

Contents

List of Contributors *XI*

Preface *XV*

1	Entropy and Renormalization in Chaotic Visibility Graphs	1
	<i>Bartolo Luque, Fernando Jesús Ballesteros, Alberto Robledo, and Lucas Lacasa</i>	
1.1	Mapping Time Series to Networks	2
1.1.1	Natural and Horizontal Visibility Algorithms	4
1.1.2	A Brief Overview of Some Initial Applications	8
1.1.2.1	Seismicity	8
1.1.2.2	Hurricanes	8
1.1.2.3	Turbulence	9
1.1.2.4	Financial Applications	9
1.1.2.5	Physiology	9
1.2	Visibility Graphs and Entropy	10
1.2.1	Definitions of Entropy in Visibility Graphs	10
1.2.2	Pesin Theorem in Visibility Graphs	12
1.2.3	Graph Entropy Optimization and Critical Points	19
1.3	Renormalization Group Transformations of Horizontal Visibility Graphs	26
1.3.1	Tangent Bifurcation	29
1.3.2	Period-Doubling Accumulation Point	31
1.3.3	Quasi-Periodicity	32
1.3.4	Entropy Extrema and RG Transformation	34
1.3.4.1	Intermittency	35
1.3.4.2	Period Doubling	35
1.3.4.3	Quasi-periodicity	35
1.4	Summary	36
1.5	Acknowledgments	37
	References	37
2	Generalized Entropies of Complex and Random Networks	41
	<i>Vladimir Gudkov</i>	
2.1	Introduction	41

2.2	Generalized Entropies	42
2.3	Entropy of Networks: Definition and Properties	43
2.4	Application of Generalized Entropy for Network Analysis	45
2.5	Open Networks	53
2.6	Summary	59
	References	60
3	Information Flow and Entropy Production on Bayesian Networks	63
	<i>Sosuke Ito and Takahiro Sagawa</i>	
3.1	Introduction	63
3.1.1	Background	63
3.1.2	Basic Ideas of Information Thermodynamics	64
3.1.3	Outline of this Chapter	65
3.2	Brief Review of Information Contents	66
3.2.1	Shannon Entropy	66
3.2.2	Relative Entropy	67
3.2.3	Mutual Information	68
3.2.4	Transfer Entropy	69
3.3	Stochastic Thermodynamics for Markovian Dynamics	70
3.3.1	Setup	70
3.3.2	Energetics	72
3.3.3	Entropy Production and Fluctuation Theorem	73
3.4	Bayesian Networks	76
3.5	Information Thermodynamics on Bayesian Networks	79
3.5.1	Setup	79
3.5.2	Information Contents on Bayesian Networks	80
3.5.3	Entropy Production	83
3.5.4	Generalized Second Law	84
3.6	Examples	86
3.6.1	Example 1: Markov Chain	86
3.6.2	Example 2: Feedback Control with a Single Measurement	86
3.6.3	Example 3: Repeated Feedback Control with Multiple Measurements	89
3.6.4	Example 4: Markovian Information Exchanges	91
3.6.5	Example 5: Complex Dynamics	94
3.7	Summary and Prospects	95
	References	96
4	Entropy, Counting, and Fractional Chromatic Number	101
	<i>Seyed Saeed Changiz Rezaei</i>	
4.1	Entropy of a Random Variable	102
4.2	Relative Entropy and Mutual Information	104
4.3	Entropy and Counting	104

4.4	Graph Entropy	107
4.5	Entropy of a Convex Corner	107
4.6	Entropy of a Graph	108
4.7	Basic Properties of Graph Entropy	110
4.8	Entropy of Some Special Graphs	112
4.9	Graph Entropy and Fractional Chromatic Number	116
4.10	Symmetric Graphs with respect to Graph Entropy	119
4.11	Conclusion	120
	Appendix 4.A	121
	References	130
5	Graph Entropy: Recent Results and Perspectives	133
	<i>Xueliang Li and Meiqin Wei</i>	
5.1	Introduction	133
5.2	Inequalities and Extremal Properties on (Generalized) Graph Entropies	139
5.2.1	Inequalities for Classical Graph Entropies and Parametric Measures	139
5.2.2	Graph Entropy Inequalities with Information Functions f^V, f^P and f^C	141
5.2.3	Information Theoretic Measures of UHG Graphs	143
5.2.4	Bounds for the Entropies of Rooted Trees and Generalized Trees	146
5.2.5	Information Inequalities for $I_f(G)$ based on Different Information Functions	148
5.2.6	Extremal Properties of Degree- and Distance-Based Graph Entropies	153
5.2.7	Extremality of $I_{f^1}(G), I_{f^2}(G), I_{f^3}(G)$ and Entropy Bounds for Dendrimers	157
5.2.8	Sphere-Regular Graphs and the Extremality Entropies $I_{f^2}(G)$ and $I_{f^3}(G)$	163
5.2.9	Information Inequalities for Generalized Graph Entropies	166
5.3	Relationships between Graph Structures, Graph Energies, Topological Indices, and Generalized Graph Entropies	171
5.4	Summary and Conclusion	179
	References	180
6	Statistical Methods in Graphs: Parameter Estimation, Model Selection, and Hypothesis Test	183
	<i>Suzana de Siqueira Santos, Daniel Yasumasa Takahashi, João Ricardo Sato, Carlos Eduardo Ferreira, and André Fujita</i>	
6.1	Introduction	183
6.2	Random Graphs	184
6.3	Graph Spectrum	187
6.4	Graph Spectral Entropy	189

6.5	Kullback–Leibler Divergence	192
6.6	Jensen–Shannon Divergence	192
6.7	Model Selection and Parameter Estimation	193
6.8	Hypothesis Test between Graph Collections	195
6.9	Final Considerations	198
6.9.1	Model Selection for Protein–Protein Networks	199
6.9.2	Hypothesis Test between the Spectral Densities of Functional Brain Networks	200
6.9.3	Entropy of Brain Networks	200
6.10	Conclusions	200
6.11	Acknowledgments	201
	References	201
7	Graph Entropies in Texture Segmentation of Images	203
	<i>Martin Welk</i>	
7.1	Introduction	203
7.1.1	Structure of the Chapter	203
7.1.2	Quantitative Graph Theory	204
7.1.3	Graph Models in Image Analysis	205
7.1.4	Texture	206
7.1.4.1	Complementarity of Texture and Shape	206
7.1.4.2	Texture Models	207
7.1.4.3	Texture Segmentation	208
7.2	Graph Entropy-Based Texture Descriptors	209
7.2.1	Graph Construction	210
7.2.2	Entropy-Based Graph Indices	211
7.2.2.1	Shannon's Entropy	212
7.2.2.2	Bonchev and Trinajstić's Mean Information on Distances	212
7.2.2.3	Dehmer Entropies	213
7.3	Geodesic Active Contours	214
7.3.1	Basic GAC Evolution for Grayscale Images	214
7.3.2	Force Terms	215
7.3.3	Multichannel Images	216
7.3.4	Remarks on Numerics	216
7.4	Texture Segmentation Experiments	217
7.4.1	First Synthetic Example	217
7.4.2	Second Synthetic Example	218
7.4.3	Real-World Example	220
7.5	Analysis of Graph Entropy-Based Texture Descriptors	221
7.5.1	Rewriting the Information Functionals	221
7.5.2	Infinite Resolution Limits of Graphs	222
7.5.3	Fractal Analysis	223
7.6	Conclusion	226
	References	227

8 Information Content Measures and Prediction of Physical Entropy of Organic Compounds 233

Chandan Raychaudhury and Debnath Pal

- 8.1 Introduction 233
- 8.2 Method 236
 - 8.2.1 Information Content Measures 236
 - 8.2.2 Information Content of Partition of a Positive Integer 240
 - 8.2.3 Information Content of Graph 243
 - 8.2.3.1 Information Content of Graph on Vertex Degree 245
 - 8.2.3.2 Information Content of Graph on Topological Distances 246
 - 8.2.3.3 Information Content of Vertex-Weighted Graph 251
 - 8.2.4 Information Content on the Shortest Molecular Path 251
 - 8.2.4.1 Computation of Example Indices 252
- 8.3 Prediction of Physical Entropy 253
 - 8.3.1 Prediction of Entropy using Information Theoretical Indices 254
- 8.4 Conclusion 256
- 8.5 Acknowledgment 257
- References 257

9 Application of Graph Entropy for Knowledge Discovery and Data Mining in Bibliometric Data 259

André Calero Valdez, Matthias Dehmer, and Andreas Holzinger

- 9.1 Introduction 259
 - 9.1.1 Challenges in Bibliometric Data Sets, or Why Should We Consider Entropy Measures? 260
 - 9.1.2 Structure of this Chapter 261
 - 9.2 State of the Art 261
 - 9.2.1 Graphs and Text Mining 262
 - 9.2.2 Graph Entropy for Data Mining and Knowledge Discovery 263
 - 9.2.3 Graphs from Bibliometric Data 264
 - 9.3 Identifying Collaboration Styles using Graph Entropy from Bibliometric Data 266
 - 9.4 Method and Materials 266
 - 9.5 Results 267
 - 9.6 Discussion and Future Outlook 271
 - 9.6.1 Open Problems 271
 - 9.6.2 A Polite Warning 272
- References 272

Index 275

