pressure, equation (1-82) is used.

$$p = \Sigma p_1 + p_2 + \ldots + p_k \tag{1-82}$$

p	total pressure [N/m^2; Pa; bar]
p_1, p_2, p_k	partial pressures

Example 1.7: [1.6]
How many liters does 1 gram water comprise at 1 Torr and +15 °C?

Solution:
As water vapor of 1 Torr begins to condense only at –17 °C, it can be considered as an ideal gas at 15 °C.

The calculation occurs acc. to equation (1-17).

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

m = 1 gram = 0.001 [kg]
T = 288 [K]
P = 1 Torr = 1.36 mbar = $1.36 \cdot 10^2 \left[\frac{N}{m^2}\right]$

$$R_o = 8314 \left[\frac{J}{kmol \cdot K}\right]$$

$$M_{H_2O} = 18.02 \left[\frac{kg}{kmol}\right]$$

From

$$R_o = R \cdot M \tag{1-67}$$

results

$$R_{H_2O} = \frac{R_o}{M_{H_2O}} = \frac{8314 \frac{J}{kmol \cdot K}}{18{,}02 \frac{kg}{Kmol}} = 461{,}4 \frac{J}{kg \cdot K} = 461{,}4 \left[\frac{Nm}{kg \cdot K}\right]$$

$$V = \frac{m \cdot R \cdot T}{p} = \frac{0.001\,\text{kg} \cdot 461.4 \frac{\text{Nm}}{\text{kg} \cdot \text{K}} \cdot 288\,\text{K}}{1.36 \cdot 10 \frac{\text{N}}{\text{m}^2}} = 0.977\,\text{m}^3 = 977\,[\text{l}]$$

Example 1.8: [1.6]

Which gas volume results from an oil spot with a mass of 1 mg, if it is exposed to an absolute pressure of 10^{-8} Torr and a temperature of 127 °C?

Solution:

Surmised molecular weight of oil is 200 kg/kmol.

The calculation of the gas volume occurs acc. to equation (1-17).

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

$$V = \frac{m \cdot R \cdot T}{p} = \frac{\text{kg} \cdot \frac{\text{Nm}}{\text{kg} \cdot \text{K}} \cdot \text{K}}{\frac{\text{N}}{\text{m}^2}} \left[\text{m}^3\right]$$

$m = 0.000001\,[\text{kg}]$

$T = 400\,[\text{K}]$

$p = 10^{-8}\,\text{Torr} = 1.36 \cdot 10^{-8}\,\text{mbar} = 1.36 \cdot 10^{-6} \frac{\text{N}}{\text{m}^2}$

$$\text{From } R_o = RM, \tag{1-67}$$

results

$$R_{\text{oil}} = \frac{R_o}{M_{\text{oil}}} = \frac{8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}}{200 \frac{\text{kg}}{\text{kmol}}} = 41.57 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 41.57 \left[\frac{\text{Nm}}{\text{kg} \cdot \text{K}}\right]$$

$$V = \frac{m \cdot R \cdot T}{p} = \frac{0.000001\,\text{kg} \cdot 41.57 \frac{\text{Nm}}{\text{kg} \cdot \text{K}} \cdot 400\,\text{K}}{1.36 \cdot 10^{-6} \frac{\text{N}}{\text{m}^2}}.$$

$V = 12227\text{m}^3 = 12227000\,[\text{l}]$

Under the conditions mentioned above the oil spot will expand to a volume of 12,227,000 liters.

Example 1.9: [1.6]

In a vessel, there is a pressure of 10^{-10} Torr at a temperature of 0 °C.

How many molecules are there in one cm^3?

Solution:

The number of particles can be determined according to equation (1-80).

$$n = \frac{p}{k \cdot T} = \frac{\frac{N}{m^2}}{\frac{Nm}{K} \cdot K} \left[\frac{1}{m^3}\right] \tag{1-80}$$

$p = 10^{-10}$ Torr $= 1.36 \cdot 10^{-8} \frac{N}{m^2}$
$k = 1.38 \cdot 10^{-23} \frac{Nm}{K}$
$T = 273.15$ K

$$n = \frac{1.36 \cdot 10^{-8} \frac{N}{m^2}}{1.38 \cdot 10^{-23} \frac{Nm}{K} \cdot 273.15 K} = 3.607 \cdot 10^{12} \left[\frac{1}{m^3}\right]$$

$$n = 3.607 \cdot 10^{12} \frac{1}{m^3} \cdot \frac{1}{1000000} \cdot \frac{m^3}{cm^3} = 3.607 \cdot 10^{6} \left[\frac{1}{cm^3}\right]$$

There are $3.607 \cdot 10^6$ molecules in 1 cm^3.

1.5.3
Standard temperature and pressure

Standard temperature and pressure is the state of a liquid, solid or gaseous substance as defined in DIN 1343 Standard issued in November 1975.

The standard temperature $T_n = 273.15$ K, $\delta = 0°$ [C].
The standard pressure $p_s = 101325$ Pa $= 1013.25$ [mbar].

In order to enable the comparison of gases, a standard temperature and a standard pressure are taken as a basis. The standard volume is the volume a gas takes up at a certain pressure ($p = 760$ Torr $= 1.01325 \cdot 10^5$ Pa) and a certain temperature ($t = 0$ °C $= 273.15$ K). Therefore, the standard volume (also defined as Nm^3) is an exactly defined quantity of gas.

Example 1.10:
The mass of the standard volume of 10 Nm^3 of air is to be determined.

Solution:
The mass is calculated according to equation (1-17).

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

$$m = \frac{V \cdot p}{R \cdot T}$$

$V = 10$ [Nm^3]
$P_s = 1.013 \cdot 10^5$ [N/m^2]
$T_n = 273.15$ [K]

$$R_{air} = 0.2871\,\frac{kJ}{kg{\cdot}K} = 287.1\,\left[\frac{Nm}{kg{\cdot}K}\right] \qquad \text{(from table 1-15)}$$

$$m = \frac{10\ m^3{\cdot}1.013{\cdot}10^5\,\frac{N}{m^2}}{287.1\,\frac{Nm}{kg{\cdot}K}{\cdot}273.15\ K} = 12.92\ [kg]$$

According to the Avogadro law, a mole of any gas takes up the same volume at the same pressure and the same temperature. This is the molar volume.

For ideal gases, the molar volume at standard temperature and pressure is

V_m= 22.41 [m^3/kmol]

Example 1.11

The density of a substance at standard temperature and pressure is to be determined from molar weight and molar volume.

Solution:

$$\rho_n = \frac{M}{V_m} \tag{1-70}$$

ρ_n density at standard temperature and pressure [kg/m^3].

According to this, the density of nitrogen is calculated as follows:

$$M_{N2} = 28.01\left[\frac{kg}{kmol}\right] \qquad \text{(from the table 1-15)}$$

$$\rho_n = \frac{28.01\,\frac{kg}{kmol}}{22.41\,\frac{m^3}{kmol}} = 1.249\left[\frac{kg}{m^3}\right]$$

1.5.4
Real gases and vapors

In real gases and vapors, the forces of attraction between the particles cannot be neglected as it is the case with ideal gases that do not dispose of specific volumes and the particles of which do not exercise interaction forces on each other. Compared to the volume of the vacuum vessel provided for the gases and vapors, the specific volume of real gases and vapors is not very low and cannot be neglected.The state equation according to Van der Waal [comp. equation (1-83 and (1-84)] records this state. [1.1]

$$\left[p + \left(\frac{a{\cdot}m^2}{V^2}\right)\right](V - m \cdot b) = \frac{N{\cdot}mu}{M}\cdot\ R_o \cdot T \tag{1-83}$$

or for practical application:

$$\left[p + \left(\frac{a \cdot m^2}{V^2}\right)\right](V - m \cdot b) = m \cdot R \cdot T \tag{1-84}$$

The specific volume and the forces of attraction of the particles are considered by the constants a and b.

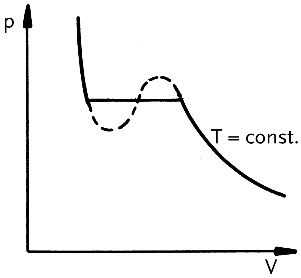


Figure 1-26. Isotherm curve of a real gas [1.1]

The curve trace (fig. 1-26) corresponds to the equation 1-83. However, in practice this kind of curve trace is hardly observed, more likely is a trace along the horizontal straight line caused by the condensation of gas. The trace of the broken line occurs only partly and in exceptional cases. The curve trace drawn in with the aid of a broken line corresponds to the instable behavior of supercooled vapor or overheated liquid.

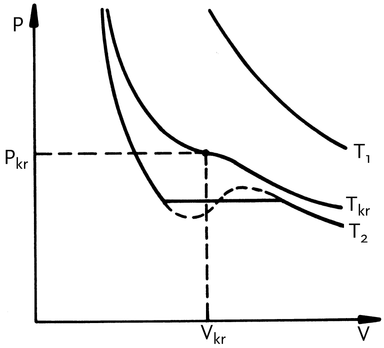


Figure 1-27. Complete p,V-diagram of a uniform chemical substance [1.1]

The curve trace in fig. 1-27 corresponds to a chemically homogeneous substance that does not decompose within the relevant temperature range.

With increasing temperature the distances between the minimum and the maximum diminish continuously and the inflectional tangent is getting flatter (fig. 1-27). At the critical temperature T_{kr}, the inflectional tangent is horizontal, and the inflection point still exists. This is the critical isotherm with the critical point p_{kr} appearing as inflection point. The inflection point has the coordinates $V=V_{kr}$ and

$p = p_{kr}$. Above the critical point, the gas cannot be condensed anymore, and the isotherms above the critical temperature show hyperbola characteristics. With increasing temperatures, the isotherms adopt more and more the trace corresponding to the state equation for ideal gases. The constants a and b can be calculated from the critical point data.

1.5.5
Phase transitions and their descriptions

For the transition from one phase to another, transition heat (latent heat quantity) is required or released. Their descriptions are given in the table 1-10.

Table 1-10. Phase transitions and their respective transition heat [1.1]

Phase transition from/to	Description of phase transition	Description of transition heat
Liquid/solid	congealing (for water: freezing)	solidification heat
Solid/liquid	melting	heat of fusion
Solid/gaseous	subliming	sublimation heat
Liquid/gaseous	vaporizing	vaporization heat
Gaseous/liquid	condensing	condensation heat
Gaseous/solid	condensing	condensation heat

Transition heats usually depend on temperature. For instance, vaporization heat decreases with increasing temperature until it reaches the value zero when reaching the critical point. Over the solid or liquid phase of a substance, there is always its gaseous phase. In the state of equilibrium, the gaseous phase adopts the saturation vapor pressure (fig. 1-28) which depends on the prevailing temperature acc. to equation (1-85) of Clausius and Clapeyron [1.1]. The quantity of heat required for phase transition or released through it at constant pressure corresponds to the enthalpy change and is specific for each material.

$$\frac{dp_s}{dT} = \frac{\Delta H}{T \cdot \Delta V} \tag{1-85}$$

p_s saturation vapor pressure
T thermodynamic temperature
ΔH transition heat = enthalpy change
ΔV volume change during the phase transition

The application of vacuum technology in process engineering requires the knowledge of vapor pressure of e.g. substances that are to be thermally treated. Comprehensive tables of temperature pressure and diagrams of pressure temperatures are available in specialist literature. In fig. 1-29, a graph shows pressure/temperature curves.

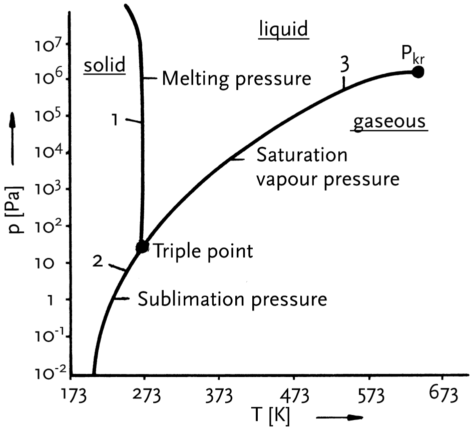


Figure 1-28. Dependence of melting temperature, sublimation and saturation temperature on pressure (water curves)
1) melting curve 2) sublimation curve 3) saturation curve

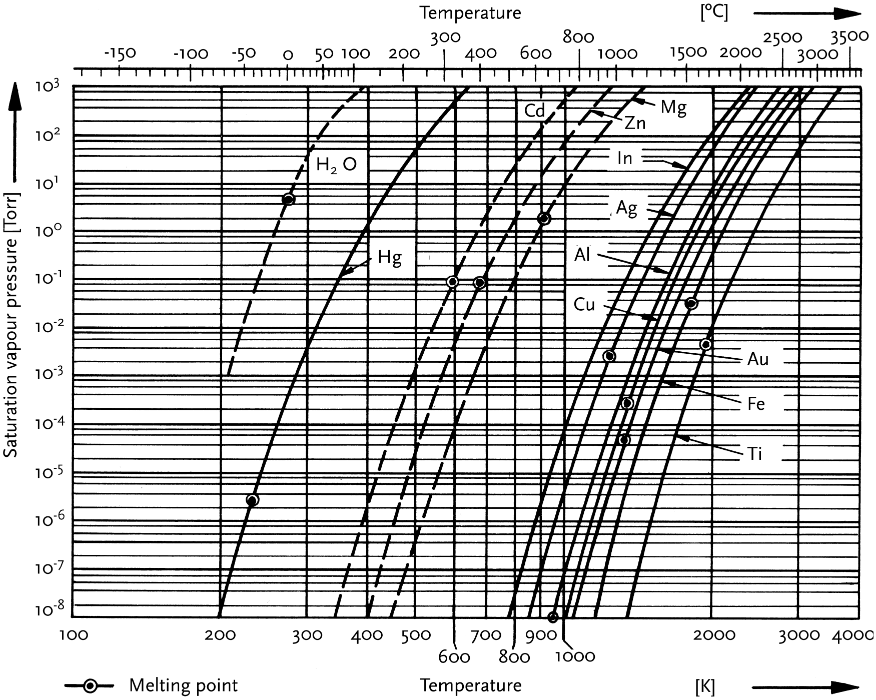


Figure 1-29. Pressure curves for saturated vapor of different substances [1.8].

Figure 1-29 contains the metals important in vacuum technology as well as those that should be avoided due to their high inherent vapor pressure (cadmium, zinc and magnesium). These metals are often contained in alloys and because of their high vapor pressure especially in high vacuum they can evaporate, contaminate the plant and deteriorate the vacuum. That is why brass in vacuum should never be exposed to temperatures over 200 °C, as otherwise zinc will evaporate.

The drawing up of vapor pressure curves as shown in fig.1-29 is often time-consuming and difficult. The depiction in the diagram, however, according to the relation

$$\lg p = f\left(\frac{1}{T}\right) \tag{1-86}$$

which is based on equation (1-85) by Clausius & Clapayron [1.2] is a simplification. Mostly, it is possible to depict the vapor pressure curves with sufficient accuracy for practical purposes as straight lines according to equation (1-87), (comp. fig. 1-30).

$$\lg p = -\frac{B}{T} + A \tag{1-87}$$

p [mbar]

A and B are material-specific values and are mostly (comp. tab. 1-11) found in literature.

$$A = \lg p_o \tag{1-88}$$

p_o vapor pressure at standard temperature and pressure

$$B = \frac{\Delta H \cdot \lg e}{m \cdot R} \tag{1-89}$$

ΔH transition heat
m gas mass
R specific gas constant

Table 1-11. Values A and B for lg p = f (l/T)-diagram [1.2].

Kind of gas/steam	Formula	t (°C)	A (A_s)	B (B_s)
bromobenzene	C_6H_5Br	155.6	8.20 (8.90)	–2230 (–2700)
heptane	C_7H_{16}	98	8.11	–1890
propyl alcohol	C_3H_7OH	97.20	9.01	–1800
water	H_2O	100	9.01 (10.52)	–2244 (–2660)

t evaporation temperature at 1013 bar
A, B for evaporation range
A_s, B_s for sublimation range

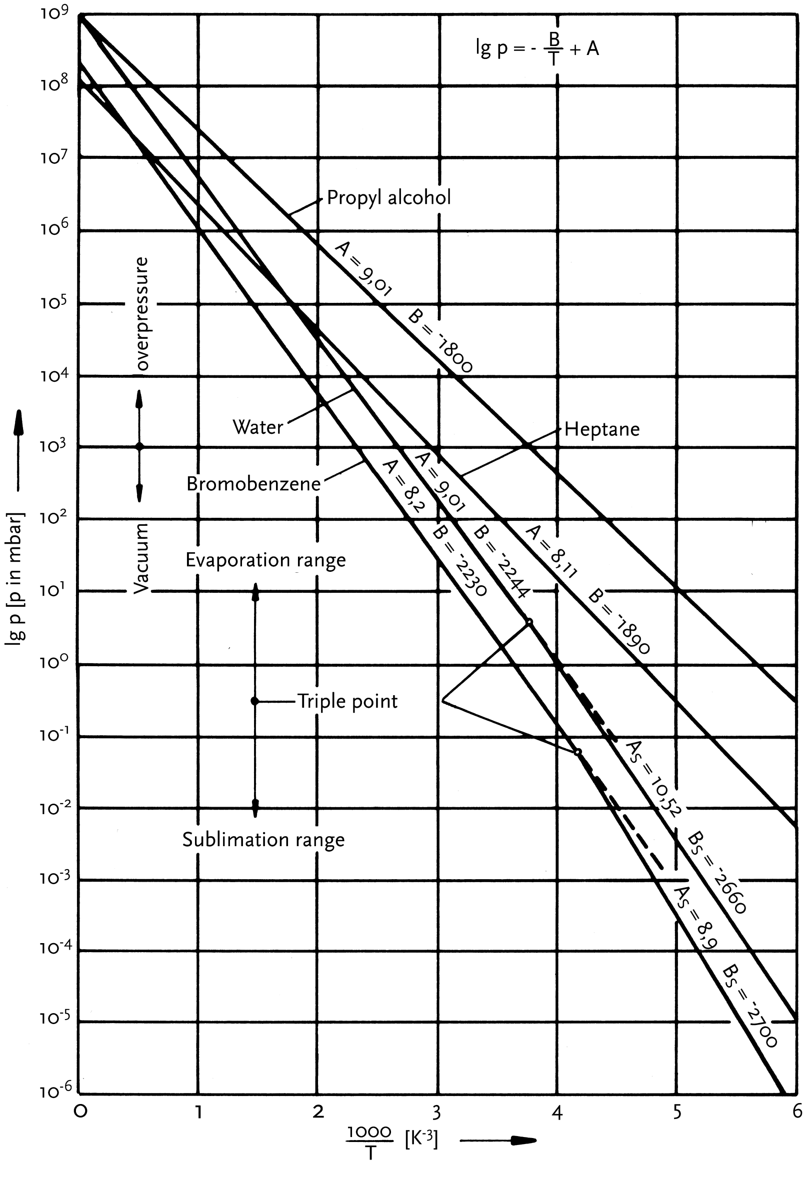


Figure 1-30. lg p = f (l/T)-diagram acc. to Clausius–Clapeyron [1.2]

1.6
Mixtures of ideal gases [1.9]

In the following, mixtures of ideal individual gases are dealt with that

a) do not chemically react with each other
b) exist in mechanical and thermal equilibrium.

Generally it can be said that a mixture of ideal individual gases behaves like an ideal gas. For instance, air is a gas mixture consisting mainly of the individual gases nitrogen and oxygen.

For the calculation of gas mixtures, volumes and mass portions of individual components as well as their partial pressures, molar masses and gas constants must be known.

In practice, the determination of the individual gas portions in a gas mixture is particularly interesting. The interrelations and different solutions are described below.

1.6.1
Mass composition

If a mixture consists of

$m_1 = 3\ \text{kg}\ H_2$
$m_2 = 1\ \text{kg}\ N_2$
$m_3 = 4\ \text{kg}\ CH_4$

then the total mass is

$$m = m_1 + m_2 + m_3 \qquad (1\text{-}90)$$

If a gas mixture consists of *k* components, the following can be generally formulated:

$$m = m_1 + m_2 + \ldots + m_k \qquad (1\text{-}91)$$

$$m = \sum_{i=1}^{k} m_i \qquad (1\text{-}92)$$

The composition of gas mixtures can be stated e. g. in mass portions ξ.

Referring to the total mass of a gas mixture, the mass portion of a gas has the value ξ, and can be determined as follows:

$$\xi_i = \frac{m_i}{m} \qquad (1\text{-}93)$$

ξ_i mass portion
m_i mass of components

m total mass of gas mixture

With three different gases, for the mixture results:

$$\frac{m_1}{m_1 + m_2 + m_3} + \frac{m_2}{m} + \frac{m_3}{m} = \frac{m}{m} \tag{1-94}$$

$$m = m_1 + m_2 + m_3$$

$$\xi_1 + \xi_2 + \xi_3 = 1 \tag{1-95}$$

Or in general terms

$$\sum_{i=1}^{k} \xi_i = 1 \tag{1-96}$$

This confirms that the addition of all mass portions existing in a gas mixture has the value 1.

1.6.2
Molar composition

The composition of a gas mixture can also be determined by the quantity of the substance n, e.g. in kmol resulting in the following relation:

$$n = \sum_{i=1}^{k} n_i \tag{1-97}$$

ψ_1 is taken as a base for molar portion:

$$\psi_1 = \frac{n_i}{n} \tag{1-98}$$

and

$$\sum_{i=1}^{k} \psi_i = 1 \tag{1-99}$$

corresponding to the examination of the mass portions.

1.6.3
Volumetric composition

Ideal gas mixtures are often calculated according to the volume percentage r. The following consideration refers to a mixture of three different gas quantities at the same pressure and temperature. The gases are ideal gases (fig. 1-31).

Before mixing, the three gas quantities are divided by three thin walls, however all three quantities are under exactly the same pressure of e.g. $p = 0.3$ mPa, and all three percentages have the same temperature of $T = 300$K.The three individual percentages can be regarded as partial volume V_1, V_2, and V_3. As temperatures and

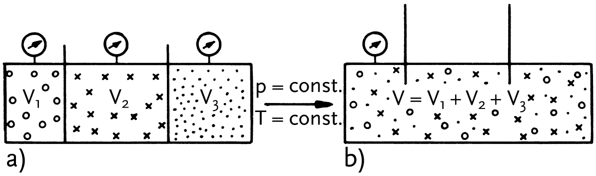


Figure 1-31. Composition of the ideal gas mixture [1.9]
a) before mixing, b) after mixing

pressures of the three components are the same, the volume of a quantity of substances existing in the mixture will have the partial volume described above after the dividing walls have been removed.

$$V = V_1 + V_2 + V_3 = \text{total volume} \tag{1-100}$$

Therefore, the volume percentages r is calculated according to the ratio of the volume proportions:

$$r_1 = \frac{V_1}{V} \tag{1-101}$$

$$r_2 = \frac{V_2}{V} \tag{1-102}$$

$$r_3 = \frac{V_3}{V} \tag{1-103}$$

Generally:

$$r_i = \frac{V_i}{V} \tag{1-104}$$

and

$$\sum_{i=1}^{k} r_1 = 1 \tag{1-105}$$

After mixing, the total pressure of the mixture is $P = 0.3$ MPa and the mixture temperature is $T = 300$ K. Therefore, the total pressure is equal to the pressure of each component in the mixture.

1.6.4
Ideal gas mixtures and general equation of gas state

Examining the individual gas proportions during the mixing of different gases it can be seen that the proportions expand from their partial volume V_i to the volume V of the gas mixture, which causes the pressure of each proportion to drop from its original value p before mixing to the partial pressure p_i after mixing (fig 1-32).

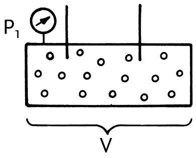


Figure 1-32. State of gas component 1 in the mixture [1.9]

Before mixing the three gases mentioned above, component 1 is under pressure p and has the partial volume V_1. According to the state equation for ideal gases, for the partial volume V_1 or partial volumes V_2or V_3 the following is applicable:

$$p \cdot V_1 = m_1 \cdot R_1 \cdot T \tag{1-106}$$

and the partial volume is

$$V_1 = \frac{m_1 \cdot R_1 \cdot T}{p} \tag{1-107}$$

During the mixing process, each single gas expands to the total available volume V. This process corresponds to the *Dalton Law.*

The pressure of component 1 decreases to partial pressure p_1. For component 1, the following state equation results:

$$p_1 \cdot V = m_1 \cdot R_1 \cdot T \tag{1-108}$$

and the partial pressure is

$$p_1 = \frac{m_1 \cdot R_1 \cdot T}{V} \tag{1-109}$$

This finding can be formulated as follows:

The partial pressure is the pressure the ideal individual gas would exert onto the walls, if this gas alone filled the whole volume V at the mixing temperature T.

Partial pressures cannot be directly and individually measured. The measurable total pressure p is equal to the sum of the individual partial pressure p_i.

$$p = \sum p_1 + p_2 + p_3 \tag{1-82}$$

Generally:

$$p = \sum_{i=1}^{k} p_i \tag{1-110}$$

as well as

$$p \cdot V_1 = p_1 \cdot V \tag{1-111}$$

and

$$\frac{V_1}{V} = \frac{p_1}{p} \qquad (1\text{-}112)$$

and also

$$\frac{V_1}{V} = r_1 \qquad (1\text{-}101)$$

The following ensues

$$r_1 = \frac{V_1}{V} = \frac{p_1}{p} \qquad (1\text{-}113)$$

Generally

$$r_i = \frac{p_i}{p} = \frac{V_i}{V} \qquad (1\text{-}114)$$

or

$$p_i = r_i \cdot p \qquad (1\text{-}115)$$

The partial pressure = volume percentage · total pressure

$$p \cdot V_2 = p_2 \cdot V \qquad (1\text{-}116)$$

goes also for component 2 and

$$\frac{V_1}{V_2} = \frac{p_1}{p_2} = \frac{r_1}{r_2} \qquad (1\text{-}117)$$

The result is: in a mixture of ideal gases, the partial pressures behave as the volume percentage.

1.7
Gas mixtures and their calculation [1.9] [1.10]

In thermodynamic calculations, gas mixtures can be treated like simple gases, as from gas mixtures the average values of densities, specific volumes, molar masses, gas constants, specific heat capacities and other quantities of the percentage components can be calculated (see also chapter 6.6.).

1.7.1
Density of an ideal gas mixture

The total mass

$$m = m_1 + m_2 + \cdots + m_k \tag{1-91}$$

or

$$V \cdot \rho_m = V_1 \cdot \rho_1 + V_2 \cdot \rho_2 + \cdots + V_k \cdot \rho_k \tag{1-118}$$

$$\rho_m = \left(\frac{V_1}{V}\right) \cdot \rho_1 + \left(\frac{V_2}{V}\right) \cdot \rho_2 + \cdots + \left(\frac{V_k}{V}\right) \cdot \rho_k \tag{1-119}$$

$$\rho_m = r_1 \cdot \rho_1 + r_2 \cdot \rho_2 + \cdots + r_k \cdot \rho_k \tag{1-120}$$

ρ density
ρ_m gas mixture density

and generally,

$$\rho_m = \sum_{i=1}^{k} r_i \cdot \rho_i \tag{1-121}$$

The densities ρ_i of the individual gases refer to the total pressure p, e.g. acc. to equation (1-106) it results:

$$p = \rho_1 \cdot R_1 \cdot T \tag{1-122}$$

Generally

$$\rho_i = \frac{p}{R_i \cdot T} \tag{1-123}$$

The density ρ_m of an ideal gas mixture is equal to the sum of the products of the volume percentage and the density of the individual gases.

1.7.2
Molar mass of gas mixture

The mean molar mass M_m is a pure operand value without physical meaning, as molecules of gas mixtures do not exist. The mean or apparently molar mass of an ideal gas mixture is calculated in compliance with the following equation:

$$\rho = \frac{M}{V_m} \tag{1-70}$$

V_m = molar volume at 0 °C and 1013 mbar

For the ideal gas mixture, the following is applicable:

$$\frac{M_m}{\bar{v}_m} = \frac{r_1 \cdot M_1}{\bar{v}_1} + \frac{r_2 \cdot M_2}{\bar{v}_2} + \cdots + \frac{r_k \cdot M_k}{\bar{v}_k} \tag{1-124}$$

The specific quantities are indicated with the symbol of the extensive value, however in small letters. Molar quantities are indicated in small letters with a line above.

$\bar{v}_m$	specific molar volume of a gas mixture
$\bar{v}_1, \bar{v}_2, \bar{v}_k$	specific molar volumes of individual gases

Acc. to Avogadro, molar volumes of all ideal gases are equally high at the same pressure and the same temperature.

The result is:

$$\bar{v}_m = \bar{v}_1 = \bar{v}_2 = \ldots = \bar{v}_k \tag{1-125}$$

and

$$M_m = r_1 \cdot M_1 + r_2 \cdot M_2 + \cdots + r_k \cdot M_k \tag{1-126}$$

or, generally

$$M_m = \sum_{i=1}^{k} r_i \cdot M_i \tag{1-127}$$

The mean molar mass of an ideal gas mixture is equal to the sum of the products of volume percentage and molar mass of each individual gas.

With standard temperature and pressure of a gas mixture

$$\rho_{mn} = \frac{M_m}{V_m} \tag{1-128}$$

ρ_{mn} mean standard density at 0 °C and 1013 mbar

1.7.3
Gas constant of an ideal gas mixture

The universal gas constant R_o has the same value for all gases.

The product $M \cdot R$ represents the gas constant R_o referred to a kmol.

The following applies

$$R_o = R \cdot M \tag{1-67}$$

and for the gas mixture, the relation $R_o = R_m \cdot M_m$ or rather

$$R_m = \frac{R_o}{M_m} \tag{1-129}$$

is applicable.

R_m = gas constant of ideal gas mixture

After mixing, the following relationship results if the mass proportions ξ_i, e.g. of three gases are known:

$$p_1 \cdot V = m_1 \cdot R_1 \cdot T \tag{1-108}$$

$$p_2 \cdot V = m_2 \cdot R_2 \cdot T \tag{1-130}$$

$$p_3 \cdot V = m_3 \cdot R_3 \cdot T \tag{1-131}$$

through addition, it results

$$(p_1 + p_2 + p_3) \cdot V = (m_1 \cdot R_1 + m_2 \cdot R_2 + m_3 \cdot R_3) \cdot T \tag{1-132}$$

$$p = p_1 + p_2 + p_3$$

$$p \cdot V = m\left(\frac{m_1 \cdot R_1}{m} + \frac{m_2 \cdot R_2}{m} + \frac{m_3 \cdot R_3}{m}\right) \cdot T \tag{1-133}$$

For the gas mixture the following applies:

$$p \cdot V = m \cdot R_m \cdot T \tag{1-134}$$

and

$$R_m = \xi_1 \cdot R_1 + \xi_2 \cdot R_2 + \xi_3 \cdot R_3 \tag{1-135}$$

or generally

$$R_m = \sum_{i=1}^{k} \xi_i \cdot R_i \tag{1-136}$$

In an ideal gas mixture, the molar proportion ψ_i is equal to the volume percentage r_i, i.e. in the equations (1-115), (1-121) and (1-127) r_i can be replaced by ψ_i.

The mean gas constant of an ideal gas mixture is equal to the sum of the products of the mass proportions and the gas constant of each individual gas.

1.7.4
Relation between mass proportions and volume percentage

$$m_1 = \rho_1 \cdot V_1 \tag{1-137}$$

$$m = \rho_m \cdot V \tag{1-138}$$

consequently:

$$\frac{m_1}{m} = \frac{\rho_1}{\rho_m} \cdot \frac{V_1}{V} \tag{1-139}$$

or generally

$$\frac{m_i}{m} = \frac{V_i}{V} \cdot \frac{\rho_i}{\rho_m} \tag{1-140}$$

and

$$\xi_i = \frac{r_i \cdot \rho_i}{\rho_m} \tag{1-141}$$

For the density ratio in equation (1-141), the following equations can be elaborated:

$$\rho_m = \frac{M_m}{\bar{v}} \tag{1-142}$$

$$\rho_i = \frac{Mi}{\bar{v}} \tag{1-143}$$

and

$$\frac{\rho_i}{\rho_m} = \frac{M_i}{M_m} \tag{1-144}$$

thus

$$\xi_i = \frac{r_i \cdot M_i}{M_m} \tag{1-145}$$

For gas 1 the following is applicable:

$$p_1 \cdot V = m_1 \cdot R_1 \cdot T \tag{1-108}$$

and for the gas mixture

$$p \cdot V = m \cdot R_m \cdot T \tag{1-134}$$

Thus

$$\frac{p_1}{p} = \frac{m_1}{m} \cdot \frac{R_1}{R_m} \tag{1-146}$$

and

$$r_1 = \frac{\xi_1 \cdot R_1}{R_m} \tag{1-147}$$

generally

$$\xi_i = \frac{r_1 \cdot R_m}{R_i} \tag{1-148}$$

1.7.5
Gas laws and their special application in vacuum technology

Clapeyron's state equation for ideal gases

$$V_{total} = \frac{R_o \cdot \left(\frac{m_1}{M_1} + \frac{m_2}{M_2} + \cdots + \frac{m_k}{M_k} \right) \cdot T}{p_{total}} \qquad (1\text{-}149)$$

For calculation purposes, gases and vapors are supposed to be ideal.

V_{total}	Total volume of gas and/or vapor mixture [m^3]
R_o	molar (general) gas constant $= 83.14$ [(mbar · m^3)/(kmol · K)]
m_1, m_2	mass of the different gases or vapors [kg]
M_1, M_2	molar mass (relative molecule masses, molecular weight) [kg/kmol]
T	temperature of the gas-vapor mixture [K]
p_{total}	total pressure of the mixture [mbar]

$p_{total} = p_1 + p_2 + \ldots + p_k$

Partial pressure in a gas and/or vapor mixture

The formula is to be used for vapors only in case of the liquids not being soluble in each other.

$$p_1 = p_{total} \cdot \frac{\frac{m_1}{M_1}}{\frac{m_1}{M_1} + \frac{m_2}{M_2} + \cdots + \frac{m_k}{M_k}} \qquad (1\text{-}150)$$

p	pressures [mbar]
m	masses [kg]
M	molar mass [kg/kmol]

Mass of an individual gas or vapor in a gas-vapor mixture

$$m_1 = m_2 \cdot \frac{M_1}{M_2} \cdot \frac{p_1}{p_2} \qquad (1\text{-}151)$$

p_1, p_2	partial pressures [mbar]
m_1, m_2	masses [kg]
M_1, M_2	molar masses [kg/kmol]

Dalton's Law

This law is also applicable for gas/vapor mixtures.

$$\frac{V_1}{V_2} = \frac{p_1}{p_2} = \frac{p_{total} - p_2}{p_2} \qquad (1\text{-}152)$$

$$p_{total} = p_1 + p_2 \tag{1-153}$$

V_1, V_2 volumes [m^3]
p_1, p_2 partial pressures [mbar]
p_{total} total pressure [mbar]

Example 1.12

For a fuel gas with the following composition in vol. % or rather volume percentages it has to be determined:

1) density at standard temperature and pressure ρ_{mn},
2) fuel gas composition in mass proportions ξ_i,
3) molar mass M_m
4) gas constant R_m
5) standard volume flow rate in m^3/h with the fuel gas flowing through a pipe having a diameter of $d_i = 200$ mm at $t = 20\,°C$, $p = 2.5$ bar, $c = 18$m/s

The standard density ρ_{in} of the individual gases can be taken from table 1-12.

Fuel gas composition

No.	Gas	Vol. %	r_i
1	H_2	42.0	0.420
2	CO	14.0	0.140
3	CH_4	33.0	0.330
4	C_2H_4	3.5	0.035
5	CO_2	3.0	0.030
6	N_2	4.5	0.045
		100.0 vol. %	1.000

Table 1-12. Standard densities of some gases

Gas	H_2	CO	CH_4	C_2H_4	CO_2	N_2
Standard density ρ_n [kg/m^3]	0.09	1.25	0.717	1.26	1.977	1.25

Solution:

$$1)\ \rho_{mn} = \sum_{i=1}^{k} r_i \cdot \rho_{in} \tag{1-121}$$

$$2)\ \xi_i = \frac{r_i \cdot \rho_{in}}{\rho_{mn}} \tag{1-141}$$

Results for 1) and 2)

No.	**Gas**	$\mathbf{r_i}$	ρ_{in} **[kg/m³]**	$r_i \cdot \rho_{in}$ **[kg/m³]**	$\xi_i = \frac{r_i \cdot \rho_{in}}{\rho_{mn}}$
1	H_2	0.420	0.090	0.0378	$\xi_i = \frac{0.0378}{0.6091} = 0.0621$
2	CO	0.140	1.250	0.1750	$\xi_2 = \frac{0.1750}{0.6091} = 0.2873$
3	CH_4	0.330	0.717	0.2366	$\xi_3 = \frac{0.2366}{0.6091} = 0.3884$
4	C_2H_4	0.035	1.260	0.0441	$\xi_4 = \frac{0.0441}{0.6091} = 0.0724$
5	CO_2	0.030	1.977	0.0593	$\xi_5 = \frac{0.0593}{0.6091} = 0.0974$
6	N_2	0.045	1.250	0.0563	$\xi_6 = \frac{0.0563}{0.6091} = 0.0924$
		$\Sigma r_i = 1.000$	$\rho_{mn} = 0.6091 [\text{kg/m}^3]$		$\Sigma \xi_i = 1.0000$

3) The molar mass can be determined acc. to equation (1-127).

$$M_m = \sum_{i=1}^{k} r_i \cdot M_i \tag{1-127}$$

For the gas mixture at standard temperature and pressure, the molar mass is easier calculated acc. to equation (1-128):

$$\rho_{mn} = \frac{M_m}{V_m} \tag{1-128}$$

and

$$M_m = \rho_{mn} \cdot V_m$$

$$\rho_{mn} = 0.6091 \left[\frac{\text{kg}}{\text{m}^3}\right]$$

$$V_m = 22.41 \left[\frac{\text{m}^3}{\text{kmol}}\right]$$

$$Mm = 0.6091 \frac{\text{kg}}{\text{m}^3} \cdot 22.41 \frac{\text{m}^3}{\text{kmol}} = 13.649 \frac{\text{kg}}{\text{kmol}}$$

4) Acc. to equation (1-129), the gas constant can be determined:

$$R_m = \frac{R_o}{M_m} \tag{1-129}$$

$$R_m = \frac{8.314\frac{kJ}{kmol \cdot K}}{13.649\frac{kg}{kmol}} = 0.6091\left[\frac{kJ}{kg \cdot K}\right]$$

5) The volume flow rate at current conditions is

$$\dot{V} = F\,(m^2) \cdot v\left(\frac{m}{s}\right) \cdot \frac{3600 \cdot s}{1h}\left[\frac{m^3}{h}\right] \qquad (1\text{-}154)$$

$$\dot{V} = \frac{0.20^2 \cdot \pi}{4} m^2 \cdot 18\left(\frac{m}{s}\right) \cdot \frac{3600 \cdot s}{h} = 2034.72\left[\frac{m^3}{h}\right]$$

at p = 2.5 bar and t = 20 °C.

The conversion into standard temperature and pressure values is carried out according to the thermal state equation (1-166) for ideal gases.

$$\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2} \qquad (1\text{-}166)$$

$p_1 = 1013$ [mbar]
$T_1 = 273.15$[K]
$p_2 = 2500$ [mbar]
$T_2 = 293$ [K]

$$\dot{V}_2 = 2034.72\left[\frac{m^3}{h}\right]$$

$$\dot{V}_1 = \frac{p_2 \cdot \dot{V}_2 \cdot T_1}{T_2 \cdot p_1}$$

$$\dot{V}_1 = \frac{2500\,mbar \cdot 2034.72\frac{m^3}{h} 273.15K}{293K \cdot 1013\,mbar}$$

$\dot{V}_1 = 4681.32\,[m^3/h]$ at 1013 mbar and 0 °C.
$\dot{V}_1 = 4681.32\,[Nm^3/h]$

Example 1.13 [1.6]
In table 1-13, the composition of dry atmospheric air in volume percents is shown.

1) Partial pressures are to be calculated for the given individual gases with 760 Torr and 1 Torr total pressure.
2) In the air of 760 Torr, a partial pressure of water vapor of 12.3 Torr is assumed. Which are the partial pressures of the other individual gases?
3) The mass of dry atmospheric air consists of vol. % values according to table 1-13. How much is the molar mass of the air?

Table 1-13. The composition of atmospheric air

	air	H_2O	nitrogen	oxygen	argon	carbon dioxide
Vol. %	100.00	0	78.09	20.95	0.93	0.03

Solution:

1) Total pressure $p = 760\,\text{Torr} = p_1 + p_2 + p_3 + p_4$

$$\frac{V_1}{V} = \frac{p_1}{p} \tag{1-112}$$

or rather

$$\frac{p_2}{p} = \frac{V_2}{V}$$

Partial pressure can be calculated, for instance, for nitrogen as follows

$$p_1 = \frac{V_1}{V} \cdot p$$

$$p_1 = \frac{78.09}{100} \cdot \text{Torr} = 593.484\,\text{Torr} = \text{partial pressure of } N_2$$

Dry air	V_1 nitrogen N_2	V_2 oxygen O_2	V_3 argon Ar	V_4 carbon dioxide CO_2
Vol. %	78.09	20.95	0.93	0.03
Total pressure	p_1	p_2	p_3	p_4
p = 760 [Torr]	593.484 [Torr]	159.22 [Torr]	7.068 [Torr]	0.228 [Torr]
Total pressure	p_1	p_2	p_3	p_4
p = 1 [Torr]	0.7809 [Torr]	0.2095 [Torr]	0.0093 [Torr]	0.0003 [Torr]

2) Total pressure $p = 760\,\text{Torr} = p_1 + p_2 + p_3 + p_4 + p_5$
Water vapor partial pressure: $p_1 = 12.30\,\text{Torr}$
$p - p_1 = 760 - 12.30 = 747.70\,\text{Torr}$

$$\frac{P_2}{P} = \frac{V_2}{V}$$

$$P_2 = \frac{V_2}{V} \cdot p = \frac{78.09}{100} \cdot 747.70\,\text{Torr} = 583.878\,\text{Torr} = \text{ partial pressure of } N_2$$

Air containing water vapor	V_1 water vapor H_2O	V_2 nitrogen N_2	V_3 oxygen O_2	V_4 argon Ar	V_5 carbon dioxide CO_2
Dry air, vol. %	–	78.09	20.95	0.93	0.03
Total pressure	p_1	p_2	p_3	p_4	p_5
p = 760 [Torr]	12.3 [Torr]	583.878 [Torr]	156.643 [Torr]	6.953 [Torr]	0.224 [Torr]

3) The mean molar mass of an ideal gas mixture is calculated according to the relation

$$M_m = \sum_{i=1}^{k} r_i \cdot M_i \qquad (1\text{-}127)$$

or rather acc. to formula

$$M_m = r_1 \cdot M_1 + r_2 \cdot M_2 + \cdots + r_k \cdot M_k \qquad (1\text{-}126)$$

Thus, for the product $r_i \cdot M_i$, for nitrogen the following value results:

$$r_1 \cdot M_1 = \frac{78.09}{100} \cdot 28 \frac{\text{kg}}{\text{kmol}} = 21.865 \left[\frac{\text{kg}}{\text{kmol}}\right]$$

Dry air	**V_1 nitrogen N_2**	**V_2 oxygen O_2**	**V_3 argon Ar**	**V_4 carbon dioxide CO_2**
Rel. molecular weight, [kg/kmol]	28	32	40	44
Vol. %	78.09	20.95	0.93	0.03
$r_i \cdot M_i$, [kg/kmol]	21.865	6.704	0.372	0.0132

Therefore, the molar mass of air is

$$M_m = \sum_{i=1}^{4} r_i \cdot M_i = 21.865 + 6.704 + 0.372 + 0.0132 \left[\frac{\text{kg}}{\text{kmol}}\right]$$

$$M_m = 28.954 \left[\frac{\text{kg}}{\text{kmol}}\right]$$

1.8
Discharge of gases and vapors

During the discharge of liquids, their volumes do not change with occurring discharge pressures, i.e. liquids are incompressible. Gases, however, change their volumes when being discharged according to the arising pressures. An ideal gas with mass m and molar mass M fulfils the state equation of ideal gases at pressure p, volume V and absolute temperature T.

1.8.1
General state equation of gas

$$p \cdot V = m \cdot \frac{R_o}{M} \cdot T \qquad (1\text{-}65)$$

General gas constant $R_o = 8314\,[\text{J}/(\text{kmol} \cdot \text{K})]$

For mixtures of ideal gases, the mean molar mass M_m of the mixture is inserted into the equation.

The ideal state equation is valid only for lower pressures where the intermolecular forces and the specific volume need not be considered. The lower the pressure of the gas is, the better these prerequisites are fulfilled. At a pressure of 3 MPa (30 bar), the state equation for ideal gases deviates from the real value by about ± 1% to 2%.

1.8.2
Real gas factor Z

At high pressures, gas type, temperature and pressure in the state equation must be included by the so-called real gas factor Z. Then the state equation goes as follows [1.3]:

$$p \cdot V = Z \cdot m \cdot \frac{R_o}{M} \cdot T \tag{1-155}$$

In ideal gas,

$$\frac{p \cdot v}{R \cdot T} = 1 \tag{1-156}$$

and in real gas

$$\frac{p \cdot v}{R \cdot T} = Z \tag{1-157}$$

$Z = 1$ ideal behaviour
$Z \gtrless 1$ real behaviour

The deviation from the ideal state increases the more the gas state approaches the range of gas condensation. The real gas factor of nitrogen is $Z = 0.99955$ at a pressure $p = 1013$ mbar and a temperature $T = 273$ K. Even at this low pressure, this gas does not behave according to the formula for ideal gas. However, the deviation from factor $Z = 1.000$ is insignificant.

In table 1-14, some real gas factors $Z = f_{(p,\ T)}$ for hydrogen and air are given.

Table 1-14. Real gas factors for hydrogen and air [1.9]

p [bar]	Z values for H_2				Z values for dry air		
	t [°C]				t [°C]		
	–100	0	100	200	0	100	200
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19.62	1.0130	1.0122	1.0098	1.0078	0.9895	1.0027	1.0064
49.05	1.0345	1.0307	1.0246	1.0196	0.9779	1.0087	1.0168
98.10	1.0756	1.0625	1.0492	1.0392	0.9699	1.0235	1.0364

The table above shows that for H_2 and air, $Z \approx 1$ at medium and low pressures. Thus in practice, the state equation for ideal gases is applicable for air and hydrogen with sufficient accuracy at pressures of up to approx. 20 bar. The ideal state equation can be applied to practically all gases without correction if it is about gases with very low condensation temperatures and with a gas state temperature being significantly higher than the condensation temperature of the gas. This applies also to gases such as nitrogen, oxygen, natural gas, town gas, helium, carbon dioxide or methane.

Consequently, the state equation for ideal gases can also be applied to all really existing gases, if the stated prerequisites are fulfilled.

The ideal gas state equation:

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

or

$$p \cdot V = m \cdot \frac{R_o}{M} \cdot T \tag{1-65}$$

Into the state equation, absolute pressures in Pa or N/m^2 and absolute temperatures in K have to be inserted.

1.8.3
General gas constant

From the state equation for an ideal gas according to equation (1-156), results

$$\frac{p \cdot v}{T} = R \tag{1-158}$$

with

$$V = m \cdot v \tag{1-66}$$

and

$$v = \frac{V}{m} \tag{1-159}$$

it results

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

and when introducing molar quantities, the gas mass (m) will be replaced by

$$m = N \cdot M \tag{1-160}$$

resulting in

$$p \cdot V = N \cdot M \cdot R \cdot T \tag{1-161}$$

The molar volume:

$$V_m = \frac{V}{N} \tag{1-162}$$

Dividing equation (1-161) by N, it results

$$p \cdot V_m = M \cdot R \cdot T \tag{1-163}$$

With

$$R_o = R \cdot M \tag{1-67}$$

results

$$p \cdot V_m = R_o \cdot T \tag{1-164}$$

For standard temperature and pressure, the following is applicable:

$$p_s \cdot V_m = R_o \cdot T_s \tag{1-165}$$

The product $R \cdot M$ is the gas constant R_o referring to a kmol and is called universal gas constant depending on the gas type. It has the same value for all ideal gases.

p	pressure [N/m^2]
V	volume [m^3]
m	mass [kg]
N	particle quantity,
T	thermodynamic temperature [K]
M	molar mass [kg/kmol]
R_o	general molar gas constant [Nm/(kmol · K)]
R	specific gas constant depending on molar mass [Nm/(kg · K)]
v	specific volume [m^3/kg]
V_m	molar volume 22.41[m^3/kmol]
p_s	standard pressure = 101325 [N/m^2]
T_s	standard temperature = 273.15 [K]

Example 1.14

The general (molar) gas constant is to be determined.

Solution:

The general state equation for 1 kmol at standard temperature and pressure:

$$p_s \cdot V_m = R_o \cdot T_s \tag{1-165}$$

$$R_o = \frac{p_s \cdot V_m}{T_s} = \frac{1.01325 \cdot 10^5 \frac{N}{m^2} \cdot 22.42 \frac{m^3}{kmol}}{273.15\ K}$$

$$R_o = 8314 \left[\frac{Nm}{kmol \cdot K}\right]$$

1.8.4

The special gas constant depending on the type of gas

Example 1.15

Determination of the special gas constant for air.

The molar mass for air is given in table 1-15.

Solution:

From the general gas constant, the special gas constant for air is calculated as follows:

$$R_o = R \cdot M \tag{1-67}$$

$$R = \frac{R_o}{M}$$

$$M_{air} = 28,96 \left[\frac{kg}{kmol}\right]$$

$$R_o = 8314 \frac{Nm}{kmol{\cdot}K} = 8314 \left[\frac{J}{kmol{\cdot}K}\right]$$

$$R_{air} = \frac{8314 \frac{J}{kmol{\cdot}K}}{28.96 \frac{kg}{kmol}} = 287.10 \left[\frac{J}{kg{\cdot}K}\right] = 0.2871 \left[\frac{kJ}{kg{\cdot}K}\right]$$

Table 1-15. Gas table (extract) [1.9]

Gas type	M [kg/kmol]	V_m [m^3 /kmol]	ρ_s [kg/m^3]	R [kJ/kgK]
H_2	2.016	22.43	0.0899	4.1243
He	4.003	22.43	0.1785	2.0042
N_2	28.01	22.41	1.250	0.2968
O_2	32.00	22.39	1.429	0.2598
Air	28.96	22.40	1.293	0.2871
CO	28.01	22.41	1.250	0.2968
CO_2	44.01	22.26	1.977	0.1889
H_2O vap.	18.02	22.40	0.804	0.4615
SO_2	64.07	21.89	2.926	0.1298
NH_3	17.03	22.08	0.771	0.4883
CH_4	16.04	22.38	0.717	0.5184
C_2H_2	26.04	22.17	1.175	0.3193
C_2H_4	28.05	22.26	1.260	0.2964
C_2H_6	30.07	22.17	1.357	0.2765
C_3H_8	44.10	21.94	2.010	0.1886
n-C_4H_{10}	58.12	21.28	2.732	0.1430

M – molar mass, V_m – molar volume, ρ_s – standard density at $t_s = 0\,°C$ and $p_s = 1.013 \cdot 10^5$ Pa, R – special gas constant

1.8.5
Thermal state equation for ideal gases

state 1 state 2

$$\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2} \qquad (1\text{-}166)$$

The formula is applicable for the same gas type, i.e. the special gas constant R is the same for both states 1 and 2, as well as the masses $m_1 = m_2$.

In state 1: $\frac{p_1 \cdot V_1}{T_1} = R$

In state 2: $\frac{p_2 \cdot V_2}{T_2} = R$

The Boyle & Marriot gas law $p_1 \cdot V_1 = p_2 \cdot V_2$ is applicable here at constant temperature, i.e. if $T_1 = T_2$.

$$\frac{p_2}{p_1} = \frac{V_1}{V_2} \qquad (1\text{-}167)$$

or rather

$$p \cdot V = \text{constant} \qquad (1\text{-}61)$$

The 1st law of Gay-Lussac is valid if p remains constant.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (1\text{-}168)$$

The 2nd law of Gay-Lussac is applicable in a special case when $V_1 = V_2$ remains unchanged.

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1\text{-}169)$$

Example 1.16 [1.9]

An oxygen cylinder with a capacity of 15 l has a temperature of 20 °C and an overpressure of 15 bar = 16 [bar (abs)], air pressure ≈ 1 bar.

1) How many kilograms of oxygen does the cylinder contain?
2) Which is the specific volume of the enclosed oxygen?
3) How high does the pressure rise, if, owing to insulation, the temperature of the gas rises to 67 °C?

Given: $V = 0.015\,[m^3]$, $T_1 = 293\,[K]$, $p_e = 15\,[bar]$, $p_1 = 16\,[bar]$, $t_2 = 67\,[\,°C]$, $T_2 = 340\,[K]$

Solution:

1) The weight is determined through equation 1-17:

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

$$R_o = 0.2598 \frac{kJ}{kg \cdot K} = 259.8 \left[\frac{Nm}{kg \cdot K}\right] \qquad \text{(from table 1-15)}$$

$$m = \frac{p \cdot V}{R \cdot T} = \frac{16 \cdot 10^5 \frac{N}{m^2} \cdot 0.015 m^3}{259.8 \frac{Nm}{kg \cdot K} \cdot 293.15 K} = 0.315\ [kg]$$

2) The specific volume is determined acc. to equation 1-159:

$$v = \frac{V}{m} \tag{1-159}$$

$$v = \frac{0.015\, m^3}{0.315\, kg} = 0.0476 \left[\frac{m^3}{kg}\right]$$

3) At constant volume, the pressure depending on temperature can be calculated acc. to equation (1-169):

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \tag{1-169}$$

$$V_1 = V_2$$

$$T_1 = 293.15\ [K]$$

$$T_2 = 340\ [K]$$

$$p_2 = \frac{p_1 \cdot T_2}{T_1} = \frac{16 \cdot 10^5 \frac{N}{m^2} \cdot 340\ K}{293.15\ K} = 18,56 \cdot 10^5 \left[\frac{N}{m}\right]$$

$p_2 = 18.56$ [bar (abs), resp. 17.56 bar overpressure].

1.8.6
Suction of dry gases and saturated air-water vapor mixture by liquid ring vacuum pumps

In this section, the suction behavior of liquid ring pumps under different influences is to be examined. With the aid of calculation examples, the operation behavior of those machines while pumping of dry and humid gas-vapor mixtures at different operating and coolant temperatures is explained. Possibilities to correct suction characteristics possibly appearing during operation are shown on the basis of generally valid or empirically determined factors. An example describes the application of a condenser installed on the suction side of the pump.

An overview of some physical laws applicable to the calculations is given below.

The general state equation

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

R specific gas constant [J/(kg · K)]

or

$$p \cdot V = m \cdot \frac{R_o}{M} \cdot T \tag{1-65}$$

R_o general universal gas constant = 8314 [J/(kmol · K)]

Clapeyron's state equation for ideal gases

$$V_{total} = \frac{R_o \cdot \left(\frac{m_1}{M_1} + \frac{m_2}{M_2} + \cdots + \frac{m_k}{M_k}\right) \cdot T}{p_{total}} \tag{1-149}$$

V_{total} total volume of a gas and/or vapor mixture [m^3]
R_o the molar (general) gas constant = 83.14 [(mbar · m^3)/(kmol · K)]
m_1, m_2 masses of different gases or vapors [kg]
M_1, M_2 molar masses (relative molecule masses, molecular weight) [kg/kmol]
T temperature of gas-vapor mixture [K]
p_{total} total pressure of the mixture [mbar]
$p_{total} = p_1 + p_2 + \ldots + p_k$

Partial pressure in a gas and/or vapor mixture

$$p_1 = p_{total} \cdot \frac{\frac{m_1}{M_1}}{\frac{m_1}{M_1} + \frac{m_2}{M_2} + \cdots + \frac{m_k}{M_k}} \tag{1-150}$$

For vapors only valid for liquids non soluble in one another.

p pressure [mbar]
m mass [kg]
M molar mass [kg/kmol]

Mass of an individual gas or an individual vapor in a gas–vapor mixture

$$m_1 = m_2 \cdot \frac{M_1}{M_2} \cdot \frac{p_1}{p_2} \tag{1-151}$$

p_1, p_2 partial pressure [mbar]
m_1, m_2 mass [kg]
M_1, M_2 molar mass [kg/kmol]

Dalton's Law

$$\frac{V_1}{V_2} = \frac{p_1}{p_2} = \frac{p_{total} - p_2}{p_2} \tag{1-152}$$

$$p_{total} = p_1 + p_2 \tag{1-153}$$

V_1, V_2	volumes [m^3]
p_1, p_2	partial pressures [mbar]
p_{total}	total pressure [mbar]

This law is also applicable for gas-vapor mixtures as follows:

$$\frac{V_G}{V_{vap}} = \frac{p_G}{p_{vap}} = \frac{p_{total} - p_{vap}}{p_{vap}} \tag{1-170}$$

$$\frac{V_{vap}}{V_G} = \frac{p_{vap}}{p_{total} - p_{vap}} \tag{1-171}$$

g	dry gas
vap	vapor

$p_{total} = p_G + p_{vap}$

p_G, p_{vap} partial pressures

The total volume of a gas-vapor mixture can be calculated acc. to equation (1-172).

$$V_{total} = \frac{83.14 \cdot \left(\frac{m_G}{M_G} + \frac{m_{vap}}{M_{vap}}\right) \cdot T}{p_{total}} \tag{1-172}$$

Individual volumes are also calculated using the same formula:

$$V_G = \frac{83.14 \cdot \frac{m_G}{M_G} \cdot T}{p_{total}} \tag{1-173}$$

$$V_{vap} = \frac{83.14 \cdot \frac{m_{vap}}{M_{vap}} \cdot T}{p_{total}} \tag{1-174}$$

Basically, with $V_{vap}/V_G \geqq 1$, the probability of vapor condensation in front of the pump should be controlled.

Example 1.17

Given: Dry-saturated mixture of gas and water vapor to be sucked.

$\dot{m}_{\text{total}} = 92.5$ [kg/h]
Individually consisting of:
$\dot{m}_1 = 40$ [kg/h], methanol CH_4O
$\dot{m}_2 = 41.3$ [kg/h], air
$\dot{m}_3 = 11.2$ [kg/h], water vapor, H_2O

$$m = m_1 + m_2 + m_3 \tag{1-90}$$

Suction pressure $p_2 = 65.7$ [mbar]
Suction temperature: $t_2 = 12$ [°C]
Compression pressure $p_1 = 1013$ [mbar]

The adequate liquid ring vacuum pump is searched. A selection can only be made on the basis of the suction capacity [m^3/h]. For the determination of the suction capacity the following two possibilities are shown:

a) *solution by determination of total molar volume*

Molecular weights acc. to tables

Kind of gas/steam	**Molecular weight M**
Methanol	32.04 [kg/kmol] = M_1
Air	28.96 [kg/kmol] = M_2
Water vapor	18.02 [kg/kmol] = M_3

At first, the molar quantity of the individual gas-vapor proportions has to be determined. This is possible through rearranging equation (1-160).

$$m = N \cdot M \tag{1-160}$$

m gas mass [kg]
N particle quantity [kmol]
M molecular weight [kg/kmol]

$$N = \frac{m}{M}$$

respectively

$$\dot{N}_{M_1} = \frac{\dot{m}_1}{M_1}$$

or

$$\dot{N}_{M_k} = \frac{\dot{m}_k}{M_k}$$

consequently:

$$\dot{N}_{M_1} = \frac{40}{32.04}\frac{\frac{kg}{h}}{\frac{kg}{kmol}} = 1.248\left[\frac{kmol}{h}\right]$$

$$\dot{N}_{M_2} = \frac{41{,}3}{28.96}\frac{\frac{kg}{h}}{\frac{kg}{kmol}} = 1.426\left[\frac{kmol}{h}\right]$$

$$\dot{N}_{M_3} = \frac{11.2}{18.02}\frac{\frac{kg}{h}}{\frac{kg}{kmol}} = 0.621\left[\frac{kmol}{h}\right]$$

total particle quantity $\dot{N}_M = 3.295\left[\frac{kmol}{h}\right]$

molar volume $V_m = 22.41\left[\frac{m^3}{kmol}\right]$ acc. to equation (1-68)

At standard temperature and pressure, the total volume of the existing mixture to be sucked can be calculated by rearranging equation (1-162):

$$V_m = \frac{V}{N} \tag{1-162}$$

$V = N \cdot V_m$

V volume [m^3]

N particle quantity [kmol]

V_m specific volume of 1 kmol [m^3/kmol]

$$\dot{V}_{tot} = \dot{N}_M \cdot V_m$$

$$\dot{V}_{tot} = 3.295\frac{kmol}{h} \cdot 22.41\frac{m^3}{kmol}$$

$$\dot{V}_{tot} = 73{,}84\left[\frac{m^3}{h}\right] \text{ at } 0\,°C \text{ and } 1013 \text{ mbar}$$

$$\dot{V}_{tot} = 73.84\left[\frac{Nm^3}{h}\right]$$

Converted to the current suction state ($p_2 = 65.70$ mbar, $t_2 = 12\,°C$), the following suction volume $\dot{V}_2$ to be sucked by the pump per hour can be determined.

Calculation applying the thermal state equation for ideal gases:

$$\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2} \tag{1-166}$$

$T = 273.15\,K + t\,°C\,[K]$

Original mixture at standard temperature and pressure	Mixture in suction state
$p_1 = 1013$ [mbar]	$p_2 = 65.7$ [mbar]
$\dot{V}_1 = 73.84\left[\mathrm{Nm}^3/\mathrm{h}\right]$	$\dot{V}_2 = ?$
$T_1 = 273.15$ [K]	$T_2 = 273.15\ \mathrm{K} + 12\ ^\circ\mathrm{C} = 285.15$[K]

$$\dot{V}_2 = \frac{1013\ \mathrm{mbar}\cdot 73.84\frac{\mathrm{m}^3}{\mathrm{h}}\cdot 185.15\ \mathrm{K}}{273.15\ \mathrm{K}\cdot 65.7\ \mathrm{mbar}} = 1188.50\left[\frac{\mathrm{m}^3}{\mathrm{h}}\right]$$

b) *solution applying the general state equation for ideal gases*

An alternative calculation using the general state equation for the ideal gas mixture leads to the same result.

Acc. to equation (1-134), the mean gas constant R_m of the gas mixture has to be determined at first.

$$p \cdot V = m \cdot R_m \cdot T \qquad (1\text{-}134)$$

$$R_m = \sum_{i=1}^{k} \xi_i \cdot R_i \qquad (1\text{-}136)$$

$$\zeta_i = \frac{m_i}{m} \qquad (1\text{-}93)$$

Gas constant of each gas in the mixture can be taken from the tables.

Kind of gas/steam	Specific gas constant R_i	
	$\left[\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right]$	$\left[\frac{\mathrm{Nm}}{\mathrm{kg\cdot K}}\right]$
Methanol	0.2594	259.48
Air	0.2871	287.10
Water vapor	0.4615	461.50

With the equations (1-93) and (1-136), the gas constant R_m is determined.

Kind of gas/steam	$\dot{m}_i$ [kg/h]	$\xi_i = \dot{m}_i/\dot{m}$	R_i [Nm/kg·K]	$\xi_i \cdot R_i$ [Nm/kg·K]
Methanol	$\dot{m}_1 = 40$	$40 : 92.5 = 0.4322$	259.48	112.14
Air	$\dot{m}_2 = 41.3$	$41.3 : 92.5 = 0.4460$	287.10	128.05
Water vapor	$\dot{m}_3 = 11.2$	$11.2 : 92.5 = 0.1210$	461.50	55.85
$Rm = \sum_{i=1}^{k} \xi i \cdot Ri$				$= 296.04 \left[\frac{Nm}{kg \cdot K}\right]$

$$\dot{m} = \dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 92.5 \text{ [kg/h]}$$

From equation (1-134), we get:

$$\dot{V} = \frac{\dot{m} \cdot R_m \cdot T}{p}$$

$\dot{V}$ volume per time unit in suction state [m^3/h]
$\dot{m}$ gas mass per time unit [kg/h]
R_m mean gas constant [Nm/(kg · K)]
T thermodynamic temperature in suction state [K]
p pressure in suction state [N/m^2]

$$\dot{V} = \frac{92.5 \frac{kg}{h} \cdot 296.04 \frac{Nm}{kg \cdot K} \cdot 285.15\ K}{6.57 \cdot 10^3 \frac{N}{m^2}}$$

$$\dot{V} = 1188.50\ [m^3/h] \quad \text{in suction state}$$

Based on the suction capacity determined under a) respectively b), the suitable vacuum pump can be chosen.

The characteristic curves of the respective pump are shown in fig. 1-33. The operation data of the pump are stated as follows:

- Suction pressure = 65.70 [mbar]
- Suction capacity $\dot{S} = 1188.50$ [m^3/h]
- Rotational speed n = 1130 [min^{-1}]
- Max. pump power input P = 36.50 [kW]

In this case, vapor condensates and water can be used as operating liquid, for instance.

Data of the operating liquid on the pump inlet:

- Temperature t = 15 [°C]
- Density $\rho = 1000$ [kg/m^3]
- Viscosity $\eta = 1$ [mPas]

Example 1.18:

An air mass $\dot{m}_{air} = 12\,[kg/h]$ is to be sucked.

Gas mixture to be sucked: air, saturated with water vapor.

- Suction pressure: 100 [mbar]
- Suction temperature: 35[°C]

Molar masses:

- air: $M_{air} = 29\,[kg/kmol]$
- water vapor, $M_{WV} = 18\,[kg/kmol]$

To be determined:

a) partial pressure of the air
b) proportion of vapor volume
c) vapor mass to be sucked
d) total volume to be sucked
e) influence of the temperature of the operating liquid on the suction capacity of the pump
f) suction capacity of the pump owing to the condensation effect
g) condensation in the vacuum pump and air volume at the pump outlet
h) condensation in the condenser before the vacuum pump

Solution:

a) partial pressure of the air is to be determined acc. to equation (1-153):

$$p_{total} = p_1 + p_2 \tag{1-153}$$

$$p_{air} = p_{total} - p_{WV}$$

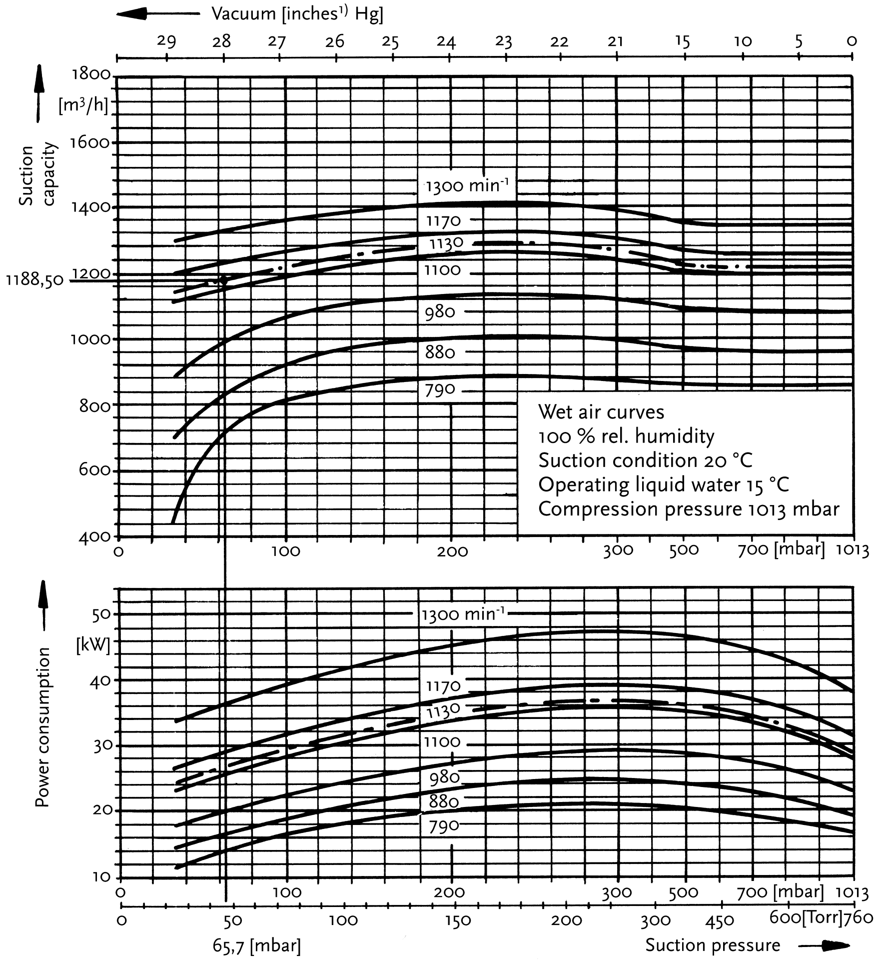
p_{air} partial pressure of the air
p_{WV} partial pressure of the water vapor
p_{total} total pressure = suction pressure = 100 mbar
$p_{WV} = 56.23$ mbar at 35 °C (from the tables)
$p_{air} = 100$ mbar – 56.23 mbar = 43.77 mbar

b) The proportion of the vapor volume is determined acc. to equation (1-171):

$$\frac{V_{WV}}{V_{air}} = \frac{p_{WV}}{p_{total} - p_{WV}} \tag{1-171}$$

$$\frac{V_{wv}}{V_{air}} = \frac{56.23\,\text{mbar}}{100\,\text{mbar} - 56.23\,\text{mbar}} = 1.28$$

V_{WV} water vapor volume
V_{air} air volume



1) 1 inch = 25,4 mm acc. to DIN 4890

Figure 1-33. Characteristic curve field with wet-air curves of a single-stage liquid ring vacuum pump

Therefore, the volume of the water vapor has a value 1.28 times as high as the volume of the dry air.

c) The vapor mass results from equation (1-151):

$$\dot{m}_{WV} = \dot{m}_{air} \cdot \frac{M_{WV}}{M_{air}} \cdot \frac{p_{WV}}{p_{air}} \tag{1-151}$$

$$\dot{m}_{WV} = 12\,\frac{kg}{h}\cdot\frac{18\frac{kg}{kmol}}{29\frac{kg}{kmol}}\cdot\frac{56.23\,mbar}{43.77\,mbar} = 9.56\left[\frac{kg}{h}\right]$$

$\dot{m}_{WV}$ water vapour mass
$\dot{m}_{air}$ air mass

Therefore, 9.56 kg water vapor is to be sucked in one hour.

d) The total occurring mixture volume before the pump is determined acc. to equation (1-172).

$$\dot{V}_{total} = \frac{83.14\cdot\left(\frac{\dot{m}_{air}}{M_{air}}+\frac{\dot{m}_{WV}}{M_{WV}}\right)\cdot T}{p_{total}} \tag{1-172}$$

$T = 273.15\,K + 35\,°C = 308.15\,K$

$$\dot{V}_{total} = \frac{83.14\frac{mbar\cdot m^3}{kmol\cdot K}\cdot\left(\frac{12\frac{kg}{h}}{29\frac{kg}{kmol}}+\frac{9.56\frac{kg}{h}}{18\frac{kg}{kmol}}\right)\cdot 308.15\,K}{100\,mbar}$$

$$\dot{V}_{total} = 241.84[m^3/h]$$

The liquid ring vacuum pump must be designed for a suction quantity of 241.84 m^3/h at 100 mbar suction pressure.

e) Initially, the influence of the temperature of the operating liquid on the suction capacity of the vacuum pump is described for the suction of dry gas. The suction capacity of a liquid ring vacuum pump changes according to the temperature of the operating liquid, as a part of the impeller chamber is filled with vaporizing operating liquid until it is saturated. For the discharge of gas this part of the volume is lost.

For water as operating liquid, this change of the suction capacity can be determined according to the diagram shown in fig. 1-34. Usually the characteristic catalogue curves of liquid ring vacuum pumps are based on inflow temperatures of the pressure water of 15 °C.

With a suction pressure of 100 mbar (74.1 Torr) and an assumed water inflow temperature of 25 °C, a correction factor $\dot{S}_{x1}/\dot{S}_k$ of 1.20 is determined. In this case, the suction capacity of the pump to be used should be 1.2 times higher than at the water inflow temperature of 15 °C in order to achieve the suction quantity of 241.84 m^3/h.

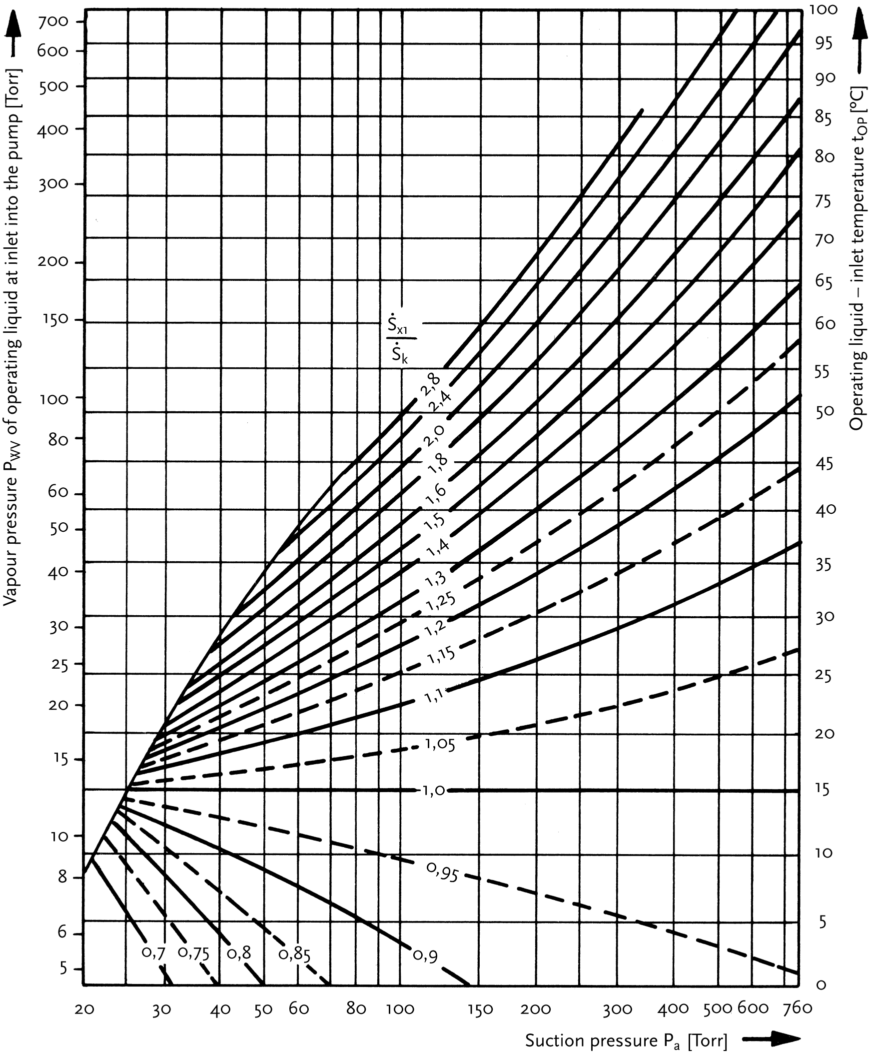


Figure 1-34. Changing of the suction capacity of liquid ring vacuum pumps depending on suction pressure and inflow temperature of the operating liquid. [1.11]
Operating liquid: water

$\dot{S}_k$ Suction capacity of a vacuum pump acc. to dry-air curve at suction temperature of 20 °C and inflow temperature of 15 °C of the operating liquid (water)

$\dot{S}_{x1}$ Required suction capacity of a vacuum pump at an inflow temperature of the operating liquid (water) of t_{op} °C

With the sucked mixture in the present example d) being dry, the required pump suction capacity would be

$$\dot{S}_{x1} / \dot{S}_k = f_1$$

$$\dot{S}_{x1} = 1.2 \cdot \dot{S}_k$$

$$\dot{S}_{x1} = 1.2 \cdot 241.84 \text{ m}^3/\text{h}$$

$$\dot{S}_{x1} = 290.21 \text{ m}^3/\text{h}$$

f) The condensing effect can occur during the pumping of a gas-vapor mixture. If a liquid ring vacuum pump is used for pumping vapors, ring liquid will vaporize in the impeller cells only until the state of saturation is reached, i. e. equilibrium is established. If the pump sucks gas already saturated with operating liquid, the vaporization of the operating liquid is theoretically impossible. Here, the suction capacity of the vacuum pump is higher by the proportion of the operating liquid otherwise vaporizing. If the temperature of the gas-vapor mixture to be sucked is higher than the temperature of the ring liquid, then cooling takes place in the vacuum pump when both fluids meet and the vapor will condense after having reached the saturation vapor pressure. Thus the vacuum pump has the function of a mixing condenser. Owing to condensation, an additional suction effect arises which is also called condensation effect. With liquid ring machines disposing of characteristic curves based on the suction of dry gases at 20 °C, a correction of the suction capacity similar to the correction curves given in fig. 1-35 is possible for the pumping of dry saturated air-water vapor mixtures within a temperature range of 10 °C and 50 °C and with water as operating liquid.

The exact determination of the corrective factors is to be made acc. to the geometric ratios of the pump. Therefore, the correction diagram should be used only for information purposes.

For a suction temperature of 35 °C and an inlet temperature of the ring liquid of 21 °C the actually arising suction quantity of the vacuum pump is to be determined for a dry saturated air-water vapor mixture acc. to example d). With a presumable inlet temperature of the operational water of 21 °C and on the assumption that dry gas is used, according to the curves in fig. 1-34 the corrective factor $\dot{S}_{x1} / \dot{S}_k = 1.1 = f_1$ results at a suction pressure of 100 mbar (= 74.1 Torr).

This means that the suction capacity of the pump to be used shall be larger by this factor compared to a pump with an inlet temperature of the operating water of 15 °C.

In the case of the given saturated air-water vapor mixture and the mentioned operating conditions the occurrence of the condensation effect at the intake of the vacuum pump has to be considered.

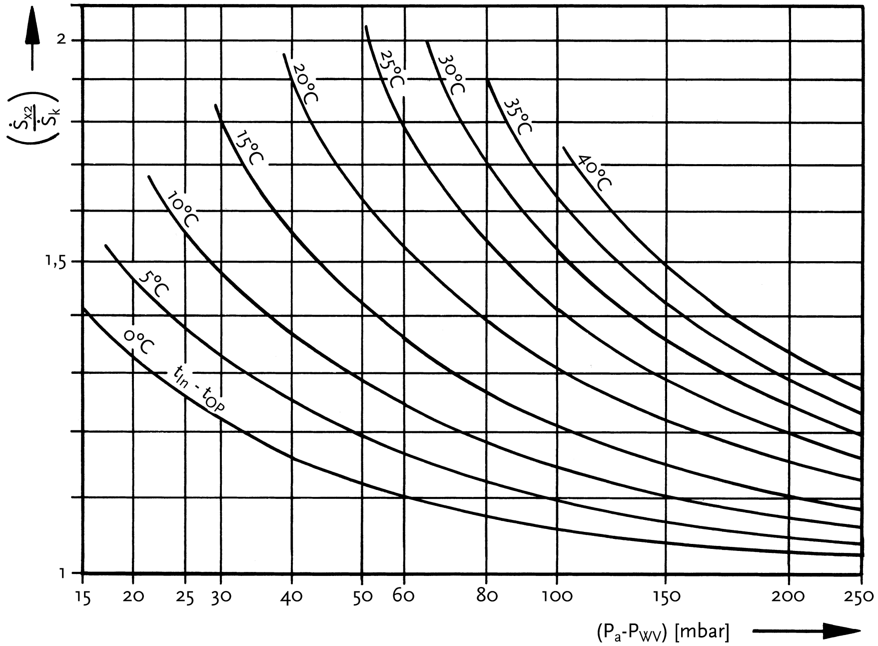


Figure 1-35. Improvement of the suction capacity of liquid ring vacuum pumps due to the condensing effect during the pumping of saturated air-water vapor mixtures [1.10]
Diagram valid for lower limit values in the range of 10 °C ≦ t ≦ 50 °C
Operating liquid: water
p_a suction pressure
p_{WV} vapor pressure of the operating liquid
t_{in} temperature of air-water-vapor mixture at pump's suction flange
t_{op} inflow temperature of the operating liquid (water)
$\dot{S}_{x2}$ suction capacity of the vacuum pump resulting from condensation effect
$\dot{S}_k$ suction capacity of the vacuum pump acc. to the dry-air curve at a suction temperature of 20 °C and an inflow temperature of the operating liquid (water) of 15 °C

Acc. to the curves in fig. 1-35, another corrective factor results which is determined as follows:

$t_{in} = 35\ °C$
$t_{op} = 21\ °C$
$t_{in} - t_{op} = 35\ °C - 21\ °C = 14\ °C$
$p_{WV} = 24.86$ mbar at 21 °C (from tables)
$p_a - p_{WV} = 100\ \text{mbar} - 24.86\ \text{mbar} = 75.14\ \text{mbar}$

From the diagram the following factor results:

$\dot{S}_{x2}/\dot{S}_k = f_2$
$\dot{S}_{x2}/\dot{S}_k = 1.27$

which confirms the improvement of the suction capacity of the vacuum pump.

As described above, the suction capacity of liquid ring vacuum pumps depends on the type of suction flow, the features of the operating liquid, the suction pressure, as well as on the temperature of the pumped gas or gas mixture and the inlet temperature of the operating liquid. Based on the correction curves acc. to fig. 1-34 and fig. 1-35, the actual suction capacity of those machines can be approximately determined according to equation (1-175) considering the existing operating conditions.

$$\dot{S}_{\mathrm{act}} = \dot{S}_{\mathrm{k}} \cdot \frac{f_2}{f_1} \qquad (1\text{-}175)$$

$\dot{S}_{\mathrm{act}}$	actual suction capacity
$\dot{S}_{\mathrm{x1}}$	required suction capacity due to the inlet temperature of the operating water
$\dot{S}_{\mathrm{x2}}$	arising suction capacity resulting from condensation effect
$\dot{S}_{\mathrm{k}}$	suction capacity acc. to the dry air curve and an inlet temperature of the operating liquid of 15 °C.
$f_1 = (\dot{S}_{\mathrm{x1}} / \dot{S}_{\mathrm{k}})$	Factor influence of the inlet temperature of the operating water
$f_2 = (\dot{S}_{\mathrm{x2}} / \dot{S}_{\mathrm{k}})$	Factor influence of the condensation effect

In the present example, the increase in the suction quantity required under the influence of the increased temperature of the operating liquid could be achieved by increasing the rotational speed or selecting a larger vacuum pump. If, however, the pump ($\dot{S}_{\mathrm{k}} = 241.84\ \mathrm{m^3/h}$)selected according to the dry air curve is kept at the same rotational speed (see fig. 1-36), the suction capacity deteriorates by the factor f_1. However, considering the pump condensation effect arising in the pump the suction capacity of this machine will increase, as the following calculation shows:

$$\dot{S}_{\mathrm{act}} = \dot{S}_{\mathrm{k}} \cdot \frac{f_2}{f_1} \qquad (1\text{-}175)$$

$\dot{S}_{\mathrm{k}} = 241.84\,[\mathrm{m^3/h}]$ (determined in example d)

$$\dot{S}_{\mathrm{act}} = 241.84 \frac{\mathrm{m^3}}{\mathrm{h}} \cdot \frac{1.27}{1.10} = 279.21 \left[\frac{\mathrm{m^3}}{\mathrm{h}}\right]$$

Therefore, the actual suction capacity of the vacuum pump is slightly higher than the required pumping quantity. In practice, this is usually tolerated as a safety margin. The calculated operating points are entered in the characteristic curve field (fig. 1-36).

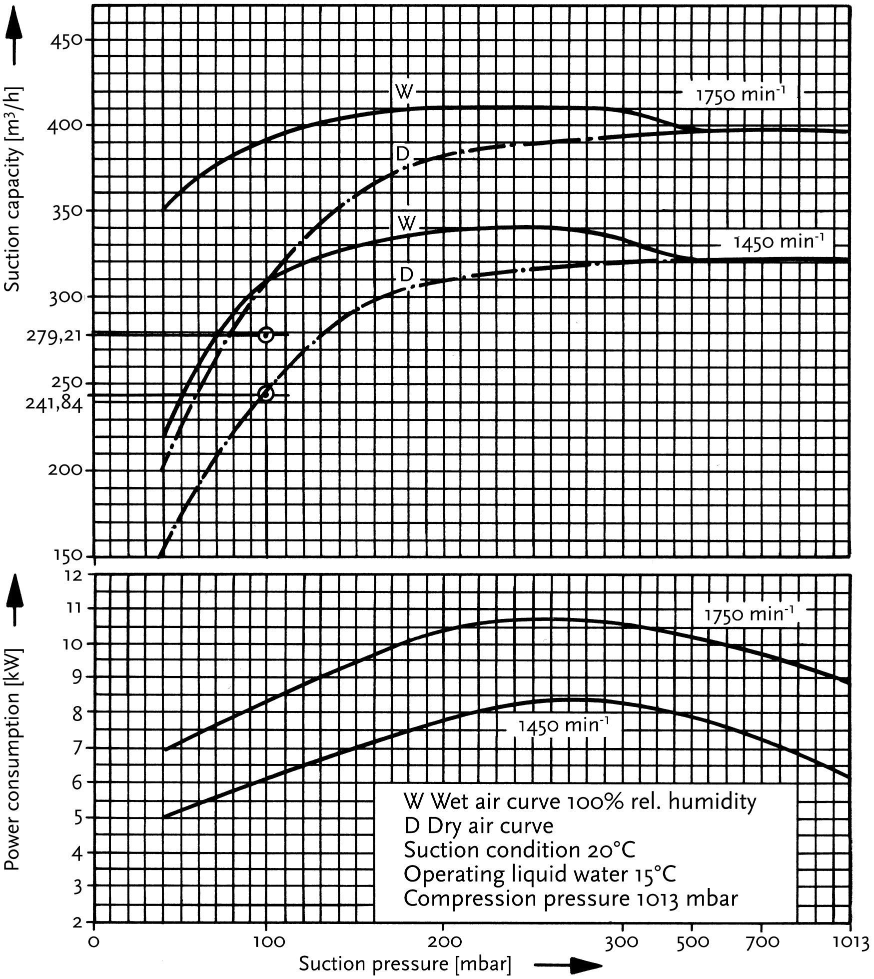


Figure 1-36. Characteristic curve field with dry and wet-air curves of a single-stage liquid ring vacuum pump

279.21 m^3/h	Operating point at the inflow temperature of the operating liquid (water) of 21 °C and a saturated air/-water-vapor mixture of 35 °C at pump's suction flange
241.84 m^3/h	Operating point at the inflow temperature of the operating liquid (water) of 15 °C and dry gas mixture at pump's suction flange

g) During compression to 1013 mbar, a certain quantity of water vapor condenses in the pump. As the temperature of the gas mixture and the temperature of the ring liquid in the pump equalize, we assume that based on a suction temperature of 35 °C and a chosen inlet temperature of the pressure water of 15 °C the common temperature at the pump outlet will be 20 °C.

The proportion of water vapor still existing in the gas mixture at the pump outlet port can be determined according to equation (1-151):

$$\dot{m}_{WV} = \dot{m}_{air} \cdot \frac{M_{WV}}{M_{air}} \cdot \frac{p_{WV}}{p_{air}} \tag{1-151}$$

p_{WV} partial pressure of the water vapor
p_{air} partial pressure of the air
$\dot{m}_{WV}$ water vapor existing at pump outlet
$p_{WV} = 23.37$ mbar at 20 °C (from tables)
$p_{air} = 1013$ mbar – 23.37 mbar = 989.63 mbar

$$\dot{m}_{WV} = 12\,\frac{kg}{h} \cdot \frac{18\,\frac{kg}{kmol}}{29\,\frac{kg}{kmol}} \cdot \frac{23.37\,mbar}{989.63\,mbar} = 0.175 \left[\frac{kg}{h}\right]$$

The total quantity of vapor at the pump inlet has been calculated in example c) with 9.56 kg/h.

Thus the vapor quantity condensed during the compression process in the pump is

$$9.65\,kg/h - 0.175\,kg/h = 9.385\,[kg/h]$$

The air volume of 12 kg/h at the pump outlet at 1013 mbar/20 °C is calculated acc. to equation (1-173):

$$\dot{V}_{air} = \frac{83.14 \cdot \frac{\dot{m}_{air}}{M_{air}} \cdot T}{p_{total}} \tag{1-173}$$

$T = 273.15$ K + 20 °C = 293.15 K

$$\dot{V}_{air} = \frac{83.14\,\frac{mbar \cdot m^3}{kmol \cdot K} \cdot \frac{12\,\frac{kg}{h}}{29\,\frac{kg}{kmol}} \cdot 293.15\,K}{1013\,mbar} = 9.95 \left[\frac{m^3}{h}\right]$$

h) In the condenser (see fig. 1-37), condensation will already begin before the vacuum pump.

- Suction temperature $t_1 = 35$ °C
- Suction pressure $p_1 = 100$ mbar
- Cooling water temperature for condenser $t_{in} = 25$ °C
- Suction mixture air, saturated with water-vapor
- Air mass to be sucked $\dot{m}_{air} = 12$ kg/h

The temperature t_2 after the condenser is assumed to be 30 °C. The vapor mass flow before the condenser is 9.56 kg/h acc. to the calculation in example c).

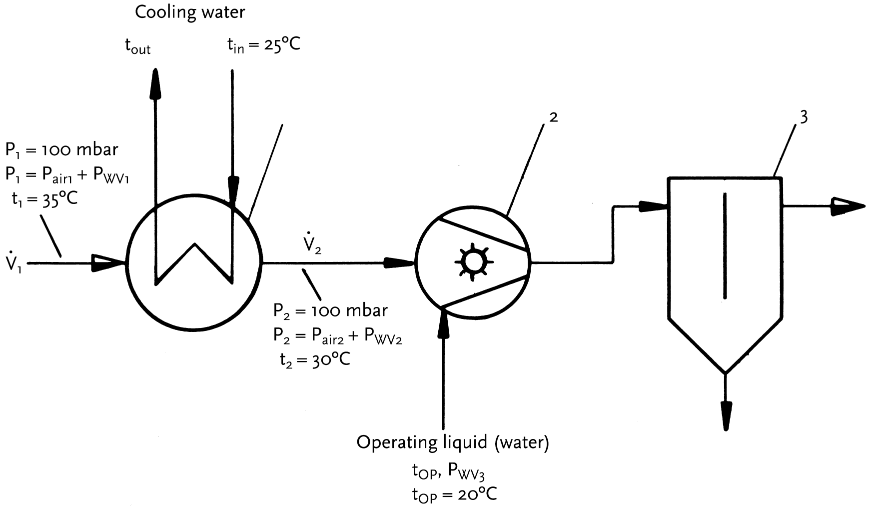


Figure 1-37. Vacuum pump with liquid separator and a condenser on the suction side
1 condenser, 2 liquid ring vacuum pump, 3 liquid separator

The condenser is assumed to show no pressure losses while pressure p_2 after the condenser corresponds to suction pressure p_1.

The vapor mass after the condenser is calculated according to equation (1-151).

$$\dot{m}_{WV_2} = \dot{m}_{air} \cdot \frac{M_{WV}}{M_{air}} \cdot \frac{p_{WV_2}}{p_{air_2}} \tag{1-151}$$

$p_2 = p_1 = 100$ mbar
$p_{total} = p_1 = p_2$
$p_{air_2} = p_2 - p_{WV2}$
$p_{WV2} = 42.43$ mbar/30 °C (from tables)
$p_{air_2} = 100\,\text{mbar} - 42.43\,\text{mbar} = 57.57\,\text{mbar}$
$\dot{m}_{WV2}$ existing vapor mass after the condenser

$$\dot{m}_{WV_2} = 12\frac{\text{kg}}{\text{h}} \cdot \frac{18\frac{\text{kg}}{\text{kmol}}}{29\frac{\text{kg}}{\text{kmol}}} \cdot \frac{42.43\,\text{mbar}}{57.57\,\text{mbar}} = 5.48\left[\frac{\text{kg}}{\text{h}}\right]$$

In the condenser 9.56 kg/h – 5.48 kg/h= 4.08 kg/h are condensing.

The accruing volume $\dot{V}_2$ before the pump is calculated acc. to equation (1-172).

$$\dot{V}_{total} = \frac{83.14 \cdot \left(\frac{\dot{m}_{air}}{M_{air}} + \frac{\dot{m}_{WV_2}}{M_{WV}}\right) \cdot T_2}{p_{total}} \tag{1-172}$$

T = 273.15 K + 30 °C = 303.15 K

$$\dot{V}_{total} = \frac{83.14\frac{mbar \cdot m^3}{kmol \cdot K} \cdot \left(\frac{12\frac{kg}{h}}{29\frac{kg}{kmol}} + \frac{5.48\frac{kg}{h}}{18\frac{kg}{kmol}} \right) \cdot 303.15\,K}{100\,mbar}$$

$$\dot{V}_{total} = \dot{V}_2 = 180.71 \left[\frac{m^3}{h}\right]$$

In example d), a total mixture volume of 241.84 m^3/h was determined without a condenser at the suction side. With a condenser installed on the suction side of the vacuum pump, the mixture volume flow in the pump is lower. This may allow the selection of a smaller pump and therefore savings in driving power.

In this case, the vacuum pump installed after the condenser can be designed for a suction capacity of 180.71 m^3/h, according to the dry-air curve at a suction pressure of 100 mbar. Due to the existing operating conditions, the actual suction capacity of this vacuum pump must be determined according to the temperature of the operating water, the suction temperature and the suction pressure, as in example f).

According to the diagram in fig. 1-34, it results:

$$\frac{\dot{S}_{x1}}{\dot{S}_k} = 1.08 = f_1$$

$p_2 = 100\,mbar\ (= 74.1\,Torr)$
$t_{op} = 20\,°C$

Through correction due to the condensation effect acc. to diagram 1-35 it results:

$$\frac{\dot{S}_{x2}}{\dot{S}_k} = 1.20 = f_2$$

$p_2 = 100\,mbar$
$p_{WV_3} = 23.37\ mbar/20\,°C$
$t_2 = 30\,°C$
$t_{op} = 20\,°C$
$p_2 - p_{WV_3} = 100\,mbar - 23.37\,mbar = 76.63\,mbar$
$t_2 - t_{op} = 30\,°C - 20\,°C = 10\,°C$

Thus, at 100 mbar suction pressure, the actual suction capacity of the vacuum pump (S_{act}) is

$$\dot{S}_{real} = \dot{S}_k \cdot \frac{f_2}{f_1} \tag{1-175}$$

$$\dot{S}_k = 180.71 \left[\frac{m^3}{h}\right]$$

$$\dot{S}_{real} = 180.71 \frac{m^3}{h} \cdot \frac{1.20}{1.08} = 200.79 \left[\frac{m^3}{h}\right]$$

The actual suction capacity of the vacuum-pump of 200.79 m^3/h is slightly higher than the accruing volume flow rate of 180.71 m^3/h measured at the outlet of the condenser at a pressure of 100 mbar.

The difference between the discharge rate of the pump and process gas quantity to be sucked can be fed to the vacuum pump through ventilation valves or through bypass via orifices, as well as via manually or automatically controlled valves.

1.8.7

Gases in mixtures with overheated vapors [1.11]

Engineering usually deals with ideal gases or gas-vapor-mixtures. With gas-vapor-mixtures it often happens that the vapors existing in the mixture are dry saturated vapors. For the calculations of these mixtures, the laws for ideal gas are applied, i. e. these mixtures are assumed to behave almost like ideal gases. In practice, even mixtures of gases and overheated vapors occur. These mixtures are also calculated roughly according to the laws applied for ideal gases.

Any mixing ratio of the respective gas proportions is possible for mixtures of ideal gases. However, the miscibility of ideal gases with vapors is subject to certain laws. The temperatures of vapors can be randomly high, i. e. higher than the boiling temperatures corresponding to their partial pressures. The occurring vapor temperatures, however, are by no means lower than the boiling temperatures belonging to their respective partial pressures. This results in a partial vapor pressure of a vapor arising in a mixture that takes on its specific vapor pressure corresponding to the temperature of the mixture. As in an ideal gas mixture the volume proportions of the existing substances behave like their partial pressures, the maximal vapor proportion in the mixture is determined by the boiling pressure of the vapor assigned to the mixing temperature. If partial pressure p_p of a vapor in the mixture already corresponds to its boiling pressure p_s at the existing temperature of the mixture, this vapor exists with its highest volume proportion. So the gas mixture contains vapor in dry saturated state. Such a mixture is also called mixture saturated with vapor.

If vapor in the mixture exists with a smaller proportion, this means that its partial pressure p_p is smaller than its saturated pressure p_s.

p_p	partial pressure
p_s	saturated pressure
φ	factor for relative humidity, without dimensions

Using the factor φ, the value of which can lie between zero and one, and which is a measure for the relative humidity the following equation results:

$$p_p = p_s \cdot \varphi \tag{1-176}$$

If p_d is smaller than p_s, the vapor is overheated, i.e. the gas mixture could take up further vapor.

If the proportion of vapor of a mixture is given in kilogram it is called absolute humidity. The quantity of vapor in kilogram contained in one kilogram of the vapor-free dry mixture is usually given with the letter

$$\kappa \left[\frac{\mathrm{kg}}{\mathrm{kg}}\right].$$

A mixture consisting of an ideal gas and an overheated vapor can be calculated according to the general state equation for ideal gases. For the proportion of vapor, however, this calculation is only approximately valid.

General state equation:

$$p \cdot V = m \cdot R \cdot T \tag{1-17}$$

Definitions for the mixture

units	**ideal gas**	**overheated vapor**
mass	m_g	m_v
partial pressure	p_g	p_v
specific gas constant	R_g	R_v
	mixture of gas and overheated vapor	
total volume	V_{total}	
total pressure	p_{total}	
temperature	T_{mix}	

The state equation for ideal gases:

$$V_{total} \cdot p_g = m_g \cdot R_g \cdot T_{mix} \tag{1-177}$$

The state equation for overheated vapors:

$$V_{total} \cdot p_v = m_v \cdot R_v \cdot T_{mix} \tag{1-178}$$

Dividing both equations, it results

$$\frac{p_v}{p_g} = \frac{m_v}{m_g} \cdot \frac{R_v}{R_g} \tag{1-179}$$

and

$$\kappa = \frac{m_v}{m_g} = \frac{R_g}{R_v} \cdot \frac{p_v}{p_g} \tag{1-180}$$

Using factor φ the result is,

$$pv = ps \cdot \varphi \tag{1-176}$$

$$p_g = p_{total} - p_v$$

resp.

$$p_g = p_{total} - p_s \cdot \varphi \tag{1-181}$$

and therefore

$$\kappa = \frac{R_g}{R_v} \cdot \frac{p_s \cdot \varphi}{p_{total} - p_s \cdot \varphi} \left[\frac{kg}{kg}\right] \tag{1-182}$$

$$\kappa \cdot \left(p_{total} - p_s \cdot \varphi\right) = \frac{R_g}{R_v} \cdot \varphi \cdot p_s$$

$$\varphi = \frac{\kappa}{\left(\frac{R_g}{R_v} + \kappa\right)} \cdot \frac{p_{total}}{p_s} \tag{1-183}$$

The largest proportion of vapor results from the maximum possible relative humidity, i. e. $\varphi = 1.0$:

$$\kappa_{max} = \frac{R_g}{R_v} \cdot \frac{p_s}{p_{total} - p_s} \left[\frac{kg}{kg}\right] \tag{1-184}$$

Example 1.19.

For the gas-vapor-mixture, i.e. humid air often occurring in practice the general equations for atmospheric pressure of

a) absolute humidity κ
and
b) relative humidity φ

are to be drawn up.

Solution:

gas constant air $R_g = 0.2871 \left[\frac{kJ}{kg \cdot K}\right]$
gas constant water vapor $R_v = 0.4615 \left[\frac{kJ}{kg \cdot K}\right]$ (from table 1.-15.)

Therefore:

$$\frac{R_g}{R_v} = \frac{0.2871 \frac{kJ}{kg \cdot K}}{0.4615 \frac{kJ}{kg \cdot K}} = 0.622$$

$p_{total} = 1013$ [mbar (abs)]
p_s [mbar (abs)]

a) from equation

$$\kappa = \frac{R_g}{R_v} \cdot \frac{p_s \cdot \varphi}{p_{total} - p_s \cdot \varphi} \tag{1-182}$$

it results:

$$\kappa = 0.622 \cdot \frac{p_S \cdot \varphi}{1013 - p_S \cdot \varphi} \left[\frac{kg}{kg}\right]$$

b) from the equation

$$\varphi = \frac{k}{\left(\frac{R_g}{R_v} + \kappa\right)} \cdot \frac{p_{total}}{p_s} \tag{1-183}$$

it results:

$$\varphi = \frac{\kappa}{0.622 + \kappa} \cdot \frac{1013}{p_S}$$

1.8.8
Condensation and cavitation

In cases with high proportions of vapor on the suction side condensing in the pump during compression it may happen that the pump lacks in sufficient inert gases, and towards the end of compression there is not enough compressible media. Then, cavitation on the pressure side occurs. As is known, cavitation on the suction side occurs when the liquid ring vacuum pump is operated at a suction pressure close to the boiling temperature of the operating liquid belonging to this pressure.

To avoid cavitation on the pressure side, the pump must always deliver as much gas – mainly inert gases – as corresponds to the suction capacity of the pump at the lowest suction pressure acc. to its characteristic curve.

1.9
Change of gas state during the compression process

In thermodynamics, two ideal borderline cases of the change of the gas state are distinguished, i.e. isothermal and adiabatic (isentropic) processes which can only be approximately realized in practice.

Distinctive quantities of these processes are pressure ratio and compression work.

1.9.1
The isothermal compression

In the isothermal compression process, the temperature of the compressed gas remains constant, i. e. the total heat accruing during compression is evacuated by respective measures. Therefore, between its initial and final state, the compressed gas behaves acc. to the Boyle-Mariotte Law (equation 1-167).

This process presupposes ideal heat conductivity.

$$\frac{p_2}{p_1} = \frac{V_1}{V_2} \qquad (1\text{-}167)$$

The volume V_1 equals, for example, the piston displacement of a piston compressor. According to the equation

$$p \cdot V = m \cdot \frac{R_o}{M} \cdot T \qquad (1\text{-}65)$$

the required isothermal compression work W_{is} depends on this pressure ratio and on the temperature, it is, however, independent from the type of gas. The compression work is calculated as follows [1.3]:

$$W_{is} = p_1 \cdot V_1 \cdot \ln\frac{p_2}{p_1} \qquad (1\text{-}185)$$

W_{is} corresponds to the heat to be evacuated in the isothermal compression process.

1.9.2
The adiabatic compression

In the adiabatic compression process, the heat accruing during compression process is not evacuated, but remains preserved by means of ideal heat insulation. The whole energy is used for increasing the gas temperature. According to S.D. Poisson, the following applies to the pressure ratio [1.3]:

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}} = \left(\frac{V_1}{V_2}\right)^{\kappa} \qquad (1\text{-}186)$$

The adiabatic compression work W_{ad} to be used is calculated acc. to equation (1-187).

$$W_{ad} = \frac{\kappa}{\kappa-1} \cdot p_1 \cdot V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] \qquad (1\text{-}187)$$

The compression work is determined by the type of gas, the temperature and the pressure ratio.

1.9.3
Adiabatic exponent κ

The adiabatic exponent κ corresponds to the ratio of the specific heat capacity of the gas at constant pressure c_p and at constant volume c_v (comp. table 1-116).

$$\kappa = \frac{c_p}{c_v} \tag{1-188}$$

c_p specific heat capacity at constant pressure
c_v specific heat capacity at constant volume
κ adiabatic exponent

As the heating-up process of a gas quantity enclosed in a chamber may proceed differently with the same change of temperature, different specific heat capacities of the same gas occur. Two borderline cases are distinguished. In one case, the gas volume is kept constant while the temperature of the gas increases. The gas pressure rises with the increasing temperature. In the other case, the pressure remains constant, temperature and volume, however increase. Between these two borderline cases, there is an unlimited number of processes with simultaneous changes of pressure and volume.

Fig. 1-38 shows the possible changes of state. [1.5].

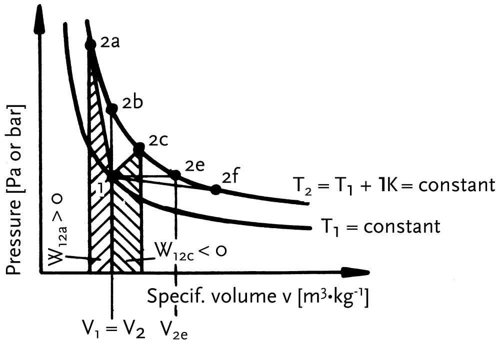


Figure 1-38. Change of the state of a gas through heating up by one degree Kelvin.

In the figure, the change of state from state 1 to state 2b is a change of state at constant volume with no external work being performed. The change of state from point 1 to point 2e takes place at constant pressure, with the output work quantity $W_{12} = p(v_{2e} - v_1)$.

In the final state 2a, the gas is supplied with a specific external work W_{12a}, in the final state 2b specific external work is neither supplied nor put out. In the final state 2c the gas puts out specific external work.

Owing to the fact that there are a lot of possibilities of change-of-state work, there are just as many specific heat capacities for one and the same gas. In practice, only two changes of state are considered:

- The change of state at constant volume and the specific heat capacity c_v and
- the change of state at constant pressure and the specific heat capacity c_p.

Although the specific heat capacities c_v and c_p are different for each gas, the ratio of c_v to c_p is the same for all biatomic gases. Those ratio values (κ) are known for most of the gases. For gaseous water in the ideal state $\kappa = 1.33$.

Table 1-16. Specific heat capacity c_p , c_v and κ-values of different technical gases [1.9]

Type of gas	$c_p \left[\frac{kJ}{kg \cdot K}\right]$	$c_v \left[\frac{kJ}{kg \cdot K}\right]$	κ
H_2	14.21	10.09	1.41
He	5.194	3.117	1.66
N_2	1.040	0.743	1.40
O_2	0.915	0.655	1.40
Air	1.004	0.718	1.40
CO	1.040	0.743	1.40
CO_2	0.818	0.628	1.30
H_2O	1.858	1.396	1.33
SO_2	0.607	0.477	1.27
NH_3	2.055	1.567	1.32
CH_4	2.158	1.640	1.32
$C_2 H_2$	1.629	1.310	1.23
$C_2 H_4$	1.496	1.200	1.24
$C_2 H_6$	1.654	1.377	1.20
$C_3 H_8$	1.512	1.325	1.14
n-C_4H_{10}	1.623	1.480	1.10

specific heat capacity c_p and c_v at p = 0 bar and t = 0 °C, $\kappa = c_p / c_v$ at t = 0 °C

The compression work to be performed in the isothermal compression process is always lower than the adiabatic compression work. With the increasing pressure ratio, this difference in compression work increases. For air, the ratio values given in table 1-17 result when compressing from p_1 to p_2 [1.3].

Table 1-17. Isothermal compression work in relation to adiabatic compression work of air
Compression from p_1 = 101 kPa to values of p_2 acc. to the table.
W_{is} = isothermal compression work
W_{ad} = adiabatic compression work

p_2 [kPa]	200	500	1000
$\frac{W_{is}}{W_{ad}}$	0.896	0.786	0.704

Real compression processes are called polytropic processes. They lie between the two borderline cases of ideal heat dissipation and ideal heat insulation. By means of cooling, a state is reached in which the gas is not so intensely heated, i.e. the convergence to an isothermal process. Processes running in cooled compressors are subject to the laws according to the equations (1-186) and (1-187), replacing the adiabatic exponent κ by the polytropic exponent n [1.3].

This change of state is subject to the law

$$p \cdot V^n = \text{const.} \tag{1-189}$$

For technically normal changes of state, n lies mostly within the range of $1 < n < \kappa$.

For isothermal, polytropic and adiabatic compression work at the same pressure ratios, the following applies:

$$W_{is} < W_{pol} < W_{ad}$$

W_{is} isothermal compression work
W_{pol} polytropic compression work
W_{ad} adiabatic compression work

The polytropic exponent of biatomic gases (e. g. air) ranges from about 1.25 to 1.35. In centrifugal compressors, the polytropic exponent n can even be larger than the adiabatic exponent κ, due to higher flow speeds and higher internal friction.

1.9.4
Especially distinguished changes of state

The four changes of states which are different according to their thermal state quantities may be regarded as special cases of polytropic curves. An overview is given in table 1-18.

Table 1-18. Changes of state in the p,V diagram [1.9]

change of state	exponent	Explanation acc. to $p \cdot V^n = \text{const.}$
isobars	$n=0$	$p \cdot V^{\circ} = \text{const.}$: $p = \text{const.}$
isotherms	$n=1$	$p \cdot V^1 = \text{const.}$: $p \cdot V = \text{const.}$
isentropes	$n=k$	$p \cdot V^k = \text{const.}$
isochores	$n = \pm\infty$	$p^{\frac{1}{n}} \cdot V = \text{const.}$: $V = \text{const.}$

n = polytropic exponent

1.10

Names and definitions in vacuum technology

(extract from DIN 28400 Part 1 – July 1979)

Absorption. Absorption is a sorption with the gas (absorbate) penetrating the solid body or liquid (absorbent).

Adsorption. Adsorption is a sorption with the gas (adsorbate) being bound on the surface of a solid body or a liquid (adsorbent).

Average particle velocity. The average particle velocity $\bar{v}$is the average amount of the particle velocities.

Characteristic flow conductivity, Intrinsic conductivity. The characteristic flow conductivity is the flow conductivity in the case of an orifice or a duct connecting two vessels with Maxwell's velocity distribution existing in each vessel. In the case of molecular flow, the characteristic flow conductivity is the product of flow conductivity at the inlet orifice of the duct and the throughput probability of the particles. In this flow range, the characteristic flow conductivity is independent from pressure. The flow conductivity usually given in catalogues corresponds to this definition.

Characteristic flow resistance. Characteristic flow resistance is the inverse value of the characteristic flow conductivity.

Chemisorption. Chemisorption is an adsorption developing chemical compounds.

Collision rate. Collision rate is the quotient of the average number of impacts of a particle with other particles during its moving through a gas within a period of time and that period of time.

Gassing off, gas release. The gassing-off is a spontaneous desorption.

Degassing. Degassing is desorption accelerated by physical processes.

Desorption. Desorption is the release of sorbed gases from a sorbent. The release may be spontaneous or accelerated by physical processes.

Diffusion coefficient. The diffusion coefficient is the absolute amount of the quotient of particle flow density and the concentration gradient. The particle flow refers to a surface element perpendicular to the direction of the gradient.

Dwell time. Dwell time is the average period within which a particle in adsorbed state remains on the surface.

Flow conductivity. The flow conductivity of an orifice in a thin wall, or of a duct or of a duct piece between two defined cross-sections is the quotient of the $p\dot{V}$-flow and the difference of the pressures existing on both sides of the orifice, or of the duct or on the two defined cross-sections of the duct piece, with the temperature being constant in the system. The obtained measured value depends on the method of pressure measurement.

Flow resistance. Flow resistance is the inverse value of the flow conductivity.

Gas (in general). Gas is a matter in a state of aggregation in which the mean distances between the particles are large in comparison to their sizes, and in which the mutual order of the individual particles changes continuously.

Gas (in the stricter sense). Gas in the stricter sense is a matter in the gaseous state of aggregation which cannot be transformed into the liquid or solid state of aggregation by compression at prevailing temperatures.

Gas consumption. Gas consumption is a sorption resulting from an electric discharge.

Gas diffusion. The gas diffusion is the movement of the gas resulting from its concentration gradient in another medium. The medium can be gaseous, liquid or solid.

Gas dynamic flow. The gas dynamic flow is the flow of a gas with comparably large changes of density characterized by high Mach-Numbers (e. g. supersonic flow in nozzle- or steam jet pumps).

Gas permeability. The gas permeability is a parameter (a quantity related to the type of matter) for permeation of a gas through a certain solid or liquid at a certain temperature. It is calculated from the quotient of the $p\dot{V}$ flow through a solid or liquid of finite thickness and a quantity being a function of the pressures existing on both sides of a solid or liquid. The kind of function depends on the physical processes taking part in the current permeability.

Gas permeation. Gas permeation is the penetration of gas through a solid or liquid of finite thickness. Permeation includes diffusion and surface effects.

Gettering. Gettering is a link of gas, mostly through chemical reactions. Frequently, getters (gettering matters) have large real surfaces.

Ideal vacuum. In vacuum technology ideal vacuum is the state in a space with the particle density being zero (up to now called “absolute vacuum”).

Knudsen flow. Knudsen flow is a gas flow in the area between viscous flow and molecular flow.

Laminar flow. The laminar flow (layer- or thread flow) is viscous flow without mixing movements at small Reynold’s numbers.

Mean free path. Mean free path is the average value of distances a particle flies through between two successive impacts with other particles.

Mass flow. Mass flow is the quotient of the mass of a gas flowing through a duct cross-section during a period of time and that period of time.

Maxwell’s velocity distribution. Maxwell’s velocity distribution is the velocity distribution according to the Maxwell-Boltzmann distribution function. This velocity distribution exists for particles of a gas in equilibrium at a certain temperature and for sufficiently large distances from the limiting wall.

Molecular effusion. Molecular effusion is a gas flow through an orifice in a wall with insignificant thickness with the largest width of the orifice being smaller than the mean free path.

Molecular flow. The molecular flow is a gas flow through a duct with the mean free path being very large compared with the largest bore size of the duct cross-section. In molecular flow, the flow resistance is independent from pressure.

Most probable particle velocity. The most probable particle velocity v_w is the particle velocity which corresponds to the maximum value of the Maxwell & Boltzmann distribution function.

Occlusion. Occlusion is the encapsulation of gas quantities in solids or liquids.

Partial pressure. Partial pressure is the pressure of a certain gas or vapor in a mixture of gases and/or vapors.

Particle flow conductivity. Particle flow conductivity of an orifice in a thin wall (wall with insignificant thickness) or of a duct or of a duct piece between two defined cross-sections is the quotient of the particle flow rate and the difference of the average particle number densities on the both sides of the orifice, or of a duct or on the both cross-sections of the duct piece.

Particle flow density. The particle flow density is the quotient of the particle flow rate through a surface element and the volume of the surface element.

Particle flow rate. The particle flow rate through a given surface element is the quotient of the difference of the average number of particles flowing through the surface element in one direction within a period of time and the average number of particles flowing though the element in the opposite direction divided by that period of time.

Particle number density (volume-related particle number). The particle number density is the quotient of the number of particles contained in a suitably selected volume element and the volume of that element.

Particle number density of a component in gas mixture. The particle number density of a component in a gas mixture is the quotient of the number of particles of this gas component contained in a suitably selected volume element and the volume of that element.

Permeation coefficient. The permeation coefficient is the product of gas permeability and the quotient of the thickness of the solid or liquid and the content of the surface on the side of the higher pressure.

Physical adsorption. Physical adsorption is an adsorption caused by physical forces which does not result in certain chemical compounds.

Poiseuille's flow. Poiseuille's flow is the special case of the laminar flow in a long pipe with a circular-cylindrical cross-section. In that case, a parabolic velocity distribution occurs.

Pore penetration; *transporisation.* Transporisation is a gas flow through a porous body due to a pressure difference.

Pressure units. The legal pressure units are Pascal as SI-unit (Pa),

1 Pascal = 1 Newton divided by 1 square meter,
$1\ \text{Pa} = 1\ \text{N} \cdot \text{m}^{-2}$,

and bar as a special unit name for 10^5 Pa,

$$1\ \text{bar} = 1000\ \text{mbar} = 10^5\ \text{Pa} = 10^5\ \text{N} \cdot \text{m}^{-2}$$

The common unit in vacuum technology is millibar.

p$\dot{V}$-flow. The p$\dot{V}$-flow is the quotient of the pV-value of a gas flowing through a pipe cross-section at the prevailing temperature within a period of time and that period of time.

pV-value. The pV-value is the product of pressure and volume of a certain quantity of a gas at the currently prevailing temperature. If the pV-value is to be a measure for the quantity of matter or the mass of a gas, then it needs to be an ideal gas with the temperature being specified.

Root of the average velocity square. The root of the average velocity square $\sqrt{\mathrm{v}}$ is the root of the average amount of squares of the particle velocities.

Saturation vapor pressure. Saturation vapor pressure is the pressure exerted by a vapor in thermodynamic equilibrium with one of its condensed phases at a certain temperature.

Sorption. Sorption is the binding of gas (sorbate) by a solid or liquid (sorbent). (Sorbents are also called sorption means).

Flow of the amount of substances, molar flow. The molar flow is the quotient of the amount of substances of a gas flowing through a duct cross-section within a period of time and that period of time.

Surface diffusion. Surface diffusion is a surface migration owing to a gradient of the chemical potential, e.g. due to a concentration gradient.

Surface migration. Surface migration is the movement of particles on a surface. The surface migration of a liquid film is called creeping.

Thermal effusion. Thermal effusion is a gas flow in the range of molecular flow through a duct system between two vessels with different temperatures. It leads to a state of equilibrium which maintains a pressure difference between the two vessels.

Total pressure. The total pressure is the sum of the existing partial pressures. This word is used when the shorter term "pressure" does not allow the clear distinction between the individual partial pressures and their sum in the given context.

Flow-through probability (Clausing factor). The flow-through probability is the probability that a gas particle penetrating a duct element through an orifice in any direction will leave it through the duct outlet without having passed the inlet again in the opposite direction. In this case, the reflexion according to the Cosine law is assumed for the interaction with the walls.

Turbulent flow. The turbulent flow is a viscous flow with mixing movement above a critical Reynold's number (for circular-cylindrical pipes, $\mathrm{Re} \approx 2300$).

Vapor. Vapor is a substance in gaseous state of aggregation existing either in thermodynamic equilibrium with its liquid or solid phase (*saturated vapor*), or which can be brought to thermal equilibrium (condensing) through compression at the prevailing temperature (*unsaturated vapor*).

Note: In vacuum engineering, the word "gas" in the broader sense is used both for a non-condensable gas and for vapor, if a distinction is not required.

Viscous flow. The viscous flow is a gas flow through a duct with the mean free path being very small compared with the smallest bore size of the pipe cross-section. The flow depends on the viscosity of the gas and can be laminar or turbulent. In case of viscous flow, flow resistance depends on the pressure.

Volume flow rate. Volume flow rate is the quotient of the volume of a gas flowing through a pipe cross-section at currently prevailing temperature and pressure during a period of time and that period of time.

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