1
Introduction

Magnetism in Medicine: A Handbook, Second Edition
Edited by Wilfried Andrä and Hannes Nowak

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1.1
The History of Magnetism in Medicine

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1.1.1
Origins

Although magnetic effects such as the ''northern lights'' in the northern hemisphere have been observed for thousands of years, it was not until the discovery of iron smelting, at around 1200 BC, that a body of knowledge on magnetism began to develop. The first effects of magnetism were observed when the smelted iron was brought close to the iron oxide in the chemical form of $FeO{\cdot}Fe_2O_3$ (Fe_3O_4), a natural iron ore which came to be known as lodestone or magnetite.

The origin of the term ''magnetite'' is unclear, but two explanations appear most frequently in the literature. In one of these, magnetite was named after the Greek shepherd Magnes, who discovered it when the nails on the soles of his shoes adhered to the ore. In the other explanation, magnetite was named after the ancient county of Magnesia in Asia Minor, where it was found in abundance.

The first treatise on magnetized needles and their properties (see Fig. 1.1) was presented by Petrus Peregrinus in 1289 (Peregrinus, 1269). This treatise clearly documented a number of magnetic properties including that: (1) magnetic forces act at a distance; (2) magnetic forces attract only magnetic materials; (3) like poles repel and unlike poles attract; and (4) north poles point north, and south poles south. Equipped with this knowledge, the medieval Europeans navigated the globe, discovering and conquering countries as they went.

Peregrinus, however, failed to note that the Earth itself is a magnet. Yet it was not until 1600 that this discovery was finally made by William Gilbert, a physician of Queen Elizabeth I. In order to arrive at this conclusion, Gilbert performed numerous experiments that separated hearsay from truth, documenting them in his book *De magnete* along with a summary of the knowledge of the time about magnetism and electricity (Gilbert, 1600). Gilbert's systematic and scientific treatise is considered by many to be one of the first great works in science (Butterfield, 1991) (see Fig. 1.2).

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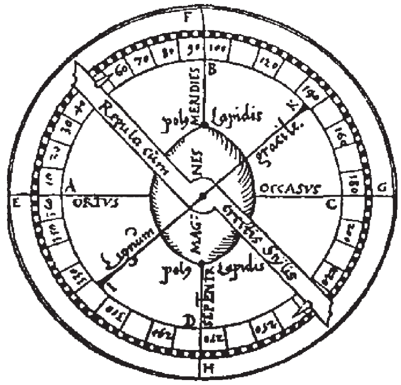

Fig. 1.1. One of Petrus Peregrinus' inventions is this "Astrolabium", an oval lodestone mounted inside a wooden box. The four points of the compass and 360 subunits were painted on the inside of the box. This instrument was placed in a bowl of water to determine the azimuth of the sun, for example, and the angle was read after the astrolabium had stopped moving.

1.1.2
First Medical Uses of Magnets

Thales of Miletus, the first Greek speculative scientist and astronomer (ca. 624–547 BC) was also the first to make a connection between man and magnet. He believed that the soul somehow produced motion and concluded that, as a magnet also produces motion in that it moves iron, it must also possess a soul. It is likely that this belief led to the many claims throughout history of the miraculous healing properties of the lodestone.

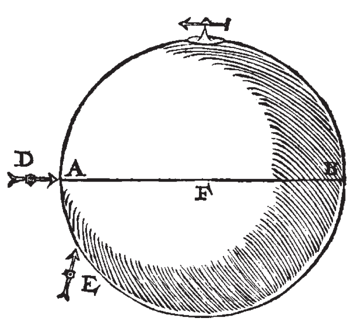

Fig. 1.2. The terrella (spherical lodestone), and the location of its poles from Gilbert's book *De Magnete*. The magnetic versorium (compass needle) on top of the sphere is pointing along a meridian circle; the versorium at D points directly to the center of the sphere and hence to the pole A, in contrast to the versorium at E.

Medical references to magnetism were made by Hippocrates of Cos (ca. 460–360 BC), who used the styptic iron oxides magnetite and hematite to stop bleeding and to control hemorrhage (Mitchell, 1932). Unraveling the true early medical applications of magnetite as described by Hippocrates and his scholars is, unfortunately, complicated by the two meanings of the same term. In particular, magnetite overlaps with the older term ''magnesite'', a magnesium carbonate with laxative properties.

In the first century, Pliny the Elder (23–79 AD), a Roman scholar, collected and condensed the entire knowledge of the time into a thirty-seven-volume encyclopedia, which was used for the next 1700 years. Amid its wealth of information lies a description of the treatment of burns with pulverized magnets. Pliny, however, failed to discriminate fact from fiction, and included much folklore and superstition in his writings. He also theorized that ''sympathies and antipathies'' were the cause of magnetic phenomena, a viewpoint which was shared by Galen of Pergamum (129–199 AD). Galen compared the lodestone to cathartic drugs which attract certain ''qualities'' such as bile and phlegm, to drugs which remove thorns and arrow-points or draw out animal and arrow-tip poisons, and to ''corn'', which is better able to draw water into itself than the sun's heat is to draw water out of it (Brock, 1916). The same attracting properties of lodestone were advocated by Dioscorides of Anazarbos in the first century in his encyclopedia of medical matter. He recommended their external use for ''drawing out gross humors'' (Gunther, 1934).

When magnetite was applied externally, this was either as the unbroken lodestone or in pulverized form, compounded with other ingredients, under the name of Emplastrum Magneticum. The usual practice seems to have been to bind the lodestone or magnetic plaster directly to the affected body part. This technique was thought to be efficacious in treating diseases such as arthritis, gout, poisoning, or baldness. Lodestones were even thought to have strong aphrodisiac potency (Mourino, 1991).

Although most ancient medical uses of magnetite were external, it was also promoted for internal applications by the Egyptian physician and philosopher Avicenna (980–1037 AD). Avicenna recommended using the magnet in doses of one grain as an antidote for the accidental swallowing of poisonous iron (rust). The pulverized magnet was often taken with milk, and the magnetite was believed to render the poisonous iron inert by attracting it and speeding up its excretion through the intestine. This remedy may have worked as a consequence not only of its intended mechanism but also because it induced vomiting (Stecher, 1995). Albertus Magnus (1200–1280), in his book *Mineralia*, recommended the same milk/magnetite mixture for the treatment of edema (Magnus, 1890).

1.1.3 Use of Attracting Forces of Magnets in Medicine

The earliest known account of the surgical use of lodestone is believed to be found in the writings of Sucruta, a Hindu surgeon who lived around 600 BC (Hirschberg, 1899). Sucruta wrote in his book *Ayur-Veda* that the magnet which is in Sanskrit called ''Ayas Kanta'' – the ''one loved by iron'' – can be used to extract an iron arrow tip. Sucruta specified that the extraction works best if ''... the piece of iron is embedded parallel to the fibers of the tissue, does not contain any ears (barbs), and the opening is wide''.

Sucruta's applications were not explored again for almost 2000 years. Gilbertus Anglicus wrote around 1290 in his earliest medical work that ''... certain surgeons apply adamant or magnet, if iron is concealed in the flesh'' (Anglicus, 1290). This concept was described in a publication from 1640 which suggested that iron in the form of iron filings should be fed to a patient with a hernia (Kirches, 1640). The appropriate placing of an externally attached magnet was then expected to attract the iron, thus drawing in and restoring the protruding intestine. The successful employment of this treatment was reported some years later by surgeon Ambrose Parè, a claim which he asked doubters to take at face value ''... on the faith of a surgeon'' (Johnston, 1678)!

Other accounts of successful magnetic extractions also appeared, including the description by G. Bartisch. In 1583, he wrote: ''A good cream, in case iron, steel or stone had leaped into your eyes, is made from 3 lots of rabbit fat, 1 lot of wax, 1 quint of yellow agstone and 1/2 a quint of lodestone. Such a cream, if applied over your eyes in form of a plaster, helps.'' Hirschberg, who cited this description (Hirschberg, 1899), added: ''Of course, it doesn't help at all!'' A similar dose of skepticism may be appropriate in the case reported by Andry and Thouret (Andry, 1779) who reported that around 1635 surgeons succeeded in bringing the point of a knife that had been swallowed accidentally to the integuments with the aid of the emplastrum magneticum. The point was then surgically removed from that location.

Gilbert cited such claims at the end of the 16th century, and categorically denied them: Lodestone ground into a plaster would not be strong enough to extract large iron objects; the same plaster applied to the head could not cure headaches; if lodestone were used with incantations it would not cure insanity; magnets applied to the head would not cause unchaste wives to fall out of bed; and lodestones would draw neither the pain out of gout nor poisons from other parts of the body (Butterfield, 1991). Gilbert believed that the only effect of this plaster was to heal ruptured tissues by drying them out. What magnetite (or iron) was good for, Gilbert maintained, was chlorosis, as patients with this disease were thought to benefit from small doses of iron filings mixed with strong vinegar. Gilbert found that this mixture also helped older patients with splenomegaly, chronic malaria and anemia – diseases not uncommon in the East Anglian swamps of England at that time.

More believable accounts of the applications of magnetic forces – at least in terms of present-day standards – began to appear in the 17th century. For example, Andreas Frisii described a case in which a needle accidentally lodged in the side of a person's throat was removed by a traveling mountebank, as ''... fools rush in where wise men fear to tread'' (Frisii, 1670):

> "When the master permitted the use of water from the spa because of health reasons, a careless maid swallowed a needle which got stuck in the inside of her throat and thus talking became difficult for her. Without doing more damage, the needle moved on to the tonsils and remained visibly stuck there for a total of nine years. Although she could feel the needle, there was no inflammation, but the maid was still afraid of a future disaster. The metal occupied several surgeons, but none of them dared in fear of an even larger misfortune, to pull out the needle by hand. Then, in the 23rd year (= 1623), a man, of the kind who still heal certain diseases even though they do not know much about medicine, appeared. They dare to promise everything to everybody, but nonetheless, their experimental knowledge is quite extensive. The one I am speaking about promised easy relief without complaints and pain. She believed it, and he began, after making an incision with a smooth knife, to pull the skin apart and place the lodestone (not the powder, as was commonly done) directly on the wound. After the ninth day the needle adhered to the stone, and the woman was relieved."

In subsequent years, medical applications of magnets came to include the removal of iron particles embedded in the eye (Quinan, 1886). The magnets used were native, or later, artificial or electromagnets. In 1627, Wilhelm Fabricius of Hildanus (1560–1634), a German physician who practiced medicine in Bern, Switzerland, documented the first case of an iron splinter being extracted from a patient's eye chamber. The following is a translation of the physician's original Latin description, titled "Of a slag splinter stuck in the cornea and its ingenious healing" (Hildanus, 1627):

> "A certain farmer from the valley St. Michelis, as they call it, near the Lake of Bienne, named Benedictus Barquin, bought steel from the trader, wanted to choose the best, and therefore beat, as it is commonly done, piece against piece. A splinter from one of them flew into the part of the cornea where the iris is visible, and this not without great pain. The relatives tried to help for several days without any result, and thus pain and inflammation increased. So he finally came on March 5th to see me in Bern. First through sensible nutrition, then through purging of the body using drugs as well as through bleeding (that is to say he had a bloodshot eye), I tried to pull out the splinter, first with instruments and then in other ways. But the splinter was so small, that it wasn't possible to pull it out. Therefore I followed another route and decided to pull out the splinter with the help of a small bag, as I described it earlier in Cent. 4, Observ. 17. But again, I lost oil and work, and my wife came upon by far the most advisable remedy. While I was holding open the eyelid with both hands, she approached the eye with a lodestone, as near as he could endure. We had to repeat this several times (it was necessary to do it like that, since he could not stand the light for long anymore). Finally, the splinter jumped, visible for us all, onto the magnet."

Over the following 300 years, increasingly complex procedures for removing metallic objects from the eye were performed and reported. The second successful case report after Fabricius was described in 1684 in a letter published by the "experienced oculist", Dr. Turberville of Salisbury, England (Turberville, 1684). The physician stated that "A person in Salisbury had a piece of iron, or steel, stuck in the

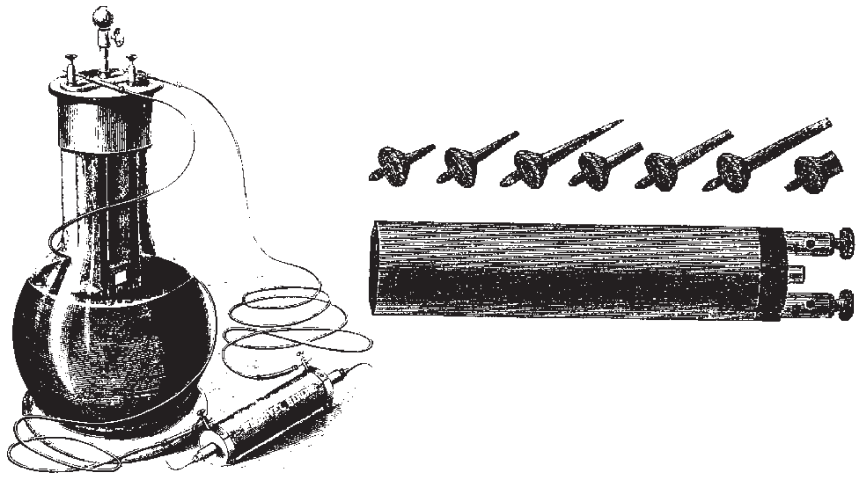

Fig. 1.3. Hand-held electromagnets used for the removal of magnetic objects from the eye. Left: The original Hirschberg magnet. Right: A further development of Dr. Hubbell. The needle-like tip is placed, preferably through the entry wound, as close as possible to the foreign iron or steel particle. The magnet is then turned on and the foreign body pulled out.

iris of the eye, which I endeavored to push out with a small spatula, but could not. But on applying a lodestone, it immediately jumped out.'' Again more than 80 years later, the use of lodestone for eye surgery was reported by Dr. Morgagni in a case involving the cornea (Morgagni, 1761). A more spectacular case involving the removal of iron fragments from behind the iris was reported by Dr. Nicolaus Meyer of Minden, Germany, in 1842. According to Hirschberg, one of the great experts in the field, this was the first case on record for the removal of pieces of iron from the interior of the eye (Hirschberg, 1883). The first case in America, was performed by Dr. Alex. H. Bayly of Cambridge, Maryland (Bayly, 1886). The magnet used in this case was an ''artificial'' or horse-shoe magnet.

Since 1879, the use of magnets has become the established procedure for the removal of magnetic objects from the interior of the eye (Tost, 1992). In that year, Dr. Julius Hirschberg reported the first ophthalmic use of electromagnets (Hirschberg, 1880). His magnet had the shape of an ''electric handmagnet'' and was used like forceps, in close proximity to the foreign metallic objects (see Fig. 1.3). Further developments in electromagnets by the Swiss ophthalmologist Otto Haab led to extractions in which the magnet was placed at greater distance from the eye (Haab, 1892). He used Rühmkorff's apparatus, a 130-kg heavy electromagnet with a small pointed horizontal protruding tip, designed at the Federal Institute of Technology in Zurich. The patient sat in front of the tip with their head fixed in a 90° cone. The cone was then moved by the physician into the extraction position. This magnet produced forces of 11.3 mT ($\sim 10^5$ dyne) at a distance of 5 mm from the tip. Due to the size of this magnet, it was later termed ''the giant magnet'' (Fig. 1.4).

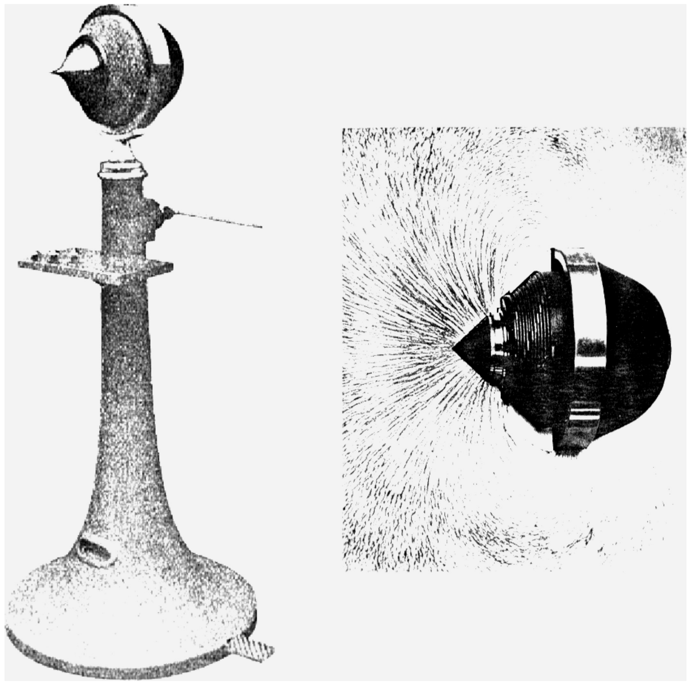

Fig. 1.4. Dr. Haab's giant magnet for the removal of iron or steel foreign bodies from a patient's eyes. The magnetic field lines around the tip of the instrument are shown to the right.

Haab and Hirschberg's different approaches to the removal of magnetic objects from the eye resulted in a 22-year-long (from 1892 to 1914) scientific battle waged through letters and articles. Their views differed with respect to the type of magnet to use, the position of the patient, and the route of removal of the foreign objects. Haab favored the direct removal of small objects through the front of the eye, ''der vordere Weg'', while Hirschberg preferred removal through the back of the eye, ''der hintere Weg''. Both methods were known to be associated with peeling of the iris, a serious side effect, although it was not until 1970 that both techniques were shown to be equally risky (Springer, 1970). It is no longer necessary to establish the superiority of either method since the use of magnets to remove objects from the eyes is currently declining, due to advancements in modern eye surgery.

Magnets have been employed to remove iron or steel objects not only from the eyes, but also from other body parts. Swallowed pins and nails are commonly extracted magnetically from the stomachs of unlucky children, and shrapnel drawn from the surface wounds of war, bomb or crime victims. The extraction of a safety pin from a child's stomach is illustrated in Figure 1.5 (Luborsky et al., 1964).

Another interesting approach to render ingested and potentially dangerous metallic objects harmless is seen in the application of magnets in veterinary medicine. Grazing cows often swallow sharp steel objects such as the barbs from barbed wire,

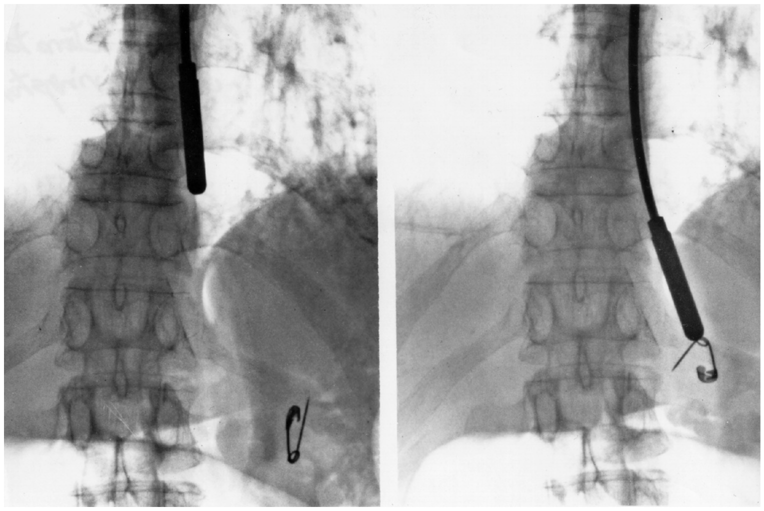

Fig. 1.5. Removal of an open safety pin from a patient's stomach. A probe is ''swallowed'' by the patient (left) and maneuvered by the physician until the tip is near the rounded end of the pin. When the tip of the probe is magnetized, it attracts the pin (right). With the pin in position, the point is less likely to do damage to the digestive tract as it is pulled out. (Photograph courtesy of F.E. Luborsky; Luborsky et al., 1964).

or pieces of wire from bales of hay. In order to prevent these sharp objects from damaging the stomach and intestinal walls, the cows are forced to swallow a ''cow magnet'', a 7 cm-long and 1 cm-diameter rod Alnico magnet covered with an anti-corrosive plastic coating. The cow magnet remains in one of the cow's stomachs, where it attracts any steel or iron objects that pass by, rendering them non-dangerous and preventing the so-called ''hardware disease''. The magnets can be easily retrieved when the cow is slaughtered, and do not appear to have any adverse side effects (Livingston, 1996).

1.1.4
Treatment of Nervous Diseases and Mesmerism

The first person to mention the topical application of a magnet in nervous diseases was Aetius of Amida (550–600 AD), who recommended this approach primarily for the treatment of hysteria, and also for gout, spasm, and other painful diseases. Some five centuries later, abbess Hildegard of Bingen (1098–1179) – who was said to have received the words for her books directly from God – described the use of plants and minerals (stones) for medical purposes and devoted a whole chapter to the lodestone. Her method of using the lodestone was somewhat new, in that the magnet had to be held in the patient's mouth to remedy fits of anger or rage, to make fasting bearable, and to keep lies and maliciousness at bay (Riethe, 1961).

Several hundred years later, the Swiss Theophrastus Bombast von Hohenheim (1493–1541), a doctor and alchemist, reasoned that since magnets have the mysterious power of attracting iron, they should also be able to attract diseases from the body. He was often criticized for his beliefs and was mockingly called "Paracelsus", which means "greater than Celsus" (Celsus was a famous Roman doctor who lived around 25 BC to 50 AD); he finally adopted the name Philippus Aureolus Paracelsus. In his work, *Volumen Medicinae Paramirum*, Paracelsus described exact procedures to transplant diseases from the body into the earth by using a magnet. The choice of magnetic pole was important for these procedures. In his treatment of epilepsy – a disease in which there is "... more nervous fluid in the brain", "... the repulsing pole of a magnet" was "... applied to the head and the spine", and "... the attracting pole to the abdominal region". Paracelsus further extended the use of magnets to leucorrhea, diarrhea and hemorrhages, for which his procedures were often successful. However, the effectiveness of his methods could probably be attributed more to the amazing powers of human imagination than to magnetism.

Reports from England during the 1740s regarding the production of strong artificial (not lodestone) magnets led to renewed interest in the use of strong magnets for healing purposes. It is unclear who was responsible for the introduction of steel magnets, but evidence points to it being either Gowin Knight, a physician; John Canton, a schoolmaster and amateur physicist; or John Michell, an astronomer. The term "horse-shoe magnet", however, came from Michell. One of the people who experimented with the new magnets was Father Maximilian Höll (1720–1792), a Jesuit priest and astronomer at the University of Vienna. In 1774, Höll became friends with the then 40-year-old physician Franz Anton Mesmer, to whom he gave some of these magnets. By applying them to his patients – who mainly had symptoms of hysterical or psychosomatic origin – Mesmer achieved many seemingly miraculous cures.

Mesmer first conjectured that the magnets worked by redirecting the flow of the universal "fluidum" from the atmosphere or the stars to the patients' bodies. He soon discovered, however, that magnets could be replaced by nonmagnetic objects such as paper, wood, stone, and even humans and animals. This led Mesmer to coin the term "animal magnetism" for the fundamental biophysical force he considered responsible for the free flow of fluidum. Disease originated from an "obstructed" flow, which could be overcome by "mesmerizing" the body's own magnetic poles and inducing a "crisis", often in the form of convulsions. The patients' health and "harmony" could thus be restored (Mourino, 1991). A graphic account of the treatment of Mesmer's first patient was given by Macklis (1993).

Mesmer's theories and, probably even more so, his rapidly gained fame soon enraged the medical faculty of Vienna. In 1777, they used the case of Maria Theresia von Paradies as the reason to expel him both from the fraternity of medicine and from the city of Vienna. Maria Theresia was a blind child piano prodigy who regained her sight after being treated by Mesmer. Unfortunately, she simultaneously lost her equilibrium as well as her musical talents. Her parents were angered and demanded that Mesmer stop the treatment. The child's reaction to the suspension of treatment was spectacular, in that she dropped immediately to the floor in convulsions, blind once again.

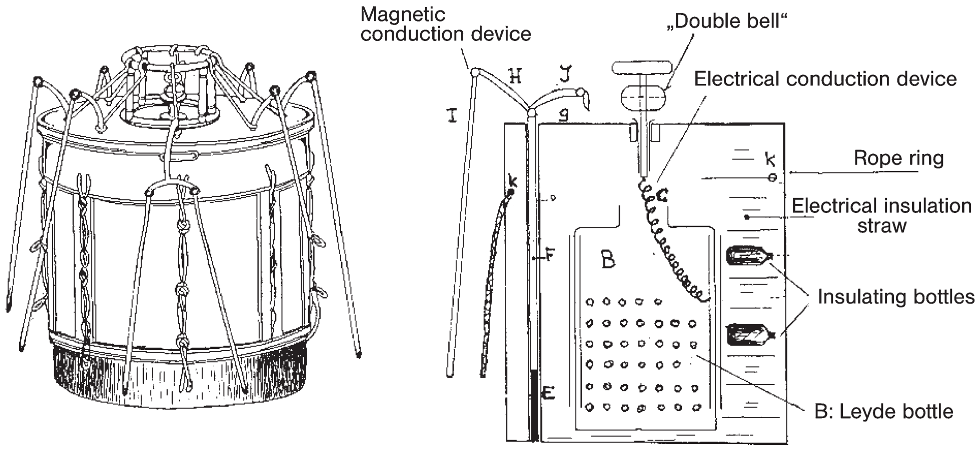

Fig. 1.6. Mesmer's tub, the first medical device from 1780 designed for the biomagnetic treatment of men and women. (Illustration courtesy of Dr. A. Dittmar, Lyon).

Evicted from Vienna, Mesmer went to Paris, where his theories of "animal magnetism" were eagerly embraced. He used his many talents in a curative, psychological way (Darnton, 1972), and his clinic soon became famous for spectacular spiritualistic sessions. In one of his best-known treatments, patients bathed in magnetized water in an oval vessel called the "Baquet de Mesmer" (Mesmer's tub) (see Figs. 1.6 and 1.7).

With time Mesmer's theories evolved from his initial teachings, developing into an empirical psychological healing science, a mix of hypnotism and psychotherapy, imaginative and psychosomatic medicine. Taken up by many healers and quacks, Mesmer's ideas were promoted in many books, and periodicals were soon crowded with reports of the successful treatment of nervous maladies. King Louis XIV of France was skeptical about these reports of animal magnetism and requested an investigation from Benjamin Franklin and Antoine Lavoisier. After having performed 16 different experiments – many of them in a blinded setup – the two scientists showed in 1784 conclusively that magnetism had nothing to do with the reported healings (Shermer, 1997). Many of the beneficial effects attributed to the use of magnets in the treatment of nervous diseases were evidently due to the increased suggestibility of the subjects to whom this novel remedy was applied. In such cases, with the necessary amount of faith, almost anything is a remedy.

Even today, magnets continue to be advertised as health-promoting, and are sold in amazing numbers and in many different forms and shapes for all purposes. Recent advertisements, for example, claim that magnetic bracelets cure headaches, and that magnetic mattresses, shoe inserts, and belts have beneficial health effects by influencing the body's magnetic field. The use of supermagnets (neodymium-iron-boron magnets) is advocated as a pseudo-scientific cancer cure. Some of these interesting claims are described in more detail by Livingston (1996).

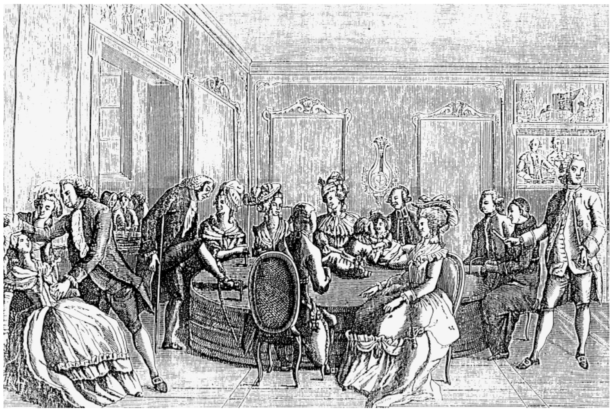

Fig. 1.7. Mesmer's tubs existed in different sizes, with large versions in great demand by the high society and the court of King Louis XVI from France during the 18th century. (Illustration from an engraving from 1779, collection of M. Gaston Tissandier; courtesy of the Lyon Historic Museum of Medicine, University Claude Bernard, Lyon).

1.1.5 Other Medical Uses of Magnets and Magnetism

During the past 20 years, the medical use of magnets has spread to fields as diverse as dentistry, cardiology, neurosurgery, oncology, and radiology, to mention only a few. The scientific advancements that made these new applications possible include the evolution and miniaturization of electromagnets, the development of superconducting electromagnets at Bell Laboratories in 1961, and the introduction of strong permanent magnets made of samarium-cobalt between 1960 and 1970 (McCaig and Clegg, 1987) and of neodymium-iron-boron (NdFeB) in 1983 (Kirchmayr, 1996; Goll and Kronmüller, 2000).

The new, much stronger magnetic materials allowed the construction of miniaturized magnets and electromagnetic coils, the smallest of which is so tiny that it could fit into the tip of a vascular catheter (Hilal et al., 1974). These small catheters permitted intravascular guidance from outside of the body with a strong magnetic field, and have been used clinically both for monitoring intracranial electroencephalograms and for producing electrothrombosis of inoperable arterial aneurysms. Furthermore, with the help of such a catheter, a discrete embolus or an intravascular adhesive can be deposited for the selective occlusion of vascular lesions. In 1979, a magnetically fixable catheter that electrically stimulated the heart was clinically tested in patients with bradycardic arrhythmia, providing temporary pacemaker therapy (Paliege et al., 1979). The design included an electrode almost identical to those of the stimulation catheters, except that its 18 mm-long and 0.9 mm-diameter tip was made from soft iron coated with gold rather than from platinum or iridium-coated NiCr-steel. It was thus ferromagnetic. Using this catheter together with an external magnet, a stable stimulation position was reached in the

Fig. 1.8. (a) The Niobe® system, a magnetic navigation system built by Stereotaxis Inc., St. Louis, Missouri, USA. The system is based on two large permanent magnets that, upon proper rotation and movement, are able to precisely direct a magnet-tipped guide wire (b) or electrophysiology mapping catheter (c) within the patient's vascular system. This system was approved by the FDA in the USA in 2003 for multiple interventional cardiology and electrophysiology procedures.

right auricle of 17 out of 19 patients, and in the right ventricle of 28 out of 32 patients. A more recent report described the successful diagnosis of a complex congenital heart disease through the use of a catheter magnetically guided through a neonate's heart (Ram and Meyer, 1991). As the distance from a baby's heart to the skin above is relatively small, an appropriately placed magnet was able to direct the magnetic catheter tip into the right ventricle, thus allowing for the injection of a contrast agent.

The magnetic guidance of catheters and similar devices in adults requires the use of higher magnetic fields and field gradients than those employed with children. One system which attains the required fields is the magnetic-implant guidance system developed for stereotactic neurosurgery (McNeil et al., 1995a,b). This system made use of very strong superconducting magnets to deliver a small magnetic NdFeB capsule within the brain with an accuracy of 2 mm. The capsule was moved by six independently controlled superconducting coils mounted in a helmet, and described to be used, in the future, to deliver radioactivity, heat, or chemotherapeutic drugs to a tumor in the brain.

During the early 2000s, the company Stereotaxis Inc., in St. Louis, Missouri, USA, further developed this system by replacing the superconducting electromagnets with easier to maintain NdFeB permanent magnets. In this new setup, the magnets are placed in a housing a few meters away from the surgical table. When the patient is ready to undergo the navigation of surgical guidewires and catheters (Fig. 1.8b and c), the two magnets in their housing are rotated into place for magnetic navigation (Fig. 1.8a). The magnetic force vector established under computer

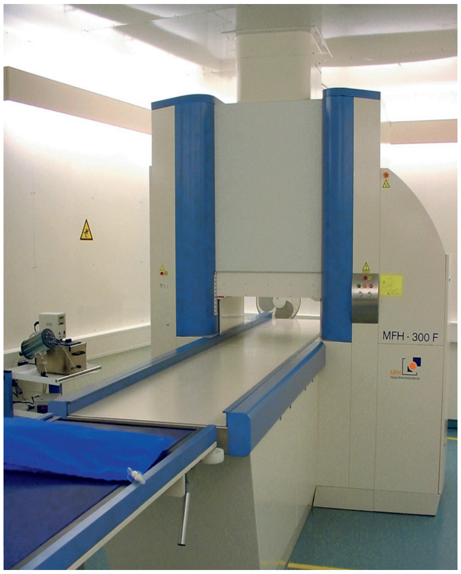

Fig. 1.9. Using an alternating magnetic field of 100 kHz, the magnetic field applicator MFH 300F (MagForce Applications GmbH, Berlin, Germany) is able to induce hyperthermia in tumors containing magnetic nanoparticles. Clinical trials are currently being performed (Jordan et al., 2001; Gneveckow et al., 2005).

and joy stick control by the surgeon then guides a catheter with a magnetic tip to chosen positions in the heart or coronary vasculature. For this purpose, the two magnets can be rotated independently and turned from one side to the other inside their housing, thus establishing precise force vectors with a 360-degree control over the catheter tip and an accuracy within 1 mm. With this system, the company hopes to improve on cardiovascular care through the performance of more complex intravascular procedures. Since 2003, when the FDA approved the Niobe® System, multiple interventional cardiology and electrophysiology procedures can now be performed. These include the placement of a catheter against the wall of a beating heart in order to record its electrical activity and to identify heart tissue that is the source of the arrhythmia. Future applications currently being investigated by Stereotaxis Inc. include the ablation of atrial fibrillation, the repair of chronic total occlusion, the placement of percutaneous cardiac bypass grafts, the repair of mitral valves, and the drug delivery of angiogenic factors to diseased areas in the heart.

Rather than using the magnetic field of a magnet to move ferromagnetic substances to a target location, a patient's own blood flow can accomplish this task. An externally applied magnet which produces a strong local magnetic field can then be employed to stop these magnetic substances at or in the target organ (e.g., a tumor). The magnetic substances – preferentially in the form of nanospheres or microspheres – thus become concentrated in the target area. The spheres, which can be filled with either chemo- or radiotherapeutic drugs, then produce their effects either by releasing the drug or by blocking the vessels and capillaries (embolization) (Poznansky and Juliano, 1984).

In addition to the embolization effect, the application of selective radiofrequency heating (similar to a microwave) to the area containing the magnetic microspheres can increase the tumor cell killing even further. First results of this approach using ferrosilicone were reported in 1976 by Rand et al. The systemic toxicity of this method is very low (Barry et al., 1981); furthermore, it can be combined with chemoembolization, as carried out by Sako for the treatment of liver tumors (Sako et al., 1985).

Developments by Jordan and Chan led to the current ''magnetic fluid hyperthermia'' (MFH) application of single domain, dextran-coated magnetite nanoparticles in tumors (Chan et al., 1993; Jordan et al., 1993). Since 2003, Jordan has been conducting a clinical Phase II trial of a combined magnetic hyperthermia and radiation therapy (Jordan et al., 2001; Gneveckow et al., 2005) using the magnetic field applicator MFH 300F built by his company MagForce Applications GmbH in Berlin, Germany (Gneveckow et al., 2004). The magnetic field applicator (Fig. 1.9) runs at 100 kHz and produces a magnetic field strength of up to 18 kA m^{-1} in a cylindrical treatment area of 20 cm diameter. The first clinical results were presented in 2004 at the 5th International Conference on the Scientific and Clinical Applications in Lyon, France. Eight patients had been treated for cervix ($n = 2$), rectal, and prostate ($n = 2$) carcinoma, chondrosarcoma, rhabdomyosarcoma, and liver metastasis. The magnetic particles were injected locally directly into the tumors. The treatment, which increased the tumor temperature to 43–50 °C, took 60 min per session and was repeated from two to eleven times. No additional applications of magnetic particles were necessary after the initial injection. The magnetic fluid hyperthermia was very well tolerated, and none of the patients stopped the treatment. There was no pain and no burns, but some discomfort was felt due to excessive tumor heating (transpiration, heat sensation). Of the eight patients, six showed local control with no recurrent growth of the tumor, while the other two showed complete remission (at 9 and 14 months after treatment, respectively). These results are very promising, however, and this topic will be discussed further in Chapter 4, Section 4.6.

In the field of dentistry, magnets are most commonly applied to aid in the retention of oral and maxillo-facial prostheses. The first treatment in orthodontics was reported in Holland in 1953 by Dr. Crefcoeur (Duterloo, 1995), since when magnets have been used for the treatment of unerupted teeth and tooth movement, as well as for the expansion, fixed retention, and correction of an anterior open bite. It seems that a prolonged constant force exerted by implanted rare-earth magnets provides effective tooth movement (Daskalogiannakis et al., 1996).

Other retention applications include the use of small rare-earth magnets to keep eyelids closed during sleep in patients suffering from facial paralysis or, conversely, to keep lids open during waking hours in patients with drooping eyelids, such as those with muscular dystrophy.

Magnetic intrauterine devices (IUD) for use in contraception have recently been developed (Livesay, 1987). The nonmagnetic versions of such devices often have a

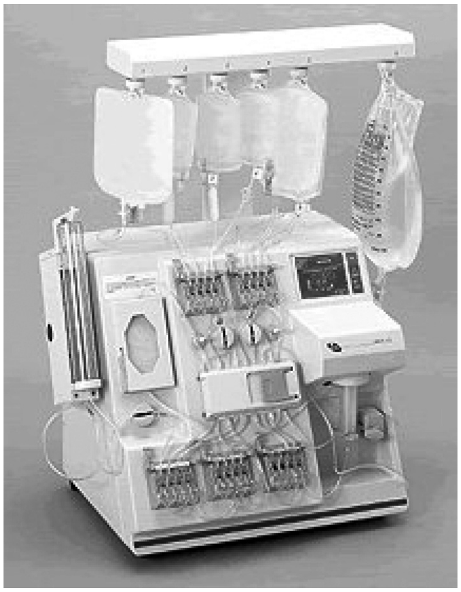

Fig. 1.10. The Isolex® 300i Magnetic Cell Selection System is the only FDA-approved product in the USA specifically for removing tumor cells in stem cell transplants.

string which extends from the uterus into the vagina; this is used by the gynecologist to remove the device. However, some studies have suggested that this string provides an entry path for bacteria and other organisms, and increases the chances of uterine infections. The addition of a small rare-earth magnet to the IUD allows for the string to be omitted. The IUD's correct position can be detected magnetically from the outside and removed using an extractor.

A recent *ex-vivo* application of magnetism in medicine is the purification of bone marrow from tumor cells with magnetic microspheres. In this procedure, the bone marrow is extracted from the patient prior to the use of conventional cancer therapy. Following high-dose treatment with radiotherapy and/or chemotherapy, the patient is rescued with an autologous bone marrow transplantation. In order to ensure that the patient's own bone marrow is free of cancer cells at the time of transplantation, a purification procedure is performed. This procedure, which was developed during the early 1980s and uses monoclonal anti-tumor antibodies conjugated to magnetic polystyrene microspheres, has now become standard (Treleaven et al., 1984; Treleaven, 1988). An initial purification system based on this technique, the Isolex 300i from Baxter (Fig. 1.10), was approved by the FDA and introduced into general therapy in 1999.

The medical use of magnets is not confined to treatment approaches, but also extends to the most powerful modern diagnostic methods such as positron emission tomography (PET) and magnetic resonance imaging (MRI). In PET, magnets are used in a cyclotron to produce short-lived radioisotopes such as ^{15}O. These radioisotopes, when injected into a patient and imaged with the PET system, allow determination of the biodistribution and biochemical functioning of different organs and tissues. In contrast, MRI utilizes the magnetic properties of the elements, and is used extensively for three-dimensional, noninvasive scans of the patient's body; indeed, it is currently the most important diagnostic method available. The history, principle and applications of MRI are covered extensively in Chapter 3.

1.1.6
The Influence of Magnetic Fields on Man

The human body is composed of atoms of different elements surrounded by water molecules. These atoms react to magnetic and electric forces and fields, and this may lead to, for example, a net-nuclear magnetization of a person when placed in a clinical MRI machine. It is therefore easy to imagine that magnetic and electromagnetic forces could alter physiologic functions, induce effects, or influence the organism in either a positive or negative way. Although the extent and importance of these phenomena has been under investigation for the past 100 years, the effects observed have generally been minimal and seldom statistically significant. A report of the American National Research Council which examined more than 500 studies spanning 17 years of research concluded, in 1996, that ''No conclusive evidence shows that exposures to residential electric and magnetic fields produce cancer, adverse neurobehavioral effects, or reproductive and developmental effects'' (National Research Council, 1997). A more succinct overview, but with the same conclusions, was provided by Tenforde (2003).

When investigating magnetic effects on humans, two different magnetic field ''types'' are generally distinguished: (1) a static magnetic field, which exists around a large magnet; and (2) a magnetic field that is pulsed at frequencies higher than 10 Hz, often abbreviated as EMF (electromagnetic fields). The study of these effects is termed ''biomagnetism'', some sub-fields of which are highly controversial, while others have already been established in medical applications (see Chapter 2).

Most scientists agree that static magnetic fields of up to 10 Tesla have no obvious effects on long-term plant growth, mouse development, body temperature, or brain activity (Barnothy et al., 1956; Barnothy and Barnothy, 1958; Maret et al., 1986). Such conclusions echo findings made more than a century ago, at which time, Mr. Kennelly – the chief electrician at the Edison Laboratory – wrote, after exposing a volunteer to 27 000 times the magnetic field of the Earth, that, ''... the human organism is in no wise appreciably affected by the most powerful magnets known to modern science; neither direct nor reversed magnetism exerts any perceptible influence upon the iron contained in the blood, upon the circulation, upon ciliary or protoplasmic movements, upon sensory or motor nerves, or upon the brain.'' (Peterson and Kenelly, 1892) (Fig. 1.11).

The lack of any apparent effects of strong magnetic fields on humans placed near powerful magnets does not imply that there are no effects at all. It would

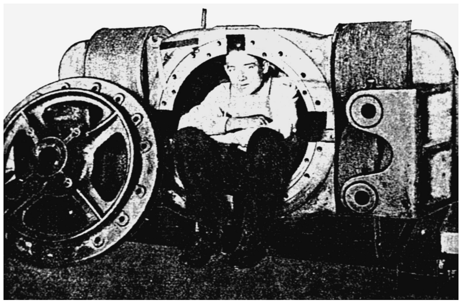

Fig. 1.11. Field magnet used in the studies of magnetic effects on dogs at the Edison laboratory (humans were not mentioned in the original legend!). The powerful attraction of bolts and chains is noticeable. The circular door at the side was made from brass.

also be foolish to conclude that humans have no magnetosensitive organs. During the past years, evidence has been mounting that not only do pigeons (Keeton, 1971), bees (Kirschvink et al., 1992a) and fin whales (Walker et al., 1992) possess magnetic receptors, but humans might also (Kirschvink et al., 1992b). Chains of magnetite particles similar to those known from magnetic bacteria and algae have been found – chains which supposedly are either a part of, or form the magnetosensitive organ itself. Several research investigations have been conducted in an attempt to show that humans have a "magnetic sense". One study reported an experiment in which students were driven around blind-folded and then asked to point in the direction of their dormitories. Those students who used only their natural "magnetic sense" had a higher success rate than those whose "magnetic sense" had been deceived by the field of a magnet attached to their heads (Baker, 1989). Clearly, further research is needed in this area as the results are often contradictory and suggest several interpretations.

Research indicates that humans are sensitive to small changes in magnetic field gradients, but not to the overall magnetic field (Rocard, 1964). Evidence supporting this has come from studies of the dowser reflex. A dowser, a person holding firmly onto a divining rod (see Fig. 1.12) will, under certain physical conditions, experience a force which results in an involuntary upward or downward movement of their rod. To skeptics the movement appears illusory, to believers it appears magical, but the effect has been consistently reported over the past 70 years by a number of authors. In the most-often performed experiment, a group of dowsers was made to walk along the same stretch of street. At points within 1 or 2 m of each other, they all had their divining rods pulled down to the earth.

Magnetic field measurements have shown that the dowser reflex occurs when the dowser passes through a region where the Earth's magnetic field is not entirely uniform. This field anomaly produces a magnetic field gradient, which must exceed 0.1 mOe m^{-1} (8 mA m^{-2}) to be detected. The speed with which the dowser

Fig. 1.12. Dowser holding a divining rod while searching for underground water. (Illustration from Abbé de Vallemont's *Treatise on the divining rod*, Paris, 1693).

passes through this field gradient also influences their magnetic reception. The dowser must pass through a 0.1 mOe m^{-1} field gradient within at least 1 s in order to detect it. Furthermore, the detection level can be increased by adding up the small differences in field gradients. Higher magnetic field gradients, however, lead to saturation and can only be detected by moving faster. Of additional interest is Rocard's notion that although most people are sensitive, a good dowser has a more accurate and rapid reflex than the bad dowser.

Physiological explanations of the dowser reflex have included the physiological induction of magnetic moments, electromagnetic currents, and nuclear magnetic resonance. None of these possibilities has, however, been able to account convincingly for the phenomenon, and thus the search for an explanation continues.

Electromagnetic machines produce fields and field gradients which are constantly changing and which have been found to influence humans. The earliest experiments to test the effects of these fields using humans were performed at the end of the 19th century. D'Arsonval's experiments were among the most spectacular (Rowbottom and Susskind, 1984). In one of these experiments, a person was completely enclosed in a large solenoid resembling a cage, and insulated from all contact with it (Fig. 1.13). Owing to the high-frequency oscillating magnetic field within the solenoid, strong currents were induced within the subject's body, and

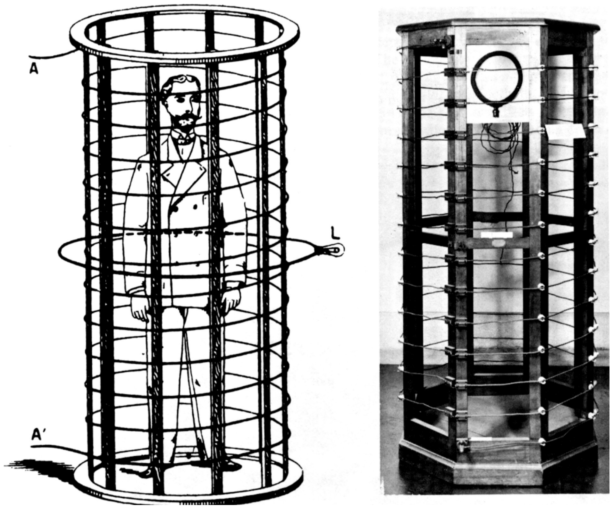

Fig. 1.13. D'Arsonval's great solenoid or cage for auto-conduction in which the person is insulated from all contact with current-carrying wire. The photograph shows the cage actually used by D'Arsonval in 1893 for his experiments.

although neither pain nor any other sensation was felt, a lamp held in the person's hands became incandescent during the procedure. D'Arsonval called this method of applying high-frequency currents to man "autoconduction".

As the 20th century began, the serious investigation of the physiologic consequences of electromagnetic fields became tainted by association with quack science and the pseudo-technology of electromedicine. Dr. Albert Adams (1863–1924), one of the controversial therapists applying electromedicine, was named "Dean of 20th century charlatans" by the American Medical Association. Adams postulated that each organ system and each patient were tuned to a characteristic electromagnetic wavelength. It should therefore have been possible to diagnose medical conditions and to deliver therapy to individuals hundred of miles away simply by using a properly tuned, radio-based device. This therapy was called "physiologic frequency manipulation", and it aroused public interest in bioelectricity and electromagnetic physiologic effects. The science community gradually lost interest in bioelectricity, but before its fall from grace, the groundwork was laid for such major clinical applications as electroconvulsive therapy, cardioversion, and transcutaneous nerve stimulation, all of which are discussed in greater detail in Chapter 4.

Between 1930 and 1960, the physiological and biological effects of electromagnetic fields were studied only minimally. Research accomplished by the small group of investigators who continued working in this area was reviewed comprehensively by Barnothy during the late 1960s (Barnothy, 1964, 1969). Although the design of many of those studies performed up to this time was flawed, some of their results have been confirmed by more stringent research. For example, results recently endorsed in a report by the National Research Council (1997) support previous findings that electromagnetic fields induce changes in the brain's electroencephalographic (EEG) activity (Bell et al., 1991), produce measurable changes in polypeptide synthesis in salivary glands (Goodman and Henderson, 1988), and are able to influence the levels of calcium and melatonin in cells exposed to high-level fields (Graham et al., 1996). Additionally, recent double-blind studies have confirmed the effects of low-frequency pulsed electromagnetic fields greater than 0.5 mT on growth induction in bone. Indeed, their use is now the treatment of choice for certain recalcitrant problems of the musculoskeletal system, including salvage of surgically resistant nonunions in children and adults and chronic refractory tendinitis (Bassett, 1989).

Available data indicate that humans are susceptible to alternating electromagnetic fields. Epidemiological studies even suggest health effects attributable to relatively small magnetic fields such as those found underneath a high-voltage line (Jauchem, 1995). The report of the National Research Council, for example, acknowledged a 1.5-fold higher incidence of childhood leukemia in homes situated close to high-voltage power lines, though the examined studies failed to show a statistically significant association between exposures and disease (National Research Council, 1997). Unless new theories for these effects are proposed on the grounds of molecular mechanisms, it will be very difficult to either prove or disprove any association between disease and the small magnetic fields produced near electric devices, machines and power lines. Even the electromagnetic fields in heavily industrialized regions amount only to a few tenths of one mTesla, which is less than 1% of the ambient terrestrial magnetic field. Most experts would not anticipate any serious effects related to these additional magnetic fields.

Current laboratory investigations employ more sophisticated techniques, more sensitive instruments and more refined statistical methods than ever before. When combined with our deeper understanding of magnetic resonance patterns in tissues (see Chapter 3), this vastly improved instrumentation should provide a strong base from which to improve our understanding of the electromagnetic field effects at the cellular and molecular levels. In time, this will likely lead to the introduction of new, magnetism-based medical techniques for diagnosis and therapy.

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1.2
Basic Physical Principles

Dmitri Berkov

1.2.1
Introduction

"Everything should be done as simple as possible, but not simpler"
(A. Einstein)

"Yes, I knew once," said Rabbit, "but I forgot."
(A.A. Milne, *Winnie the Pooh*)

In this chapter, an attempt will be made to provide an introduction to magnetic phenomena on a reasonable level. But what level do we consider to be reasonable? Clearly, we cannot expect a specialist in medical sciences to have a deep knowledge of the quantum mechanics required for a real understanding of most magnetic phenomena. Yet, at the same time, we cannot hope to explain something really interesting in terms of school-level mathematics and physics (note the quotation by Einstein, above). So, it was decided to introduce most magnetic phenomena using classical electrodynamics starting from Maxwell Equations, and to refer to quantum mechanics only if absolutely necessary – and even in such cases, the text is restricted to a qualitative description of a problem.

On the other hand, this chapter is not intended to serve as a detailed introduction to the classical field theory in general, and magnetism in particular. In fact, this chapter is best suited for those, "who knew it once, but then forgot". For this reason, we expect from the reader: (1) basic skills in the classical theory of fields (mainly definitions and basic properties of gradient, divergence and rotor operators); and (2) physical knowledge on the basic-school-textbook level (force and torque, its relation to the potential energy, concepts of electric and magnetic fields, electric charges and current). We have always tried to derive desired results from the first principles (as far as it is possible in terms of classical physics), but the reader interested only in the final results can simply skip the intermediate transformations. For those, who, in contrast, are interested in more detailed consideration, we cite the corresponding sources at the end of this section.

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1.2.2
The Electromagnetic Field Concept and Maxwell Equations

We start with the Maxwell Equations, which form the basis of classical electrodynamics. These equations were derived more than a century ago using experimental facts and theoretical ideas known at that time. Their importance for the subsequent development of physics in general – and studies of electromagnetic phenomena in particular – cannot be overestimated. The prediction of electromagnetic waves based on these equations is only one impressive example. Below, we shall see that virtually all (classical) electromagnetic phenomena can be derived from the four Maxwell Equations. Readers are referred to Feynmann et al. (1963) for a more detailed, yet simple-to-understand consideration of these equations. For a more ‘‘theoretical’’ introduction, the reader is referred to Landau and Lifshitz (1975b).

1.2.2.1
Maxwell Equations in a General Case of Time-Dependent Fields

Let us denote the electric and magnetic fields in a vacuum as $\mathbf{e}$ and $\mathbf{h}$, respectively (small letters are used to distinguish these fields from the corresponding macroscopically averaged fields in condensed matter in Section 1.2.3). We will also use the electric charge and current densities, ρ and $\mathbf{j}$, so that the total charge in the ‘‘physically infinitely small’’ volume dV is $\rho\, dV$ and the total current through the small surface dS is $\mathbf{jn}\, dS$, where $\mathbf{n}$ is a unit vector normal to this surface. In this notation, the Maxwell Equations in the so-called differential form are (c is the speed of light)

$$\operatorname{rot} \mathbf{e} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} \tag{1.1}$$

$$\operatorname{rot} \mathbf{h} = \frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} + \frac{4\pi}{c}\mathbf{j} \tag{1.2}$$

$$\operatorname{div} \mathbf{e} = 4\pi\rho \tag{1.3}$$

$$\operatorname{div} \mathbf{h} = 0 \tag{1.4}$$

The ‘‘differential form’’ means that Eqs. (1.1) to (1.4) provide the relationship between time-dependent magnetic and electric fields and their time derivatives at one and the same spatial point. Before proceeding with the explanation of the physical sense of Eqs. (1.1–1.4) (and to make this explanation more transparent), the Maxwell Equations should be derived in the integral form.

Considering an open surface S bounded by a contour L and, integrating Eqs. (1.1) and (1.2) over this surface, we obtain

$$\int_s \text{rot } \mathbf{e} \, d\mathbf{S} = -\frac{1}{c}\int_s \frac{\partial \mathbf{h}}{\partial t} d\mathbf{S} \tag{1.5}$$

$$\int_s \text{rot } \mathbf{h} \, d\mathbf{S} = \frac{1}{c}\int_s \frac{\partial \mathbf{e}}{\partial t} d\mathbf{S} + 4\pi \int_s \mathbf{j} \, d\mathbf{S} \tag{1.6}$$

According to the Stokes theorem, integrals of the field rotors over the surface S on the left-hand sides can be transformed into the integrals of the fields itself along the surface-bounding contour L. On the right, we can interchange time derivatives and integrations, thus obtaining time derivatives of the total field fluxes Φ through the surface S:

$$\int_s \frac{\partial \mathbf{h}}{\partial t} d\mathbf{S} = \frac{\partial}{\partial t}\int_s \mathbf{h} \, d\mathbf{S} = \frac{\partial \mathbf{\Phi}_h}{\partial t} \tag{1.7}$$

and the same for the corresponding term in Eq. (1.6).

Finally, in Eq. (1.6) the integral of the current density over the surface S is clearly the total current through this surface J_S. Summarizing all that, we obtain the first two Maxwell Equations in the integral form

$$\oint_L \mathbf{e} \, d\mathbf{l} = -\frac{1}{c}\frac{\partial \Phi_h}{\partial t} \tag{1.8}$$

$$\oint_L \mathbf{h} \, d\mathbf{l} = \frac{1}{c}\frac{\partial \Phi_e}{\partial t} + \frac{4\pi}{c} J_S \tag{1.9}$$

To obtain the last two required equations, Eqs. (1.3) and (1.4) are integrated over a volume V surrounded by a (closed) surface S, to obtain

$$\int_V \text{div } \mathbf{e} \, d\mathbf{V} = 4\pi \int_V \rho \, d\mathbf{V} \tag{1.10}$$

$$\int_V \text{div } \mathbf{h} \, d\mathbf{V} = 0 \tag{1.11}$$

The integral of the charge density over the volume V on the right-hand side of Eq. (1.10) is the total electrical charge Q inside this volume. Volume integrals of the field divergences on the left-hand sides can be transformed using the Gauss theorem into the integrals of the fields itself over the surrounding surface S, which provides the integral form of the other two Maxwell Equations

$$\oint_S \mathbf{e} \, d\mathbf{S} = 4\pi Q \tag{1.12}$$

$$\oint_S \mathbf{h} \, d\mathbf{S} = 0 \tag{1.13}$$

Now, we consider the physical sense of these equations. We shall immediately see, that these equations themselves (not to mention their consequences) already contain many fundamental properties of electric and magnetic fields.

The first Maxwell Equation in its differential form (Eq. 1.1) states, first of all, that the electric field can be induced by a changing magnetic field (such that $\partial\mathbf{h}/\partial t \neq 0$). Another property of the electromagnetic field which can be seen from Eq. (1.1) is that in the absence of an electric field (so that rot $\mathbf{e} = 0$) the magnetic field can be only stationary: $\partial\mathbf{h}/\partial t = 0$, or $\mathbf{h} = Const.$

Considering this equation in its integral form (Eq. 1.8), we note that the integral of the electric field over the closed contour is, by definition, the electromotive force along this contour. So this equation is nothing else but the generalized Faraday law: the electromotive force along a closed contour is proportional to the time derivative of the magnetic field flux through this contour.

According to the second Maxwell Equation (1.2), a magnetic field can be created either by a time-dependent electric field [fully analogous to Eq. (1.1)] or by an electric current, the density of which is also present on the right-hand side of the equation. Moreover, this equation implies that any current creates a magnetic field, because if $\mathbf{j} \neq 0$ then rot $\mathbf{h} \neq 0$ which is possible only when the magnetic field itself $\mathbf{h} \neq 0$. We also point out the importance of the opposite signs before the field time derivatives in Eqs. (1.1) and (1.2): the consequence is the existence of electromagnetic waves (Landau and Lifshitz, 1975b).

The integral Eq. (1.9) connects the circulation of the magnetic field over some closed contour with the time derivative of the electric field flux and the total current through the surface bounded by this contour.

The second two Maxwell Equations are also of primary importance. Equation (1.3) is the mathematical expression of the fundamental physical fact that electric charges are sources of the electric field. According to its integral form (Eq. 1.12), the total flux of the electric field through some closed surface is proportional to the total charge inside this surface. The immediate consequence of this equation is the Coulomb law (see next paragraph).

The last equation (Eq. 1.4) can be understood if we compare it with the corresponding equation for the electric field (Eq. 1.3): zero on the right-hand side of Eq. (1.4) means that there are no sources of the magnetic field, there exist no magnetic charges. Consequently the flux of the magnetic field through any closed surface is exactly zero (see Eq. 1.13).

1.2.2.2
Constant (Time-Independent) Fields: Electro- and Magnetostatics

We will very often encounter a situation where nothing (at least on the macroscopic size and time scales) in the system under consideration changes with time. In this case, the electric and magnetic fields produced by such a system are also constant, corresponding time derivatives in the Maxwell Equations vanish, and we arrive at two pairs of decoupled equations which describe electrostatic

$$\text{rot } \mathbf{e} = 0 \tag{1.14}$$

$$\text{div } \mathbf{e} = 4\pi\rho \tag{1.15}$$

and the corresponding magnetostatic

$$\text{rot } \mathbf{h} = \frac{4\pi}{c}\mathbf{j} \tag{1.16}$$

$$\text{div } \mathbf{h} = 0 \tag{1.17}$$

phenomena. Indeed, it can be seen immediately from these four equations that in a stationary case there is no connection between magnetic and electric field – a circumstance that has allowed studying magnetism and electricity separately over centuries.

Equation (1.15) immediately leads to the basic law of the electrostatics – the Coulomb law. To see this, let us consider the integral form of this equation (Eq. 1.12). If we place a point charge Q in a center of a spherical surface (with the radius R), then according to Eq. (1.12) the total flux of the electric field through this surface is $\Phi_E = 4\pi Q$. On the other hand, for the charge in a center of a sphere the field $\mathbf{e}$ at each point of the spherical surface is directed perpendicular to this surface (from the symmetry reasons), so that the total flux is given simply by the product of the field magnitude and the surface area: $\Phi_E = 4\pi R^2|\mathbf{e}|$. Comparing it with the previous expression $\Phi_E = 4\pi Q$, we immediately obtain the desired result $|\mathbf{e}| = Q/R^2$.

In a similar fashion, the first equation of magnetostatics (Eq. 1.16) leads to the Biot–Savart law. Namely, in the integral form, Eq. (1.16) reads

$$\oint_L \mathbf{h}\, d\mathbf{l} = \frac{4\pi}{c} J_S \tag{1.18}$$

(compare with Eq. 1.9), where J_S is the total current through the surface S bounded by a contour L. Let us consider a long straight wire carrying a full current J and a circular contour (radius R) around this wire so that the contour plane is perpendicular to the wire and the wire passes through the center of the circle. In this case, according to Eq. (1.18), the circulation of the magnetic field around this contour is $C_H = 4\pi J/c$. And again, from symmetry considerations the field is directed along the circle at each point of it, so that the circulation is the product of the field magnitude and the contour length (i.e., $C_H = 2\pi R h$). Comparing these two expressions for the field circulation, we obtain the magnetic field of a straight current $h = (2/c)J/R$ – the Biot–Savart law in its simplest form.

1.2.2.3
Electric and Magnetic Potentials: Concept of a Dipole

Maxwell Equations for electro- and magnetostatics have another very important property – they allow the introduction of the so-called scalar (electric) and vector (magnetic) potentials, which greatly simplifies the solution of many practical problems. In this paragraph, we introduce these important concepts following mainly the route suggested by Landau and Lifshitz (1975b).

In order to introduce a scalar potential for the electric field, we need Eq. (1.14): rot $\mathbf{e} = 0$. According to this equation, we can always find a function ϕ from which the electric field can be evaluated as $\mathbf{E} = -\text{grad}\,\phi$ (a minus sign is chosen for convenience reasons) because for any "good" function $\text{rot}\,\text{grad}\,\phi \equiv 0$. This function ϕ is called the "scalar electric potential", and is very convenient to use because it contains all information about the electrostatic field **e**. Moreover, being a scalar function, it is much easier to handle than a general vector field. The potential introduced above is not uniquely defined: $\phi' = \phi + C$, where C is an arbitrary constant, gives the same electric field as ϕ, because $\text{grad}\,C \equiv 0$. This additional constant does not really matter, because it is the field itself which has a real physical meaning (because it determines a force acting on the charges); we only need one additional condition to determine this arbitrary constant C. Usually (for finite systems), the constant is chosen so that ϕ tends to zero at the infinity.

The equation for the evaluation of ϕ also follows from the Maxwell Equations. Substituting the relation $\mathbf{e} = -\text{grad}\,\phi$ into the second Maxwell Equation (1.15), we immediately obtain

$$\Delta\phi = -4\pi\rho \tag{1.19}$$

where we have introduced the Laplace operator $\Delta(*) = \text{div}\,\text{grad}(*)$, ($(*)$ represents any function one wants the operator to operate on). This so-called Poisson Equation for the function ϕ allows us to evaluate the electric potential, when the charge distribution in our system and boundary conditions are given. Its solution for the finite system and zero boundary conditions at infinity (i.e., $\phi \to 0$ when we go away from our system) is known: the potential at the point r_0 is

$$\phi(\mathbf{r}_0) = \int_V \frac{\rho(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|}\,dV \tag{1.20}$$

where the integral is taken over the whole system volume. For a system of discrete charges, Eq. (1.20) transforms into

$$\phi(\mathbf{r}_0) = \sum_i \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|} \tag{1.21}$$

which is the obvious generalization of the Coulomb potential for a single charge.

Equation (1.21) allows us to introduce in a natural way a very important concept of electrostatics – the concept of a dipole moment. Let us choose the origin of our coordinate system inside the system of charges under study and evaluate the scalar potential far away from this system so that for any charge $r_0 \gg r_i$. In this case, we can use the first-order Taylor expansion for a function of many variables $f(\mathbf{r}_0 - \mathbf{r}_i) \approx f(\mathbf{r}_0) - \mathbf{r}_i\,\text{grad}\,f(\mathbf{r}_0)$ to approximate $1/|r_0 - r_i|$: here $f(r_0) = 1/r_0$ and its gradient $\text{grad}(1/r_0) = -\mathbf{r}_0/r_0^3$. So for the potential at a large distance we obtain

$$\phi(\mathbf{r}_0) \approx \frac{1}{r_0}\sum_i q_i + \left(\sum_i q_i \mathbf{r}_i\right)\frac{\mathbf{r}_0}{r_0^3} = \frac{Q}{r_0} + \frac{\mathbf{d}\mathbf{r}_0}{r_0^3} = \phi_p + \phi_{\text{dip}} \tag{1.22}$$

where the total system charge $Q = \sum_i q_i$ and its *dipole moment*

$$\mathbf{d} = \sum_i q_i \mathbf{r}_i \tag{1.23}$$

have been introduced. The first term (ϕ_p) in Eq. (1.22) is simply the Coulomb potential of the point charge Q. It decreases with the distance as r_0^{-1}, and thus dominates the potential for charged systems. This means that if the total system charge is not zero, then the only system parameter which is important to evaluate the field far enough from the system is the magnitude of this charge (and not its distribution inside the system).

However, for a vast majority of physical systems the total charge is exactly zero – otherwise huge Coulomb forces would make our world extremely unstable. For this reason, the first term in Eq. (1.22) is mostly zero and the electric potential of the system at the large distances is dominated by the second term ϕ_{dip}. The only system characteristic which this term depends on is its dipole moment defined by Eq. (1.23). The reason why this moment of the charge distribution is called a "dipole" moment is as follows: the simplest system with zero net charge which possesses this moment consists of two charges with the same value and opposite sign – "two poles". Such a system itself is also called an electric dipole.

The electric field of a dipole at the distances much larger than its size is (see Fig. 1.14)

$$\mathbf{e}_{\text{dip}} = -\nabla\phi_{\text{dip}} = \frac{3\mathbf{r}_0(\mathbf{d}\mathbf{r}_0)}{r_0^5} - \frac{\mathbf{d}}{r_0^3} = \frac{3\hat{\mathbf{r}}_r(\mathbf{d}\hat{\mathbf{r}}_r) - \mathbf{d}}{r_0^3} \tag{1.24}$$

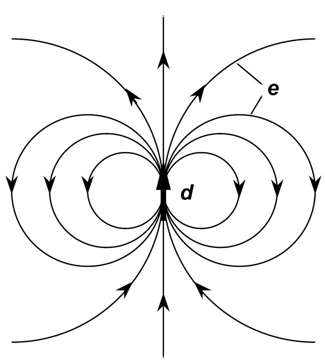

Fig. 1.14. The electric field of a dipole.

Here, ∇ denotes as usual the gradient operator and $\hat{\mathbf{r}}_r = \mathbf{r}_0/r_0$ is the unit vector in the direction of r_0; sometimes one refers to the expression (1.24) as to the field of a "point dipole".

In order to introduce analogous concepts for the magnetic field we need much more effort, and the experience gained by considering the electric field will be very useful (this is the main reason why we spent so much time describing the electric potential and dipole). The main reason why we cannot simply repeat the procedure used above for the magnetic field is evident from Maxwell Equation (1.16): in general, rot $\mathbf{h} \neq 0$, and hence the introduction of a scalar magnetic potential is impossible. However, we can use another Maxwell Equation div $\mathbf{h} = 0$ (1.17), which implies that it is always possible to define such a vector function A that the magnetic field can be evaluated as $\mathbf{h} = \operatorname{rot} \mathbf{A}$, because for any vector field $\operatorname{div} \operatorname{rot}(*) \equiv 0$. This new vector function is called the magnetic vector potential.

To determine which equation should be used to evaluate A (we are seeking a magnetic version of the Poisson Equation (1.19) for the electric potential), let us substitute the definition $\mathbf{h} = \operatorname{rot} \mathbf{A}$ into Maxwell Equation (1.16):

$$\operatorname{rot} \mathbf{h} = \operatorname{rot} \operatorname{rot} \mathbf{A} = \operatorname{grad} \operatorname{div} \mathbf{A} - \Delta \mathbf{A} = \frac{4\pi}{c} \mathbf{j} \tag{1.25}$$

where Δ denotes again the Laplace operator. To simplify this expression we note that (similar to the electric potential) the magnetic potential is not uniquely defined: $\mathbf{A} \to \mathbf{A} + \nabla f$, where f is an arbitrary scalar function, and gives the same magnetic field $\mathbf{h} = \operatorname{rot} \mathbf{A}$ (which is of real physical interest) since $\operatorname{rot} \operatorname{grad}(*) \equiv 0$. This use of "degree of freedom" in choosing A enables us to impose one additional restriction on it. It is very convenient to postulate $\operatorname{div} \mathbf{A} = 0$, so that the first term in the equation for A (Eq. 1.25) vanishes and we finally obtain

$$\Delta \mathbf{A} = -\frac{4\pi}{c} \mathbf{j} \tag{1.26}$$

This equation resembles the corresponding Eq. (1.19) for ϕ and hence its solution for the vector potential vanishing at the infinity is analogous to Eq. (1.20):

$$\mathbf{A}(\mathbf{r}_0) = \frac{1}{c} \int_V \frac{\mathbf{j}(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|} dV \tag{1.27}$$

Equation (1.27) enables us to evaluate the magnetic potential (and hence, the magnetic field) of any system with the known current distribution j.

To continue our consideration we need the following mathematical statement: the average time derivative $\overline{df(t)/dt}$ of any bounded function $f(t)$ is zero. The proof is very simple:

$$\overline{\frac{df(t)}{dt}} = \frac{1}{T_{\text{av}}} \int_0^{T_{\text{av}}} \frac{df(t)}{t} dt = \frac{f(T_{\text{av}}) - f(0)}{T_{\text{av}}} \tag{1.28}$$

which tends to zero when the averaging time T_{av} increases, because for bounded $f(t)$ the difference $f(T_{\mathrm{av}}) - f(0)$ remains finite.

Now, proceeding in the same fashion as for the electric potential we can derive the magnetic potential at large distances from the finite system of currents. Let us recall the fact that if a particle with the charge q is moving with the velocity $\mathbf{v}$, then the corresponding current density is $\mathbf{j} = q\mathbf{v}$. We shall also use the notation $\bar{f}$ for the quantity f averaged over a large period of time, T_{av}. It is important to note here that T_{av} should be large compared with typical times characterizing the movement of charges in our system. For all practically interesting cases, these typical times correspond to the microscopic charge movements on the atomic scale and hence are extremely small, so that for any reasonable measurement time $T_{\mathrm{av}}\overline{df/dt} = 0$ if f remains finite (see above).

For further consideration it is more convenient to use the discrete version of Eq. (1.27), which describes the magnetic potential created by a system of moving particles with charges q_i and velocities v_i:

$$\overline{\mathbf{A}(\mathbf{r}_0)} = \frac{1}{c}\sum_i \frac{\mathbf{j}_i}{|\mathbf{r}_0 - \mathbf{r}_i|} = \frac{1}{c}\sum_i \frac{q_i v_i}{|\mathbf{r}_0 - \mathbf{r}_i|} \tag{1.29}$$

For large distances $r_0 \gg r_i$, proceeding exactly as for the electric potential, and using the first-order Taylor expansion for $\bar{\mathbf{A}}$, we obtain

$$\overline{\mathbf{A}(\mathbf{r}_0)} = \frac{1}{cr_0}\sum_i \overline{q_i\mathbf{v}_i} + \frac{1}{cr_0^3}\sum_i \overline{q_i\mathbf{v}_i(\mathbf{r}_0\mathbf{r}_i)} = \frac{1}{cr_0}\sum_i \overline{\frac{dq_i\mathbf{r}_i}{dt}} + \frac{1}{cr_0^3}\sum_i \overline{q_i\mathbf{v}_i(\mathbf{r}_0\mathbf{r}_i)} \tag{1.30}$$

The first term, being the average of the time derivative of a bounded function (we study finite systems, so all r_i are finite), vanishes. In order to rewrite the second term in the desired form we use the following trick: we introduce a quantity

$$\frac{1}{2}\frac{d}{dt}\overline{\mathbf{r}_i(\mathbf{r}_i\mathbf{r}_0)} = \frac{1}{2}\left(\overline{\mathbf{v}_i(\mathbf{r}_i\mathbf{r}_0)} + \overline{\mathbf{r}_i(\mathbf{v}_i\mathbf{r}_0)}\right) = 0$$

which is zero because it is again a full time-derivative of a bounded quantity. Subtracting this zero from each term of the second sum in Eq. (1.30), it can be verified, using elementary vector algebra, that $\bar{\mathbf{A}}$ can be rewritten as

$$\overline{\mathbf{A}(\mathbf{r}_0)} = \frac{1}{2cr_0^3}\sum_i \overline{q_i[\mathbf{r}_0 \times [\mathbf{r}_i \times \mathbf{v}_i]]} = \frac{[\boldsymbol{\mu} \times \mathbf{r}_0]}{r_0^3} = \overline{\mathbf{A}_{\mathrm{dip}}} \tag{1.31}$$

where we have introduced the magnetic dipole moment of a system of moving particles (or a system of currents) as

$$\boldsymbol{\mu} = \frac{1}{2c}\sum_{\mathrm{i}} \overline{q_i[\mathbf{r}_i \times \mathbf{v}_i]} = \frac{1}{2c}\sum_{\mathrm{i}} \overline{[\mathbf{r}_i \times \mathbf{j}_i]} \tag{1.32}$$

For a system with a continuous current distribution the definition is

$$\boldsymbol{\mu} = \frac{1}{2c}\int_V \overline{[\mathbf{r}\times\mathbf{j}(\mathbf{r})]}\,dV \tag{1.33}$$

The similarity between the expressions for the electric (see Eq. 1.22) and magnetic potentials (Eq. 1.31) created by corresponding dipoles is obvious. However, it comes even better: the expression for the magnetic field created by a magnetic dipole

$$\mathbf{h}_{\text{dip}} = \frac{3\mathbf{r}_0(\boldsymbol{\mu}\mathbf{r}_0)}{r_0^5} - \frac{\mathbf{d}}{r_0^3} \tag{1.34}$$

is exactly the same as that for the corresponding electric field shown in Figure 1.14 (with the replacement of $\mathbf{d}$ by $\boldsymbol{\mu}$). This similarity explains the name magnetic dipole moment, because otherwise it could not be justified: there exist no magnetic charges, and hence no magnetic poles in any systems.

We point out that the concept of a magnetic dipole plays in magnetism an even more important role than its counterpart – electric dipole – in electricity, because the absence of magnetic charges makes magnetic dipoles, roughly speaking, the most "elementary" object at least in magnetostatics.

The validity of the dipolar approximation (Eq. 1.31) for the exact expression (Eq. 1.27) is based only on the assumption $r_0 \gg r_s$, where $\mathbf{r}_0$ is the distance from the measurement point to the system and $\mathbf{r}_S$ is a characteristic system size. In practice, it is usually sufficient to have $r_0 \geq 10r_s$, because the next term in the Taylor expansion of Eq. (1.27) decreases with the distance as r_0^{-3} (in contrast to r_0^{-2} for the dipole potential).

While concluding this discussion, we would like to mention for those who wish to know more about the subject, that the simplicity in performing some electro- and magnetostatic calculations is by far not the most important reason to introduce scalar and vector potentials ϕ and A. The actual reason is much deeper – these potentials are primary physical concepts and natural variables necessary for the construction of the classical field theory starting from the relativistic invariant action of a charged particle moving in an electromagnetic field (Landau and Lifshitz, 1975b).

1.2.2.4
Force, Torque and Energy in Magnetic Field

To evaluate a total average force acting in a magnetic field on a finite system of moving charges, we can use directly the expression for the Lorentz force $\mathbf{F} = (1/c)q[\mathbf{v}\times\mathbf{h}]$ acting in the magnetic field $\mathbf{h}$ on the charge q moving with the velocity $\mathbf{v}$. For a system of charges we have correspondingly

$$\bar{\mathbf{F}} = \frac{1}{c}\sum_i \overline{q_i[\mathbf{v}_i\times\mathbf{h}_i]}$$

where $\mathbf{h}_i$ is the field at the location point of the charge q_i and the averaging is performed in the same sense as in the derivation of the magnetic dipole potential above. For the homogeneous field (so that $\mathbf{h}_i = \mathbf{h}$ is the same for all particles) this expression can be "greatly simplified": taking such a field out of the sum we can rewrite the force above as

$$\bar{\mathbf{F}} = \frac{1}{c}\left[\left(\sum_i \overline{\frac{d}{dt} q_i \mathbf{r}_i}\right) \times \mathbf{h}\right] = 0$$

It is zero because every term in the sum over charges is again the average time-derivative of a finite quantity (all $\mathbf{r}_i$ are finite). This is a very important result which we would like to formulate explicitly: the total average force acting on any finite body *in a homogeneous magnetic field is zero*.

Let us consider a torque in a homogeneous field. By definition, a torque is a vector product of the particle radius-vector and the force acting on the particle $\mathbf{G} = [\mathbf{r} \times \mathbf{F}]$. Using the expression for the Lorentz force, we have for a system of charges

$$\bar{\mathbf{G}} = \sum_i \overline{[\mathbf{r}_i \times \mathbf{F}_i]} = \frac{1}{c}\sum_i q_i \overline{[\mathbf{r}_i \times [\mathbf{v}_i \times \mathbf{h}]]} = \frac{1}{c}\sum_i q_i \overline{(\mathbf{v}_i(\mathbf{r}_i\mathbf{h}) - \mathbf{h}(\mathbf{r}_i\mathbf{v}_i))} \tag{1.35}$$

The second term in the sum over particles vanishes, because $\overline{\mathbf{r}_i\mathbf{v}_i} = (1/2)\overline{d/dt(r_i^2)}$ $= 0$ for the same reason as usual. To deal with the first term we apply the same trick as in the transition from Eq. (1.30) to Eq. (1.31): we subtract from this term a zero quantity written in the form

$$0 = \frac{1}{2}\overline{\frac{d}{dt}}\mathbf{r}_i(\mathbf{r}_i\mathbf{h}) = \frac{1}{2}\overline{\mathbf{v}_i(\mathbf{r}_i\mathbf{h})} + \frac{1}{2}\overline{\mathbf{r}_i(\mathbf{v}_i\mathbf{h})}$$

After some simple vector algebra we get the desired result

$$\bar{\mathbf{G}} = -\frac{1}{2c}\sum_i q_i \overline{[\mathbf{h} \times [\mathbf{r}_i \times \mathbf{v}_i]]} = [\boldsymbol{\mu} \times \mathbf{h}] \tag{1.36}$$

where $\boldsymbol{\mu}$ is the magnetic dipole moment already familiar to us. Hence, in contrast to the force, the torque acting on a body in a homogeneous magnetic field is not zero.

The next step is the evaluation of the potential energy of a dipole in a magnetic field, whereby we can make use of the previous result for the net torque. We remind the reader that the torque projection on the axis perpendicular to an arbitrary plane (see Fig. 1.15 for the geometry) is given by $G_z = -\partial U/\partial\theta$, where U is the potential energy of the body depending on the angle θ which characterizes the rotation of the body in this plane. From Eq. (1.36) we can see that in the same geometry $G_z = -\mu h \sin\theta$, so that the potential energy can be found as

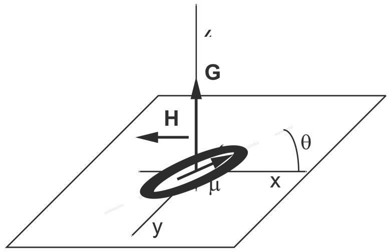

Fig. 1.15. Evaluation of the potential energy of a dipole in a magnetic field (see text for details).

$$U(\theta_0) = -\int_0^{\theta_0} N_z(\theta)\,d\theta = \int_0^{\theta_0} \mu h \sin\theta\,d\theta = -\mu h \cos\theta_0 = -\boldsymbol{\mu}\mathbf{h} \tag{1.37}$$

where by the evaluation of the integral we have omitted (as usual for the potential energy definition) a constant term ($\boldsymbol{\mu}\mathbf{h}$).

This result enables us to evaluate the energy of a magnetic dipole in a magnetic field, assuming that during the rotation in this field the magnitude of a dipole moment is held constant. This is a very important assumption because when the system is rotated in an external field, this field acts on electric charges, the movement of which provides the dipole moment of the system (see Eq. 1.32). Hence the external field, generally speaking, affects the dipole moment of the system and some additional energy source may be needed to hold it constant. A detailed discussion on this subject can be found in Feynmann et al. (1963).

Equation (1.37), or already Eq. (1.36), explains why a compass works. Indeed, a compass needle is a permanent magnet; this means (see Section 1.2.4) that it possesses a magnetic dipole moment which is constant in magnitude due to specific properties of the iron piece from which the compass needle is made. It is well known that any physical system left on its own tries to minimize its energy. And according to Eq. (1.37), the minimal energy of the magnetic dipole in an external field (which in this case is the Earth's magnetic field) is achieved when the magnetic moment points along this field. This causes the compass needle to rotate around its central point until it is directed along the Earth's magnetic field, thus pointing (approximately) in the direction of north.

With Eq. (1.37), we are now able to derive an expression for the force acting on a magnetic dipole in an inhomogeneous magnetic field (remember that in a homogeneous field such a force is zero). Using a standard connection between the force and the potential energy $\mathbf{F} = -\nabla U$, and bearing in mind that in the expression for U only the magnetic field (which is now inhomogeneous) depends on the coordinates, we obtain

$$\mathbf{F} = -\nabla U = -\nabla(\boldsymbol{\mu}\mathbf{h}) = \left(\boldsymbol{\mu}\frac{\partial \mathbf{h}}{\partial x}\right)\mathbf{i} + \left(\boldsymbol{\mu}\frac{\partial \mathbf{h}}{\partial y}\right)\mathbf{j} + \left(\boldsymbol{\mu}\frac{\partial \mathbf{h}}{\partial z}\right)\mathbf{k} = (\boldsymbol{\mu}\nabla)\mathbf{h} \tag{1.38}$$

where we have used the definition of the operator $(\mathbf{a}\nabla)(*)$ to simplify the notation.

In order to understand qualitatively the consequences of Eq. (1.38), let us consider a magnetic dipole directed along the x-axis of our coordinate system, so that $\mu_x \neq 0$ and $\mu_y = \mu_z = 0$. It can be seen from Eq. (1.38) that the only term left in the expression for the force in this case is $F_x = \mu_x(\partial h_x/\partial x)$. In equilibrium, as we already know from Eq. (1.37), the moment is directed along the field, so that $\boldsymbol{\mu}_x$ and $\mathbf{h}_x$ have the same sign. If, say, $h_x > 0$, $\mu_x > 0$ and the field magnitude increases with x, then $(\partial h_x/\partial x) > 0$. In this case $F_x = \mu_x(\partial h_x/dx) > 0$, i.e., the force acting on a magnetic moment in an inhomogeneous field is directed towards the region where this field is larger (in our example in the positive direction along the x-axis). This explains, in particular, the attraction of iron bodies to a permanent magnet – such bodies either already possess a magnetic moment or it is induced by the same magnet and is directed along the external field. Hence, such bodies move towards the region where the field is stronger.

1.2.3 Magnetic Field in Condensed Matter: General Concepts

1.2.3.1 Maxwell Equations in Condensed Matter: Magnetization

For the studies of magnetic phenomena in condensed matter, the original Maxwell Equations (1.1) to (1.4) are not suitable. The reason is that fields, charges and currents appearing in these equations are exact microscopic quantities which contain in principle the whole complexity of electromagnetic processes in condensed matter: movements of single elementary particles on a microscopic space and time scale, corresponding changes of electric and magnetic field, etc. In order to obtain equations which can provide a background for the electrodynamics of condensed matter, we must average Eqs. (1.1) to (1.4) over the microscopic fluctuations mentioned above. This can be done (Landau and Lifshitz, 1975a) using the averaging over a "physically infinitely small volume", which means a volume that is: (1) sufficiently small in the sense that all macroscopic parameters of a body over this volume can be considered as constant; yet (2) at the same time is sufficiently large that it contains a large number of atoms and the averaging over such a volume eliminates fluctuations on the microscopic (atomic) level.

To study magnetic phenomena we need the averaged versions of Eqs. (1.2) and (1.4). Averaging over the "small" volume mentioned above is denoted by angular brackets: $\langle \ldots \rangle$. For historical reasons (which means as usual that (almost?) nobody knows why), the average magnetic field $\langle \mathbf{h} \rangle$ is called *magnetic induction*, and is denoted by $\mathbf{B} : \langle \mathbf{h} \rangle = \mathbf{B}$. So the averaged Eq. (1.4) has the form

$$\operatorname{div} \mathbf{B} = 0 \tag{1.39}$$

If we consider a physical system under stationary conditions (which will be the case almost always), then the average electric field in Eq. (1.2) is constant, so that its time derivative vanishes. Hence, averaging Eq. (1.2) we obtain

$$\text{rot}\,\mathbf{B} = \frac{4\pi}{c}\langle\mathbf{j}\rangle \tag{1.40}$$

Below, we consider a body for which the integral of the current density over its arbitrary cross-section is zero:

$$\int_S \langle\mathbf{j}\rangle\, d\mathbf{S} = 0 \tag{1.41}$$

which is always true for dielectrics and also true for conducting bodies if the total current is absent. Equation (1.41) enables us to introduce a new vector field $\mathbf{M}$, which is zero outside a body and inside it is connected with the average current density as $\langle\mathbf{j}\rangle = c\,\text{rot}\,\mathbf{M}$ (the factor c is introduced for convenience). Indeed, integrating $\langle\mathbf{j}\rangle$ over the surface S bounded with the contour L outside a body, and applying the Stokes formula, we obtain

$$\int_S \langle\mathbf{j}\rangle\, d\mathbf{S} = c\int \text{rot}\,\mathbf{M}\, d\mathbf{S} = c\oint_L \mathbf{M}\, d\mathbf{l} = 0$$

because outside a body $\mathbf{M} \equiv 0$. Substituting the average current density written as $\langle\mathbf{j}\rangle = c\,\text{rot}\,\mathbf{M}$ into Eq. (1.40), we obtain the desired result – the second Maxwell Equation for condensed matter:

$$\text{rot}\,\mathbf{H} = 0 \tag{1.42}$$

where we have introduced a new vector $\mathbf{H}$ as

$$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} \tag{1.43}$$

For the same historical reasons (although it is very confusing), vector $\mathbf{H}$ is called the *magnetic field intensity*, but it should be remembered that the average value of the actual (microscopic) magnetic field intensity is denoted as $\mathbf{H}$. We realize (as do most scientists) that such a confusing notation is very annoying, but now it is too late for it to be changed, as very large numbers of books and papers would need to be rewritten, making the whole operation absolutely out of question. Fortunately, we almost never simultaneously encounter $\mathbf{h}$ and $\mathbf{H}$ (or $\mathbf{h}$ and $\mathbf{H}$) in the same problem.

The vector field $\mathbf{M}$ formally introduced above has a very important physical meaning. To determine this, we recall the definition (Eq. 1.33) of the total magnetic moment of a body and rewrite it using $\mathbf{M}$ as

$$\boldsymbol{\mu} = \frac{1}{2c}\int_V [\mathbf{r} \times \langle \mathbf{j} \rangle]\, d\mathbf{V} = \frac{1}{2}\int_V [\mathbf{r} \times \text{rot}\, \mathbf{M}]\, d\mathbf{V} \tag{1.44}$$

Here, the integration volume can be expanded to contain the body inside it because outside a body $\langle \mathbf{j} \rangle = 0$. Rewriting the last integral in Eq. (1.44) as

$$\int_V [\mathbf{r} \times \text{rot}\, \mathbf{M}]\, d\mathbf{V} = \oint_S [\mathbf{r} \times [d\mathbf{S} \times \mathbf{M}]]\, d\mathbf{V} - \int_V [[\mathbf{M} \times \nabla] \times \mathbf{r}]\, d\mathbf{V} \tag{1.45}$$

(the proof of Eq. (1.45) is a very nice exercise in vector analysis), we note that due to the mentioned expansion of the integration volume its bounding surface S is now outside the body where $\mathbf{M} = 0$ and hence the first integral in Eq. (1.45) vanishes. Finally, rewriting a double vector product in the second integral as

$$[[\mathbf{M} \times \nabla] \times \mathbf{r}] = -\mathbf{M}\, \text{div}\, \mathbf{r} + \mathbf{M} = -2\mathbf{M}$$

we obtain the desired result

$$\boldsymbol{\mu} = \frac{1}{2c}\int_V [\mathbf{r} \times \langle \mathbf{j} \rangle]\, d\mathbf{V} = \int_V \mathbf{M}\, d\mathbf{V} \tag{1.46}$$

which shows that $\mathbf{M}$ is simply the density of the magnetic moment of the body (magnetic moment per unit volume). For this reason, $\mathbf{M}$ is called the *magnetization* vector.

1.2.3.2
Classification of Materials According to their Magnetic Properties

The system of Eqs. (1.39), (1.42) and (1.43)

$$\begin{aligned} &\text{div}\, \mathbf{B} = 0 \\ &\text{rot}\, \mathbf{H} = 0 \\ &\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} \end{aligned} \tag{1.47}$$

which describes the magnetic field in a condensed matter is clearly incomplete, because we still do not know the relationship between $\mathbf{M}$ and $\mathbf{H}$ (or between $\mathbf{B}$ and $\mathbf{H}$) inside a body. This relationship depends heavily on the material from which the body under study is made. Fortunately, for an overwhelming majority of physical substances, the required relationship is very simple:

$$\mathbf{B} = \mu\mathbf{H} \quad \text{or} \quad \mathbf{M} = \chi\mathbf{H} \tag{1.48}$$

where scalar quantities μ and χ are called correspondingly magnetic permeability and susceptibility (so here μ is not the magnitude of the total magnetic moment!). From Eqs. (1.43) and (1.48), the relationship between μ and χ is $\chi = (\mu - 1)/4\pi$. In some cases (e.g., for solid monocrystalline samples), μ and χ appearing in the proportionality relationships (Eq. 1.48) are tensors of a corresponding rank. For ferromagnets, the situation is even more complicated – the relationship between **B** and **H** is nonlinear in general case and depends on the history of the sample.

The magnetic susceptibility is the most important quantity characterizing the magnetic properties of a material (Landau and Lifshitz, 1975a; Kittel, 1986). Namely, it enables us to calculate the magnetization (and hence the magnetic moment) of the body in an external field. If $\chi < 0$, then, as can be seen from Eq. (1.48), the magnetic moment induced by an external field is directed opposite to this field. Such materials are called diamagnets (e.g., inert gases, organic liquids, graphite, bismuth). According to our discussion of the force acting on a body in a nonhomogeneous magnetic field (see text following Eq. 1.38), diamagnetic bodies are repelled from the magnet.

For substances with $\chi > 0$, the induced magnetic moment caused by the external field, points in the same direction as the external field. Materials with positive but very small (for most materials $\chi \sim 10^{-6}$) susceptibility are called paramagnets (some gases, organic free radicals, most metals).

Finally, there exists a narrow class of materials for which magnetic susceptibility defined by Eq. (1.48) – when possible – is huge ($\chi \sim 10^3$, but for some specially prepared materials $\chi \sim 10^6$ can be achieved). Such substances are known as ferromagnets (iron, cobalt, nickel and their alloys, some iron and chromium oxides, etc.). It is evident that these materials are most interesting, for both theoretical studies and practical applications. We shall consider corresponding problems in the final paragraph of this section and again in Section 1.2.4. Here, it should only be mentioned that the relationships (1.48) for ferromagnets are, generally speaking, not valid – the induced magnetic moment is not simply proportional to the external field.

Diamagnetism and paramagnetism can be explained in terms of classical physics (to be more precise, in these terms we can provide explanations which appear reasonable). Let us begin with diamagnetism. The molecules of diamagnetic substances do not have their own magnetic moments; that is, they do not possess a spontaneous moment – a magnetic moment in the absence of an external field. From basic electromagnetism we are familiar with Lenz's law: when we try to change a magnetic flux through a conducting contour, then an electric current in this contour is induced in such a way, that the magnetic field created by this current opposes the change of the external magnetic flux. In other words, if we try to increase a magnetic field inside a closed contour, the magnetic moment associated with the current induced by this external field will be directed opposite to it.

From the classical point of view, electrons moving in atoms or molecules can be considered as currents. Hence, by applying a magnetic field to a body, we try to increase magnetic flux through contours formed by these currents (electron orbits).

According to Lenz's law, this increase leads to changes in corresponding currents, with the result that the magnetic moment of the body induced by this change is directed opposite to the external field, which means diamagnetism.

Molecules of paramagnetic substances already possess their own dipole moments. When we apply an external field, these moments tend to align themselves along this field, because it would minimize their magnetic energy according to Eq. (1.37). The chaotic thermal motion tries to prevent such an alignment, but an average magnetic moment nevertheless appears and points in the field direction, leading to paramagnetism ($\chi > 0$).

1.2.3.3
Mean Field Theory of Ferromagnetism

The existence of ferromagnetism is one of (not very many) the macroscopic phenomena which, in principle, cannot be explained in terms of classical physics. To demonstrate this (Kittel, 1986), it is sufficient to estimate the magnitude of interactions between atomic magnetic moments which are responsible for the ferromagnetic phenomena using the following arguments. The main manifestation of ferromagnetism is the existence of the spontaneous magnetization – that is, a ferromagnetic sample can possess a spontaneous macroscopic magnetic moment. This means that there exists some strong interaction which results in the parallel alignment of all atomic magnetic moments inside the body. The magnitude of this spontaneous magnetic moment decreases if the sample temperature increases, because the thermal movement (thermal fluctuations) acts against any order trying to destroy it. At some temperature, T_c, which depends on the material and is termed the "critical temperature" or "Curie point", the spontaneous magnetization vanishes, and for temperatures $T > T_c$ our body behaves like a paramagnet.

The interaction energy, E_{fm}, for the interaction type responsible for the ferromagnetism should be of the same order of magnitude as the thermal energy at the Curie point: $E_{\text{fm}} \sim kT_c$. The only interaction known in classical physics which could cause the alignment of magnetic moments is the magneto dipole interaction between them. The interaction energy of two magnetic dipoles E_{dip} can be estimated according to Eq. (1.37) as $E_{\text{dip}} \sim \mu H_{\text{dip}}$, where the order of magnitude of the dipole field is (see Eq. 1.34) $H_{\text{dip}} \sim \mu/r^3$, so that $E_{\text{dip}} \sim \mu^2/r^3$. Substituting in this expression typical values of the atomic magnetic moment $\mu \sim \mu_B \approx 10^{-20} erg/Gauss$ (μ_B is a so-called Bohr magneton which is a very convenient unit for measuring atomic magnetic moments) and the interatomic distance ($\sim$ lattice constant in a typical crystal) $r \sim (2 \ldots 3) \cdot 10^{-8}$ cm, we obtain for the interaction energy $E_{\text{fm}} = E_{\text{dip}} \sim 10^{-17} erg$. The value of the Boltzmann constant is $k \approx 1.4 \cdot 10^{-16} erg/K$, so the critical temperature for a typical Ferro magnet should be $T_c = E_{\text{fm}}/k \sim 0.1$ K. This value has nothing in common with the experimentally measured Curie points which, for most ferromagnets, are of the order $T_c \sim 10^3$ K (e.g., for iron, $T_c = 1043$ K). Hence, ferromagnetism cannot be explained by the magneto dipolar interaction, and in classical physics we have nothing else at our disposal.

For many decades, all attempts to develop a reasonable theory of ferromagnetism failed. The first phenomenological theory which succeeded in explaining some aspects of this phenomenon was suggested by Weiss (1907). Weiss postulated that: (1) there exists some (unknown) effective interaction field $\mathbf{H}_E$ which tends to align atomic magnetic moments parallel to each other; and (2) the magnitude of this effective field is proportional to the average magnetization: $\mathbf{H}_E = \lambda\langle\mathbf{M}\rangle$. These assumptions, together with the well-known expression (the so-called Langevin function) for the average magnetization of a system of noninteracting magnetic moments in an external field as a function of the temperature T and field $\mathbf{H}$ (Kittel, 1986) (which in Weiss' theory should be set to the sum of the external $\mathbf{H}_0$ and effective $\mathbf{H}_E$ fields), allowed Weiss to deduce the temperature dependence of the spontaneous magnetization. The result demonstrated a remarkable agreement with experimental data, which was more than acceptable for such a simple theory. However, as mentioned earlier, the existence of a ferromagnetic interaction itself was postulated by Weiss, so the nature of this interaction still required an explanation.

Such an explanation could be provided only after the appearance of quantum mechanics (for an excellent historical review, see Mattis, 1965). Here, an attempt will be made to provide a brief description of how ferromagnetism follows from its basic postulates. (Note: Should the reader feel uncomfortable when confronted with words such as "quantum", the following explanation may be missed out by simply accepting that permanent magnets do exist.)

Ferromagnetism occurs due to the collective behavior of electrons in some materials. Every electron possesses its own angular momentum $\mathbf{S}$ (called spin) which, being expressed in units of the so-called Planck constant (Feynmann et al., 1963; Landau and Lifshitz, 1971) is exactly $S = 1/2$. According to one of the basic principles of quantum mechanics – the Pauli principle – two particles with the spin $1/2$ *cannot occupy one and same quantum state* (Feynmann et al., 1963; Landau and Lifshitz, 1971) which, for our purposes, can be reformulated as "two particles having the same spin direction cannot occupy one and same space region". In other words, if the spins of two electrons do not have the same direction, then the distance between them can, in principle, be very small, but electrons with parallel spins must be "far away" from each other.

This means, in turn, that the energy of a system of two electrons with different spin directions can be very large, because two close electrons exhibit a huge electrostatic repulsion as two charges of the same sign. Moreover, the electrostatic energy of two electrons with parallel spins should be quite small because such electrons must avoid each other due to the Pauli principle (please don't ask when have the electrons read any textbook on quantum mechanics!). For this reason, the state where spins of two electrons – and their magnetic moments! – are parallel is strongly preferred from the energy point of view, because the (average) electrostatic energy in this state is much lower! And this preferred state with all electron spins parallel is exactly what we want – the ferromagnetic state, where all electron magnetic moments are aligned and hence the body possesses macroscopic spontaneous magnetization. The phenomenon just described is called the "exchange interaction", because its quantitative description is based on the so-called exchange integrals (Landau and Lifshitz, 1971). We realize that this reference does not make the things clearer, but the discussion on what these integrals are and why are they called "exchange integrals" is far too complicated to be presented here.

The explanation given above indeed accounts for the Curie temperatures observed experimentally. In this physical picture it is the strong Coulomb (electrostatic) interaction which is responsible for the appearance of ferromagnetism – not the weak magnetodipole forces. If we estimate T_c using the arguments given above, we simply determine the correct order of magnitude.

Of course, this is a very long way from our brief description of this basic idea to a real theory of the ferromagnetic phenomena (to see this, it is sufficient to note that if our arguments would represent the whole truth in all cases, then all substances would be ferromagnetic because there are some electrons in all materials!). But at least we have shown the beginning of the way that can lead to an explanation of ferromagnetism.

1.2.4
Magnetic Field in Condensed Matter: Special Topics

In this section, we consider some special topics dealing mainly (but not only) with ferromagnetic materials: various contributions to the magnetic energy of such materials, magnetic domains and domain walls, hysteresis phenomena, very small (so-called single-domain) ferromagnetic particles, and irreversible magnetic relaxation. Further, we briefly review the energy dissipation in alternating magnetic fields and discuss the possibility of a reconstruction of the magnetization distribution inside a body from magnetic field measurements outside it.

1.2.4.1
Magnetic Energy Contributions

There are several contributions to the total magnetic energy of the body arising from various interaction types between elementary magnetic moments (Chikazumi, 1964; Kittel, 1986; Landau and Lifshitz, 1975a). Below, we restrict ourselves to the phenomenological consideration of these contributions, and always assume (unless mentioned otherwise) that the temperature is much lower than the Curie point of the ferromagnets under study: $T \ll T_c$.

Exchange Energy

The first and most important energy contribution comes from the exchange interaction, which was introduced above as a purely quantum mechanical effect responsible for the alignment of atomic magnetic moments in a ferromagnetic body. The assumption $T \ll T_c$ means that the energy of temperature fluctuations is negligible compared with the exchange energy, so that adjacent magnetic moments are (almost) parallel. Hence, the magnitude of the magnetization $\mathbf{M}$ of the body (magnetic moment per unit volume) can be considered as constant $|\mathbf{M}| = M_s$; this constant is called the saturation magnetization of a ferromagnetic material. For low temperatures, only the magnetization direction can be varied inside a body under an additional condition that the distance where the magnetization direction varies considerably is much larger than the lattice constant (or mean interatomic distance for amorphous ferromagnets) of the material. The latter circumstance allows us to introduce a unit vector $\mathbf{m} = \mathbf{M}/M_s$ along the magnetization direction; its spatial distribution fully describes the magnetization structure of a ferromagnetic body.

Let us now write down the phenomenological expression for the exchange energy using simple general arguments (Landau and Lifshitz, 1975a). First, we recall that in ferromagnetic materials the exchange interaction ''prefers'' the parallel alignment of magnetic moments. This means that the corresponding energy is minimal for the magnetization configuration where all magnetic moments of a body are parallel to each other – the so-called homogeneous magnetization state. We set the exchange energy of such a state to zero, thus using it as a reference point. We also point out that the exchange interaction energy does not change when the magnetization configuration is rotated as a whole with respect to a ferromagnetic body.

We hope that it is clear from the consideration above that the exchange energy density (exchange energy per unit volume) e_{exch} can depend only on the spatial variation of the magnetization, thus being a function of its spatial derivatives $\partial M_i/\partial x_k$, $(i, k = 1, 2, 3)$, where M_i denotes Cartesian components of the magnetization and $x_k = x, y, z$. Moreover, e_{exch} can be a function only of a product of even numbers of such derivatives, because M_i itself and hence – $\partial M_i/\partial x_k$ – changes sign due to the time inversion operation $t \to -t$ (this is because $\mathbf{M} \sim [\mathbf{r} \times \mathbf{v}] = [\mathbf{r} \times d\mathbf{r}/dt]$, see Eq. 1.32) and the energy does not. The simplest expression which satisfies this condition and another condition mentioned above – that the exchange energy is invariant with respect to the rotation of the magnetization configuration as a whole – is

$$e_{\text{exch}} = \frac{1}{2}\sum_{i,k,l} \alpha_{ik} \left(\frac{\partial M_l}{\partial x_i}\right)\left(\frac{\partial M_l}{\partial x_k}\right) \tag{1.49}$$

Here, α_{ik} are the components of a (symmetrical) tensor of exchange coefficients which means that these components form a symmetrical 3×3 matrix. In the simplest case of a crystal with the cubic symmetry $\alpha_{ik} = \alpha\delta_{ik}$ (δ_{ik} is a Kronecker symbol: $\delta_{ik} = 1$ if $i = k$ and zero otherwise) and Eq. (1.49) takes the form

$$\begin{aligned} e_{\text{exch}} &= \frac{\alpha}{2}\sum_i \left(\frac{\partial \mathbf{M}}{\partial x_i}\right)^2 = \frac{\alpha}{2}\left[\left(\frac{\partial \mathbf{M}}{\partial x}\right)^2 + \left(\frac{\partial \mathbf{M}}{\partial y}\right)^2 + \left(\frac{\partial \mathbf{M}}{\partial z}\right)^2\right] \\ &= \frac{A}{2}\left[\left(\frac{\partial \mathbf{m}}{\partial x}\right)^2 + \left(\frac{\partial \mathbf{m}}{\partial y}\right)^2 + \left(\frac{\partial \mathbf{m}}{\partial z}\right)^2\right] \end{aligned} \tag{1.50}$$

where we have introduced a new exchange constant $A = \alpha M_s^2$ using the unit magnetization vector $\mathbf{m}$ defined above. The total exchange energy of a ferromagnetic body can be evaluated, as usual, as an integral of the corresponding density Eq. (1.49) (or Eq. 1.50) over the body volume:

$$E_{\text{exch}} = \int_V e_{\text{exch}}(\mathbf{r})\, dV = \frac{A}{2}\int_V \left[\left(\frac{\partial \mathbf{m}}{\partial x}\right)^2 + \left(\frac{\partial \mathbf{m}}{\partial y}\right)^2 + \left(\frac{\partial \mathbf{m}}{\partial z}\right)^2\right] dV \tag{1.51}$$

Magnetic Anisotropy Energy

As stated above, the exchange energy is invariant under the rotation of the magnetization configuration as a whole, which means that it does not depend on the orientation of the total magnetic moment of the body with respect to its crystallographic axes. On the other hand, there exists a well-known experimental fact that if a sample represents a single crystal, then for many ferromagnets it is much easier to magnetize it in certain directions than in some other directions. This means that there exists an energy contribution which depends heavily on the magnetization orientation relative to crystallographic axes.

This energy contribution is termed the *magnetic anisotropy energy*. Its physical origins are: (1) interaction between atomic magnetic moments with the electric field of a crystal lattice (spin-orbit); and (2) direct magnetic (spin–spin) interaction between atomic moments. For both types of interaction, the corresponding energies depend heavily on the orientation of magnetic moments relative to each other and to the crystal lattice, thus providing the desired orientation dependence of the anisotropy energy. The characteristic magnitude of this energy is usually much less than that of the exchange energy because, according to Eq. (1.32), atomic magnetic moments contain a small factor v/c, where v is the velocity of atomic electrons and c is the speed of light (a more detailed discussion of this question can be found in Landau and Lifshitz, 1975b).

To identify the nature of the anisotropy energy density, it is again sufficient to use general symmetry considerations (Landau and Lifshitz, 1975a). First, the anisotropy energy should depend on the magnetization orientation itself (and not on its spatial derivatives as the exchange energy), which means that it depends directly on the magnetization components m_i $(i = x, y, z)$. The second idea is the same as for the exchange energy – the anisotropy energy density is invariant with respect to the time inversion operation, and hence can be only an even function of such components. Considering first the simplest possible case – products of two m_i-components – we obtain the anisotropy energy density e_{an} in the form

$$e_{\text{an}} = \sum_{i,k} K_{ik} m_i m_k \tag{1.52}$$

where K_{ik} is also a symmetrical tensor.

The remainder of the information required for complete determination of the anisotropy energy density is provided by the symmetry of the crystal lattice of a ferromagnet under study. In the simplest case of a uniaxial crystal, let us choose the z-axis of our coordinate system along the main crystal symmetry axis. For such a crystal there exists only one independent component of K_{ik} for which the corresponding combination of the magnetization components is orientation-dependent (there is also another K-component corresponding to the combination of m-values, namely $m_x^2 + m_y^2 + m_z^2 = 1$ which does not depend on anything). For such crystals, the anisotropy energy density is

$$e_{\text{an}} = K(m_x^2 + m_y^2) = K \sin^2 \theta \tag{1.53}$$

where θ is the angle between the magnetization vector and the z-axis (uniaxial magnetic anisotropy). If $K > 0$, then the anisotropy energy is minimal for the magnetization along the symmetry axis ($\theta = 0$) – ''easy axis'' anisotropy. For $K < 0$, the minimal energy is achieved if the magnetization lies in the plane perpendicular to this axis – the ''easy-plane'' case.

Quite often, terms of higher orders in m_i-values are needed. In hexagonal crystals, such terms only modify Eq. (1.53) as

$$e_{\text{an}} = K_1 \sin^2 \theta + K_2 \sin^4 \theta \tag{1.54}$$

though in cubic crystals they provide the first nonvanishing contributions to the anisotropy energy density at all:

$$e_{\text{an}} = K(m_x^2 m_y^2 + m_x^2 m_z^2 + m_y^2 m_z^2) \tag{1.55}$$

This means that in cubic crystals there exist either three (for $K > 0$, energy minima for m along the cube edges, as in iron) or four ($K < 0$, minima for m along the cube space diagonals, e.g., in nickel) equivalent easy magnetization axis.

Magnetic Dipole Interaction (Demagnetizing) Energy

Another important energy contribution occurs due to the dipolar interaction of magnetic moments: any magnetic dipole creates the magnetic field (Eq. 1.34); if another dipole is placed into this field, then it possesses an energy according to Eq. (1.37). The energy of a system of two dipoles can be written either as an energy of the first dipole $\boldsymbol{\mu}_1$ in the field of the second one at the location point of the first $\mathbf{h}_{21}$, or vice versa:

$$E = -\boldsymbol{\mu}_1 \mathbf{h}_{21} = -\boldsymbol{\mu}_2 \mathbf{h}_{12} \tag{1.56}$$

Rewriting this expression in a symmetrical form

$$E = -\frac{1}{2}(\boldsymbol{\mu}_1 \mathbf{h}_{21} + \boldsymbol{\mu}_2 \mathbf{h}_{12}) \tag{1.57}$$

we can immediately generalize it to a system of many dipoles:

$$E = -\frac{1}{2}\sum_i \boldsymbol{\mu}_i \mathbf{h}_i \tag{1.58}$$

where $\mathbf{h}_i$ means the dipole field created at the location of the i-th dipole by all other dipoles.

What we need now is the continuous version of Eq. (1.58) – that is, the energy of the magnetic dipolar interaction which exists between various parts of a ferromagnetic body if the magnetization configuration of a body $\mathbf{M}(\mathbf{r})$ is known. According to the rules explained in Section 1.2.3, for the transition to the condensed matter case exact microscopic quantities appearing in Eq. (1.58) should be replaced as follows: $\mathbf{h}_i \to \mathbf{B}_i$ and $\boldsymbol{\mu}_i \to \mathbf{M}_i \Delta V_i$. Here, we have subdivided the ferromagnetic body into a (finite) number of small parts with volumes ΔV_i, so that the index i refers now not to the point dipole [as in Eq. (1.58)] but to such a small part of a body. Passing to a continuous limit, we obtain as a generalization of Eq. (1.58) an integral expression

$$E_{\text{dip}} = -\frac{1}{2}\int_V \mathbf{MB}\, dV \tag{1.59}$$

For further use it is more convenient to rewrite the equation using the field $\mathbf{H}$. Substituting the expression $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$ (see Eq. 1.43) into Eq. (1.59) and using the fact that in ferromagnets the absolute value of the magnetization is constant ($\mathbf{M}^2 = M_s^2 = Const$), we obtain

$$E_{\text{dip}} = -\frac{1}{2}\int_V \mathbf{MH}\, dV - 2\pi\int_V \mathbf{M}^2\, dV = -\frac{1}{2}\int_V \mathbf{MH}\, dV - 2\pi M_S^2 V$$

The last term can be omitted because for the given body it is constant, and any constant in an energy expression can be omitted (just choose this constant as a reference point). The final result then is

$$E_{\text{dip}} = -\frac{1}{2}\int_V \mathbf{MH}_{\text{dip}}\, dV \tag{1.60}$$

Here, the notation $\mathbf{H}_{\text{dip}}$ points out that the field in Eq. (1.60) is the *dipole* magnetic field created by all magnetic moments of a body. We also note that despite Eq. (1.60) appearing to be simply a continuous version of Eq. (1.58) in a general case, it is valid for ferromagnets only, because by its derivation we have substantially used the condition $\mathbf{M}^2 = M_s^2 = Const$. The similarity between Eqs. (1.58) and (1.60) arises from the confusing notation mentioned above ($\langle \mathbf{h} \rangle = \mathbf{B}$ and not H!).

The dipole energy (Eq. (1.60), often also called magnetostatic energy) is always non-negative: $E_{\text{dip}} \geq 0$. To prove this, we expand the integration in Eq. (1.60) over the whole space (we can do this because outside a body $\mathbf{M} = 0$) and replace $\mathbf{M}$ by $\mathbf{M} = (\mathbf{B} - \mathbf{H})/4\pi$ using Eq. (1.43). Then, Eq. (1.60) takes the form

$$E_{\text{dip}} = -\frac{1}{2}\int \mathbf{MH}\,dV = -\frac{1}{8\pi}\int \mathbf{BH}\,dV + \frac{1}{8\pi}\int \mathbf{H}^2\,dV \tag{1.61}$$

Now, we need a so-called orthogonality theorem from vector analysis: a volume integral over the whole space of the product of a divergence-free and a rotor-free field is zero, if these fields are square-integrable functions (which means that integrals of their squares over the whole space are finite). Magnetic induction $\mathbf{B}$ is a divergence-free field (see Eq. 1.39), and magnetic field $\mathbf{H}$ is rotor-free (see Eq. 1.42). It is also easy to show that these fields, when created by any finite system (of permanent magnets or of electrical currents), are square-integrable. For this reason the orthogonality theorem states that the first integral in Eq. (1.61) is exactly zero, thus leading to the final result

$$E_{\text{dip}} = \frac{1}{8\pi}\int \mathbf{H}_{\text{dip}}^2\,dV \tag{1.62}$$

where the integral is taken over the whole space. From Eq. (1.62) it is evident that the magnetostatic energy is always non-negative, and is zero only if the dipolar field is absent at all. To handle the magnetostatic energy and corresponding field more easily, a useful concept of ''magnetic charges'' can be introduced in a following manner. Let us rewrite the condition of Eq. (1.39) $\operatorname{div}\mathbf{B} = 0$ using the relationship of Eq. (1.43) between $\mathbf{B}$, $\mathbf{M}$ and $\mathbf{H}$ as

$$\operatorname{div}\mathbf{H}_{\text{dip}} = -4\pi \operatorname{div}\mathbf{M}$$

Now, by formally defining a scalar quantity ρ_{mag} as

$$\rho_{\text{mag}} = -\operatorname{div}\mathbf{M} \tag{1.63}$$

we arrive at the equation

$$\operatorname{div}\mathbf{H}_{\text{dip}} = 4\pi\rho_{\text{mag}} \tag{1.64}$$

that exactly resembles the corresponding Maxwell Equation (1.3), which states (see discussion in Section 1.2.2.2) that the sources of the electric field are electric charges. This is the reason why the quantity ρ_{mag} defined by Eq. (1.63) is called the density of ''magnetic charges'', despite the fact that real magnetic charges (i.e., charges which would produce the real microscopic magnetic field $\mathbf{h}$) do not exist (see Eq. 1.4). ''Magnetic charges'' introduced above are simply a very convenient mathematical tool both for quick estimation and for detailed calculations dealing with the magnetostatic energy.

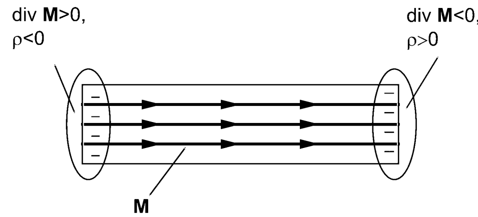

Fig. 1.16. "Magnetic charges" for a uniformly magnetized slab.

Let us present a simple example. We consider a ferromagnetic slab uniformly magnetized along its long axis, as shown in Figure 1.16. Clearly, div $\mathbf{M} = 0$ everywhere except for the regions near the ends of the slab. If we consider a small volume around, say, the slab's right-hand end, then it can be seen that there exists a total nonzero flux of the magnetization **M** *into* this volume. According to the definition of the divergence operator (which is, roughly speaking, the average flux of a vector field out of a small volume surrounding the given point divided by this volume), this means that the average divergence of **M** for the region near the right-hand slab end is negative (div $\mathbf{M} < 0$) and, according to the definition in Eq. (1.63), there is a net positive "magnetic charge" near this end. By the same token, there exists a negative magnetic charge (we omit "..." here and below, but please do remember that magnetic charges are not real physical charges!) near the left-hand end of the slab (Fig. 1.16).

It follows from Eq. (1.64) that magnetic charges are the sources of the magnetic field **H**. Thus, from the arguments presented above we can conclude that the magnetic field created by a uniformly magnetized slab (which is a very good model for a bar-shaped permanent magnet) should look exactly like the electric field of a large (not point-like!) dipole because such a slab possesses two magnetic charges with equal magnitude and opposite sign on its ends. And this similarity is indeed present, which is a well-known text-book result.

Now we are also able to explain why the magnetostatic energy is often called the "demagnetizing" energy. For the simplest case of a ferromagnetic layer uniformly magnetized perpendicular to its plane (Fig. 1.17) there exist (according to the pre-

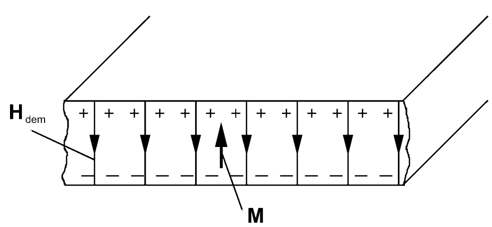

Fig. 1.17. Demagnetizing field of a layer which is uniformly magnetized perpendicular to its plane.

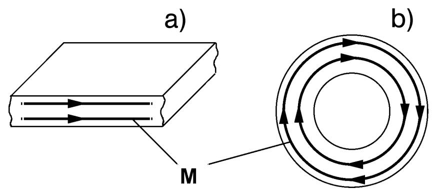

Fig. 1.18. Examples of stray-field-free magnetization configurations.

vious consideration) positive magnetic charges on its upper surface, and negative charges on its lower surface. The field induced by such a system of two charged planes is well known (it looks exactly as the electric field of a flat capacitor) – it exists only inside the layer, is homogeneous (from symmetry reasons), and is directed from positive to negative charges. This means that it is directed opposite to the magnetization thus trying to decrease it. The latter statement is true not only for the example presented above but also in a general case – the magnetic field $\mathbf{H}_{\text{dip}}$ created by some magnetization distribution tries to decrease the corresponding magnetization and hence is called the demagnetizing field [and its magnetostatic energy, Eq. (1.62) – the demagnetizing energy].

The concept of magnetic charges also enables us to determine in which cases there is no demagnetizing energy – we should simply avoid the appearance of such charges, because they create the demagnetizing field and this field always (according to Eq. 1.62) carries energy with them. Magnetic charges (Eq. 1.63) are absent if div $\mathbf{M} = 0$ everywhere – in other words, the charges are absent when the lines of the magnetization field are continuous. Some examples of corresponding magnetization configurations (magnetic layer and core) are shown in Figure 1.18.

1.2.4.2
Magnetic Domains and Domain Walls

Although some materials such as iron, cobalt, and nickel are ferromagnetic, macroscopic samples of, for example, nickel taken without special preparation either do not possess a spontaneous magnetization at all, or their total moment is much less than those expected for the magnetically saturated sample (where all atomic magnetic moments are aligned parallel to each other). The phenomenological explanation of this fact was provided by Weiss at the start of the 20th century in connection with his effective field theory (see above). For polycrystalline samples, Weiss' explanation can be reformulated as follows (Kittel, 1986). The effective field – and hence the directions of the spontaneous magnetization – may be different in different parts of a ferromagnetic sample (e.g., in different crystallites). Then, inside one crystallite all elementary magnetic moments would be parallel, so that a single crystallite would possess a net macroscopic magnetic moment. However, due to different (and essentially random) directions of these moments for various crystallites, the net magnetic moment of the sample would be very small, as is observed experimentally.

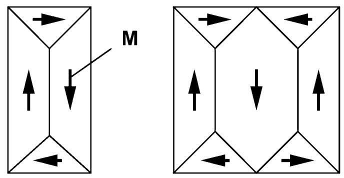

Fig. 1.19. Examples of magnetic domain configurations that do not produce any stray field.

Today, we know that such regions of a ferromagnetic sample where atomic magnetic moments are aligned parallel to each other – the *magnetic domains* – do not necessarily coincide with crystallites (in particular, a single-crystal sample may also consist of many domains). Nonetheless, the idea itself concerning the coexistence of such regions with various magnetization directions inside one sample was correct, and today we have at our disposal many methods which enable the direct observation of such domains (Chikazumi, 1964); subsequently, the so-called domain theory has become a well-established area of magnetism.

The main reason why magnetic domains do exist even in single-crystal samples is the demagnetizing energy (see Section 1.2.4.1). If all magnetic moments of a macroscopic sample were to be aligned, it would possess a huge total magnetic moment inducing a strong magnetic field. According to Eq. (1.62), a system which produces such a field would have a very large demagnetizing energy, making the corresponding fully aligned state energetically unfavorable. For this reason, in the absence of a strong external field (which would cause the alignment of all elementary moments along this field), a macroscopic ferromagnetic sample is usually divided into many domains, the magnetic moments of which are oriented in such a manner that the existence of "magnetic poles" or "magnetic charges" (which can be considered as sources of a magnetic field **H**) is avoided as far as possible (Fig. 1.19). Another important factor which determines the direction of magnetic moments inside a single domain is the magnetic anisotropy of a crystal: the magnetic moment of a domain should preferably be directed along one of the crystal anisotropy axis of a crystal lattice (to minimize the anisotropy energy).

Domains are separated from each other by boundaries called *domain walls* (Chikazumi, 1964; Kittel, 1986). The parameters of these walls (e.g., their thickness and energy) which are important in applications of magnetic materials can be calculated using the above-mentioned phenomenological domain theory. Although such calculations are far beyond the scope of this chapter, we can try to understand the main dependencies of these parameters on magnetic properties of the material, at least qualitatively.

Let us consider the simplest example of a domain wall – a wall between two domains in a uniaxial crystal where magnetization directions in these domains are opposite (along two opposite directions of the anisotropy axis) (Landau and Lifshitz, 1975a; Kittel, 1986). The transition between these two magnetization directions should be very smooth on the atomic length scale (see Fig. 1.20), because

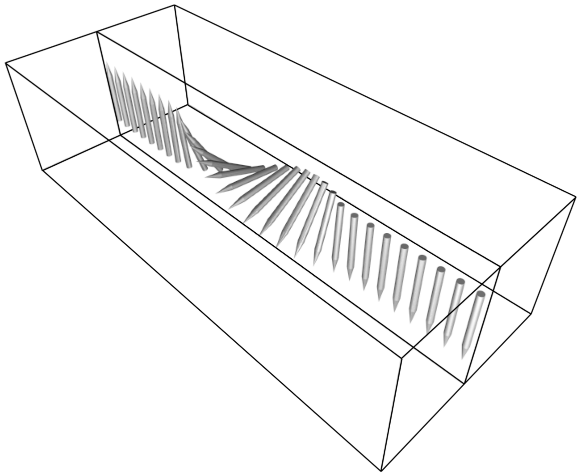

Fig. 1.20. Structure of a 180° domain wall in a uniaxial magnetic material.

abrupt changes in the magnetization direction with a large angle between magnetizations, in two adjacent atomic layers would lead to high exchange energy.

To estimate the width δ_w of this transition region (domain wall) (Landau and Lifshitz, 1975a), we first note that this width should increase with the increasing exchange constant A of the material (see Eq. 1.50) because a larger magnitude of the exchange interaction requires a smoother transition (smaller magnetization angles between two adjacent atomic layers are allowed). On the other hand, the larger the width of such a wall, the larger is its anisotropy energy, because inside a wall, the magnetic moments are not oriented along the (energetically favorable) direction of the anisotropy axes. For this reason, δ_w should decrease with an increasing anisotropy constant K (see Eqs. 1.53 and 1.55). The simplest combination of the constants A and K with the dimension of a length and with the properties just mentioned is $\delta_w \sim \sqrt{A/K}$, and this expression indeed gives the correct dependence of the domain wall width on the magnetic parameters of the material in the situation shown in Figure 1.20.

The corresponding energy dependence can be guessed using the same arguments: the domain wall energy clearly increases with both A and K, because inside a domain wall we have both energy contributions. The corresponding dimensionally correct combination (with the dimension energy per unit area) gives the desired domain wall energy dependence on the material parameters: $E_w \sim \sqrt{AK}$.

1.2.4.3 Magnetization Curves and Hysteresis Loops

If we apply an external field to a demagnetized (i.e., without a net magnetic moment) ferromagnetic sample, then a macroscopic magnetic moment $\mu = \int \mathbf{M}\, dV$ will appear in this specimen. If we increase the field magnitude, then the magni-

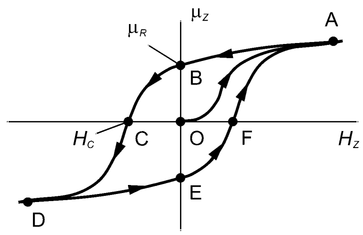

Fig. 1.21. Typical initial magnetization curve $O \to A$ and hysteresis loop. $A \to B \to C \to D \to E \to F \to A$ for ferromagnetic materials.

tude of this moment also increases until at some certain field value H_{sat}, the so-called saturated state where all microscopic moments of a sample are aligned in the field direction will be achieved. The increase of the field intensity beyond this value will clearly not result in any further moment growth. The process just described is termed the "initial magnetization process"; the corresponding dependence of the total magnetic moment projection μ_z on the field direction (which we will take as a positive direction of the z-axis) – the initial magnetization curve – is shown in Figure 1.21 as the line OA.

Within the domain picture there are two main reasons for such a continuous magnetization growth with the increasing field: (1) rotation of the magnetization inside a domain towards the external field direction; and (2) domain growth – that is, the sizes of those domains whose magnetic moments are oriented approximately along the external field will increase on the cost of less favorably oriented domains. Processes of the first type result in an increase of the anisotropy energy, because magnetic moments are forced to rotate out of the easy-axes directions (along which they were oriented in the absence of the external field). This anisotropy energy growth (see Eqs. 1.53 and 1.55) should be compensated by the decrease of the energy in the external field (Eq. 1.37), which can require quite large field magnitudes. In contrast to the magnetization rotation, the second type of process requires "only" domain wall displacements, and this can be normally be done without major effort. For this reason, the lower part of the initial magnetization curve is usually dominated by the domain growth, whereas its behavior near saturation is determined by the magnetization rotation processes.

One of the most striking phenomena in ferromagnetism is the existence of the magnetization hysteresis or the irreversibility of the magnetization processes (Chikazumi, 1964; Kittel, 1986). This means that if we decrease the field magnitude starting from the saturated sample state A (Fig. 1.21), then the field dependence of the sample magnetic moment (shown by the curve AB) does not coincide with the corresponding dependence for the increasing field (curve OA). When we decrease the field to zero, there is still some substantial net magnetic moment left – this is called the *remanent magnetic moment* μ_R. If we then reverse the field direction and again increase its magnitude, a certain (often quite large) nonzero value of this field H_c (called the coercive force) is needed to bring the net sample moment to zero (point C in Fig. 1.21). Further increase of the field amplitude in the negative direction also leads to sample saturation in this direction (state D). Finally, by decreasing the field magnitude up to zero and then increasing it again in the positive direction, the magnetization of our sample will follow the curve $D \to E \to F \to A$. So, during a complete field cycle $H_{\text{sat}} \to -H_{\text{sat}} \to H_{\text{sat}}$ the sample magnetic moment changes along the curve $A \to B \to C \to D \to E \to F \to A$, which is called the *hysteresis loop*.

The general reason for such a hysteretic behavior is the existence of so-called metastable states – those system states that correspond to local (and not global) energy minima. Such states are separated from the global minimum and other local minima by energy barriers. If the heights of these barriers are large compared to the temperature (in other words, to the energy of thermal excitations), then the system may remain in such a metastable state for a macroscopically long time, although this particular state does not correspond to the global energy minimum achievable for the given external parameters (in our case, in the given external field).

To make this explanation more transparent, let us consider the simplest example of a single-domain uniaxial ferromagnet placed in an external field with the direction along its easy anisotropy axis. We start from a large value of this field in one direction (which we will call positive) where the sample is saturated. When we reduce the external field to zero, all atomic magnetic moments remain oriented in the same direction, because this direction corresponds to one of the two possible minima of the anisotropy energy, and hence the sample remains saturated (for simplicity, we neglect the demagnetizing field). If we now apply a relatively small external field in the opposite (negative) direction, then a global energy minimum of the system would correspond to the magnetization saturated in this negative direction (because the magnetization direction would coincide not only with one of the two easy axis directions but also with the external field). However, before we applied this small negative field the magnetization was aligned in the positive direction of the anisotropy axis. It still corresponds to a (local) energy minimum because all moments are oriented along the anisotropy axis (see Eq. (1.53) and subsequent discussion) so that in order to switch to the opposite (negative) direction, they must overcome an anisotropy energy barrier.

In this situation the sample magnetic moment will remain oriented in the positive direction until a certain negative field value is achieved where the minimum corresponding to the positive magnetization orientation vanishes. In this field, the magnetic moment will "suddenly" jump to the opposite direction and the hysteresis loop for such a sample will resemble that shown in Figure 1.22. For real systems, where different easy axes as well as structural defects and demagnetizing fields are present, the resulting hysteresis loop can be much more complicated, although the main result – the presence of hysteresis due to the existence of metastable states – remains valid.

As explained above, during the hysteresis cycle metastable states vanish (under the influence of the external field) and the system jumps into energetically lower states. Because the system energy drops after such a jump (transition), an energy

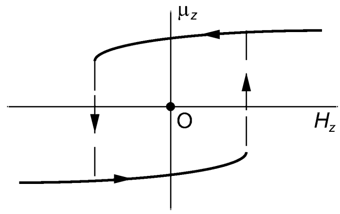

Fig. 1.22. A hysteresis loop for a Stoner–Wohlfarth particle (see text for details).

dissipation occurs when we proceed along the hysteresis loop changing the external field. The total energy loss for one complete hysteresis cycle can be calculated quite easily (Chikazumi, 1964) using general expressions for the energy of a ferromagnetic body in an external field. However, the final result can be guessed without any calculations based again on dimensionality arguments. Namely, we need some characteristics of the hysteresis loop such as that shown in Figure 1.22 which has the dimension of an energy (erg, or in magnetic quantities μH) and is dependent on the magnetization behavior during the whole remagnetization process (because energy losses can occur in principle at any point of the hysteresis loop). The simplest loop characteristic which satisfies these two conditions is its area S_{hyst} in $\mu - H$ coordinates, given by the corresponding curve integral over the complete hysteresis loop:

$$W = \oint \mu_z(H_z)\, dH_z = S_{\text{hyst}} \tag{1.65}$$

This expression, indeed, provides the correct result for the total energy W losses after one complete hysteresis cycle (such as the path $A \to B \to C \to D \to E \to F \to A$ in Fig. 1.21).

1.2.4.4
Single-Domain Particles and Superparamagnetism

Sufficiently small ferromagnetic particles possess an important property which makes them attractive for many applications: particles below a certain size are always in a so called single-domain state (Kneller, 1966; Landau and Lifshitz, 1975a). This means that the whole particle volume is occupied by a single magnetic domain – that is, all atomic magnetic moments of such a particle are aligned parallel to each other (homogeneous magnetization state).

The main reasons for the transition to a single-domain state for very small particles are: (1) the demagnetizing energy which favors closed (noncollinear) magnetization configuration decreases when the particle size decreases; whereas (2) the exchange energy of a nonhomogeneous magnetization configuration increases when the size of such a configuration (which cannot be larger than a particle size) decreases. This means that below a certain particle size, the homogeneous (single-domain) particle magnetization state which has a relatively large demagnetizing energy but very low exchange energy is energetically more favorable than some closed (multi-domain) magnetization configuration with low demagnetizing but large exchange energy.

To estimate the critical size below which a particle should be single-domain (Landau and Lifshitz, 1975a), let us study the size dependence of the energy contributions mentioned above. The demagnetizing energy of a single-domain state can be easily estimated using Eq. (1.60). The demagnetizing field $\mathbf{H}_{\text{dem}}$ inside a homogeneously magnetized spherical particle with the saturation magnetization $\mathbf{M}$ is $\mathbf{H}_{\text{dem}} = -4\pi\mathbf{M}/3$ (Kittel, 1986) which, according to Eq. (1.60), leads to the demagnetizing energy of the order $E_{\text{dem}} \sim M^2 V$, where V is the particle volume. The exchange energy density of a nonhomogeneous magnetization configuration inside a particle if large ($\sim M$) magnetization changes occur at the length scale of the particle size a is $e_{\text{exch}} \sim \alpha M^2/a^2$ (see Eq. 1.50), so that the total exchange energy is $E_{\text{exch}} \sim e_{\text{exch}} V \sim (\alpha M^2/a^2) V$ (and only large magnetization rotations leading to closed magnetization configurations can provide substantial decrease of the demagnetizing energy).

The particle "prefers" a single-domain state if the corresponding demagnetizing energy is less than the exchange energy of a closed magnetization state inside a particle: $E_{\text{dem}} < E_{\text{exch}}$, or $M^2 V < (\alpha M^2/a^2) V$. This means that the particle is in a single-domain state if its size is less than $a_{\text{cr}} \sim \sqrt{\alpha}$. For materials with a large magnetic anisotropy constant K, one should also take into account the anisotropy energy of a nonhomogeneous magnetization configuration which leads to the estimate $a_{\text{cr}} \sim \sqrt{\alpha K/M^2}$. The critical sizes for common ferromagnetic materials such as Fe or Ni are $\sim 10 \ldots 100$ nm.

The energy calculation for a single-domain particle can be greatly simplified. First, its magnetization configuration can be described by a single vector μ of its total dipole magnetic moment the magnitude of which is simply proportional to the particle volume: $\mu = M_s V$ (M_s is the saturation magnetization of the particle material). The exchange energy (see Eq. 1.50) for the homogeneous magnetization configuration is $E_{\text{exch}} = 0$. Further, for a spherical particle its demagnetizing energy does not depend on the moment orientation and hence can be omitted as any constant in the energy expression. Thus, the particle energy can be written as a sum of its magnetic moment energy in the external field $\mathbf{H}_0$ (see Eq. 1.37) and its magnetic anisotropy energy (see, e.g., Eq. 1.53):

$$E = -\mathbf{m}\mathbf{H}_0 M_s V + KV \sin^2(\mathbf{nm}) \tag{1.66}$$

Here, we have used the unit vector $\mathbf{m}$ along the particle magnetic moment and the unit vector $\mathbf{n}$ along the particle anisotropy axis.

The system of uniaxial single-domain particles each of which possesses the energy (Eq. 1.66) is known as a Stoner–Wohlfarth model (Stoner and Wohlfarth, 1948), and is widely used in fine magnetic particle theory due to its (apparently) simple properties. One of its most important features is the existence of a magnetization hysteresis in a collection of such particles (see the corresponding explanation in Section 1.2.4.3). Due to the simple energy expression (Eq. 1.66) for a single particle, many magnetic characteristics of the noninteracting Stoner–Wohlfarth model such as initial susceptibility, permanent magnetization and coercive force can be computed either analytically or by very transparent numerical calculations (Kneller, 1966).

Another interesting property of fine particle systems is that, above a certain temperature T_{sp}, such a system behaves like a paramagnetic body, although T_{sp} is still much lower than the Curie point T_c for the corresponding ferromagnetic material. To understand this behavior (Kneller, 1966), let us consider a system of uniaxial particles with the energies described by Eq. (1.66).

In the absence of an external field the magnetic moment of each particle has two equivalent equilibrium positions (states) along two opposite directions of the anisotropy axis. These states are separated by the energy barrier, with height equal to the maximal possible anisotropy energy: $E_{an}^{max} = KV$. If the system temperature is sufficiently low such that the thermal activation energy kT is much less than this barrier height, then the magnetic moment of each particle will (almost) forever stay in one of these two states depending on the previous system history (i.e., in which direction a strong external field was applied, say, several years ago). However, for sufficiently high temperatures $T > T_{sp}$, thermal transitions between the two equilibrium states may occur on the observable time scale so that after some time each moment can be found with equal probabilities in one of these two states.

For such temperatures the total magnetic moment of a fine particle system in the absence of an external field is zero (as for paramagnetic and diamagnetic substances), because each moment can be oriented with equal probabilities in two opposite directions. When a small external field is applied, then the moment orientation along that direction of the anisotropy axis that has the smallest angle with this field is preferred and the system demonstrates a net average magnetization along the applied field (as usual paramagnets do). However, the magnetic susceptibility (which characterizes the system response to the applied field) for such a system of fine ferromagnetic particles is about $10^4 \ldots 10^6$ times larger than for usual paramagnetic materials because the moment of small particles which now play the role of single atoms (molecules) of a paramagnet is much larger than any atomic or molecular magnetic moment. For this reason, the behavior of a fine particle system for $T > T_{sp}$ is known as *superparamagnetism*.

To estimate the temperature of this superparamagnetic transition T_{sp} (which is also known as a blocking temperature T_{bl}), we are reminded that, according to the Arrhenius law, for a system with the temperature T the average transition time between two states separated by an energy barrier ΔE is $t_{tr} \sim \tau_0 \exp(\Delta E/kT)$, k being the Boltzmann constant, $\Delta E \sim KV$ (see above). The prefactor τ_0 should be measured experimentally, and for magnetic phenomena under study is about $\tau_0 \sim 10^{-9}$ s. To observe the superparamagnetic behavior, the observation time t_{obs} should be at least of the same order of magnitude as t_{tr}, which leads to the relationship $t_{obs} \geq \tau_0 \exp(\Delta E/kT)$. Hence, for the given observation time t_{obs} the blocking temperature can be estimated as $T_{sp} \sim KV/\ln(t_{obs}/\tau_0)$. The corresponding value, for example for iron particles of size $\sim$10 nm and an observation time $t_{obs} = 1$ is $T_{sp} \sim 10^2$ K.

In real fine particle systems the transition to a superparamagnetic state with increasing temperature occurs gradually due to the always-present particle size and shape distribution. These distributions lead to a spread of the energy barrier heights, and this results in different transition temperatures for different particles.

1.2.4.5 Irreversible Magnetic Relaxation

The magnetic moment of a ferromagnetic system magnetized in an external field and then left on its own often changes with time. This is one of the manifestations of the so-called "irreversible magnetic relaxation phenomena", which inevitably occur at finite temperatures in any magnetic system which is not in a thermal equilibrium state.

A typical experiment to observe irreversible magnetic relaxation is as follows: a system is magnetized in an external field so that it acquires some total magnetic moment in the direction of this field. The magnitude and (or) the direction of the applied field is then suddenly changed and the time dependence of the system magnetic moment is measured. For a wide class of magnetic systems (magnetic powders, some alloys, thin films, etc.) such measurements provide a nontrivial result: magnetization relaxation is not exponential ($\mu_z \sim \exp(-t/\tau_c)$, which one would expect for the thermal relaxation of a system over an energy barrier) but rather can be described by a linear–logarithmic dependence

$$\mu_z = \mu_0 - S \ln\left(\frac{t}{t_0}\right) \quad \text{or} \quad \frac{d\mu_z}{d(\ln t)} = -S(= Const) \tag{1.67}$$

where the coefficient S is called *magnetic viscosity*. This linear–logarithmic dependence fits in many cases experimental data measured over many time decades from seconds to years (!) quite well. Such relaxation is called "anomalous" in order to distinguish it from the simple exponential relaxation.

The first phenomenological explanation of this phenomenon for magnetic systems was provided by Street and Wooley (1949). These authors suggested that such an unusual (at that time) relaxation behavior was due to the wide distributions of the energy barrier heights in the system under study. To understand why such a distribution leads to the linear–logarithmic behavior, let us consider the simplest model, namely a system of noninteracting magnetic particles each of which has two equilibrium magnetization states separated by the energy barrier E. We assume that the height of this barrier changes from particle to particle, and that the fraction of particles dN with the energy barriers in the small interval from E to $E + dE$ is $dN = \rho(E)\, dE$ (in this case $\rho(E)$ is called the distribution density of the energy barriers).

The irreversible magnetization relaxation for particles with the given energy barrier E is given by the simple exponential Arrhenius law mentioned above: $\mu_z \sim \exp(-t/\tau_c)$. This means that for each such particle the probability $p(t)$ to jump over the barrier during the time t is given by $p(t) = 1 - \exp(-t/\tau_c)$. The relaxation time τ_c also exhibits an exponential dependence on the barrier height: $\tau_c = \tau_0 \exp(E/kT)$ where the prefactor is about $\tau_0 \sim 10^{-9}$ s (see the discussion of the superparamagnetic phenomena given above).

For the observation time t, all particles with the relaxation time $\tau_c \ll t$, had already relaxed almost surely ($t/\tau_c \gg 1$, $\exp(-t/\tau_c) \approx 0$ probability to jump $p(t) \approx 1$), so that their relaxation can no longer be observed. The particles with much larger relaxation times $t/\tau_c \ll 1$ are still not yet relaxed almost surely ($t/\tau_c \ll 1$, $\exp(-t/\tau_c) \approx 1$, probability to jump $p(t) \approx 0$), hence, their relaxation could no longer be observed either. Thus, the only particles whose relaxation we measure at the observation time t are those with the relaxation time $\tau_c (= \tau_0 \exp(E/kT)) \sim t$ – that is, with the energy barriers $E \approx E_c = kT \ln(t/\tau_0)$. Due to the very strong (exponential!) dependence of the relaxation time on the energy barrier height E, only those particles with barriers in a narrow interval $\Delta E \sim kT$ around the so-called *critical energy* $E_c(t) = kT \ln(t/\tau_0)$ make any substantial contribution to the magnetic relaxation, observed at time t (see Fig. 1.23a), where the probability to jump – that is, the probability $P(E)$ to overcome the energy barrier – is shown as a function of the barrier height E. In other words, it is a very good approximation to treat the critical energy E_c, as the boundary between the already relaxed and not yet relaxed particles (Fig. 1.23b).

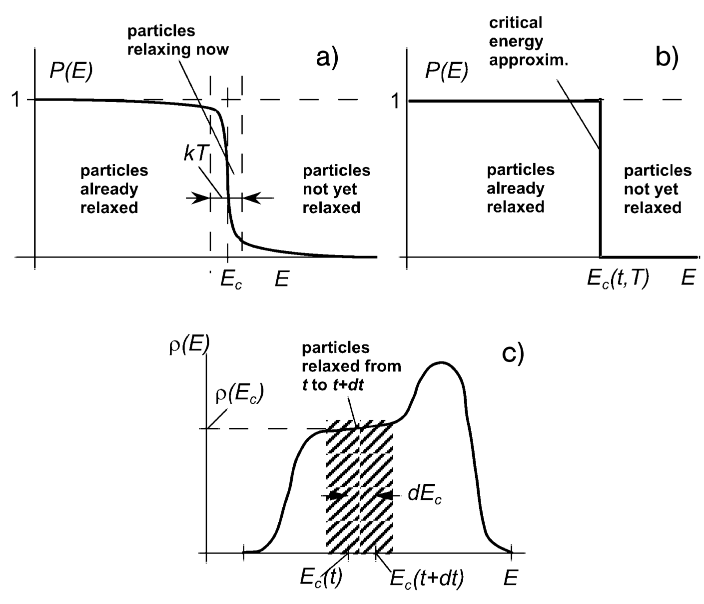

Fig. 1.23. An explanation of the linear–logarithmic time dependence of the magnetization.

The width of the energy barrier distribution $\rho(E)$ is normally much larger than the thermal energy kT. For this reason, inside the mentioned small interval $\Delta E \sim kT$ around E_c this distribution can be treated as constant. Hence, the number of relaxed particles dn (and the magnetic moment decrease $-d\mu_z$) in the time interval from t to $t + dt$ can be calculated as the product of the corresponding value $\rho(E_c)$ and the shift of the critical energy dE_c during this interval (see Fig. 1.23c; we recall that the critical energy separates relaxed and nonrelaxed particles): $d\mu_z \approx -\rho(E_c)\, dE_c(t)$. Substituting in this relationship the time dependence of the critical energy $E_c(t) = kT \ln(t/\tau_0)$, so that $dE_c(t) = kT\; dt/t$, we obtain

$$d\mu_z \approx -kT\rho(E_c)\; dt/t \quad \text{or}$$

$$t\frac{d\mu_z}{dt} = \frac{d\mu_z}{d \ln t} \approx -kT\rho(E_c) \tag{1.68}$$

which coincides with Eq. (1.67) if we set $S = kT\rho(E_c)$. This means that the magnetic viscosity is simply proportional to the value of the energy barrier distribution density for the critical energy E_c. Generally speaking, this depends on the observation time due to the time-dependence of $E_c \sim \ln t$. However, since this dependence is very weak (logarithmic), it can be neglected for nonpathological barrier densities $\rho(E)$, and this leads to the almost constant magnetic viscosity (Eq. 1.67) observed experimentally.

1.2.4.6
Reconstruction of Magnetization Distribution Inside a Body from Magnetic Field Measurements

In this last section we turn our attention to a question which, despite being somewhat aside from the main route, is extremely important for applications of magnetism in many areas, and especially in medicine. The question relates to the possibility of reconstructing the complete magnetization configuration inside a body, based on measurements of the magnetic field outside it. If this were possible, then we would obtain a powerful tool for studying areas such as current distribution inside a human brain, This in turn would lead to immense progress not only in the diagnosis of a variety of diseases but also to an understanding of how the human brain functions.

From a mathematical viewpoint, we are looking for the solution of an integral Eq. (1.27): provided that the magnetic potential $\mathbf{A}(r_0)$ outside a body is known everywhere, could we reconstruct the current distribution $\mathbf{j}(\mathbf{r})$ inside this body? This would be the same as that of reconstructing the magnetization distribution $\mathbf{M}(\mathbf{r})$ from the magnetic field measurements because $\mathbf{M}(\mathbf{r})$ can be calculated from the known current distribution using the relationship $\operatorname{rot} \mathbf{M}(\mathbf{r}) = \mathbf{j}/c$ (see Section 1.2.2.3) whereby the field can be found as $\mathbf{H} = \operatorname{rot} \mathbf{A}$.

The problem described above is known as an inverse problem of potential theory (Romanov, 1987) and, unfortunately, cannot be solved uniquely in general. To demonstrate this fact, we consider first the corresponding problem in electrostatics, namely the reconstruction of charge distribution inside a body from electric field measurements outside it. To show the nonuniqueness of the solution we turn our attention to a simple example, the electric field outside a sphere which carries a total charge Q. It is a well-known text-book result that while the charge distribution inside the sphere remains spherically symmetrical, the field outside the sphere is given by $E(\mathbf{r}) = Q/r$. Hence, this field is exactly the same if, for example: (1) there is a point charge Q in the sphere center; or (2) if the same total charge Q is uniformly distributed on the sphere's surface. Moreover, there is no way to determine the real charge distribution in a sphere unless the field inside the sphere can be measured.

The mathematical reason why such a reconstruction fails is that outside the charged bodies the electric potential satisfies the Laplace Equation $\Delta\phi = 0$ (ϕ is a harmonic function). For such functions a so-called Dirichlet problem can be formulated: find a solution of the Laplace Equation $\Delta\phi = 0$ outside some closed region Ω that satisfies some reasonable boundary condition on the surface S of Ω ($\phi(\mathbf{r} \in S) = f(\mathbf{r})$) and vanishes at the infinity. It can be shown that the solution of this problem is unique – that is, the values of the potential $\phi(\mathbf{r})$ in the whole space outside some closed surface S can be found if we know its values on this surface $\phi(\mathbf{r} \in S)$. Hence, when measuring the potential (or the field) outside a charged body we have at our disposal actually only two-dimensional (2D) information (ϕ-values on any closed *surface* surrounding a body), which is clearly insufficient to reconstruct a three-dimensional (3D) *volume* charge distribution inside the body.

The same arguments are valid for the magnetic vector potential. Indeed, this potential satisfies the vector Poisson Equation (1.26) which outside a system of currents (or magnetic samples) transforms into the vector Laplace Equation $\Delta\mathbf{A} = 0$. Again, according to the same solution of the Dirichlet problem, the values of the vector potential in the outer space are completely determined by its values on some closed surface surrounding a system under study, so there is no way to obtain more than 2D information and hence reconstruct a (generally) 3D current or magnetization distribution inside the system.

Although this is a disappointing answer in general, there exist several particular problems for which additional information about the current (magnetization) distribution is available, such that a reconstruction becomes possible. First, in the simplest case when the magnetic field is known to be created by a single point-like dipole, it is possible to reconstruct its position and the magnitude and orientation of its magnetic moment. In principle, such a reconstruction is possible for any given finite number of dipoles, but in practice its reliability falls rapidly when this number increases.

Another tractable case is when some symmetry properties of the magnetization distribution to be reconstructed are known in advance. If, for example, we know that the magnetization inside a finite cylinder is distributed in axially symmetrical fashion and we can measure the magnetic field on some closed surface surrounding this cylinder, then the reconstruction is (in principle) possible.

In concluding this discussion, we would like to mention that, apart from the principal difficulties demonstrated above, the solution of the Fredholm integral equations of the 1st type

$$f(x) = \int K(x, y) g(y)\, dy$$

(one must solve for $g(y)$ if the left-hand function $f(x)$ and the integral kernel $K(x, y)$ are known) is the so-called ''ill-conditioned problem'' (Press et al., 1992) in the Hadamard sense. This means, that small errors in the experimental data (represented here by $f(x)$) can cause arbitrary large deviations in the solution if no special precautions (the so-called ''regularization techniques'') are taken. However, this very interesting topic cannot be discuss at this point, and interested readers are referred to literature references in Press et al. (1992).

Appendix

In this Appendix, the most important expressions from this chapter are listed in Gauss units (left column) and SI units (right column) units. If expressions in SI and Gauss systems coincide, only one formula is presented. The values of the SI constants ε_0 and μ_0 appearing in these expressions are

$$\varepsilon_0 = 10^7/(4\pi c^2 \approx 8.854 \times 10^{-12} \text{ Farad m}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry m}^{-1}$$

$$\text{rot } \mathbf{e} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} \qquad \text{rot } \mathbf{e} = -\frac{\partial \mathbf{h}}{\partial t} \tag{A.1}$$

$$\text{rot } \mathbf{h} = \frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} + \frac{4\pi}{c}\mathbf{j} \qquad \text{rot } \mathbf{h} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{e}}{\partial t} + \mu_0 \mathbf{j} \tag{A.2}$$

$$\text{div } \mathbf{e} = 4\pi\rho \qquad \text{div } \mathbf{e} = \frac{\rho}{\varepsilon_0} \tag{A.3}$$

$$\text{div } \mathbf{h} = 0 \tag{A.4}$$

$$\oint_L \mathbf{e}\, d\mathbf{l} = -\frac{1}{c}\frac{\partial \Phi_h}{\partial t} \qquad \oint_L \mathbf{e}\, d\mathbf{l} = -\frac{\partial \Phi_h}{\partial t} \tag{A.8}$$

$$\oint_L \mathbf{h}\, d\mathbf{l} = \frac{1}{c}\frac{\partial \Phi_e}{\partial t} + \frac{4\pi}{c} J_S \qquad \oint_L \mathbf{h}\, d\mathbf{l} = \mu_0 \varepsilon_0 \frac{\partial \Phi_e}{\partial t} + \mu_0 J_S \tag{A.9}$$

$$\oint_S \mathbf{e}\, d\mathbf{S} = 4\pi Q \qquad \oint_S \mathbf{e}\, d\mathbf{S} = \frac{Q}{\varepsilon_0} \tag{A.12}$$

$$\oint_S \mathbf{h}\, d\mathbf{S} = 0 \tag{A.13}$$

$$\phi(\mathbf{r}_0) = \int_V \frac{\rho(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|} dV \qquad \phi(\mathbf{r}_0) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|} dV \tag{A.20}$$

$$\mathbf{e}_{\text{dip}} = \frac{3\mathbf{r}_0(\mathbf{d}\mathbf{r}_0)}{r_0^5} - \frac{\mathbf{d}}{r_0^3} \qquad \mathbf{e}_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\mathbf{r}_0(\mathbf{d}\mathbf{r}_0)}{r_0^5} - \frac{\mathbf{d}}{r_0^3}\right] \tag{A.24}$$

$$\mathbf{A}(\mathbf{r}_0) = \frac{1}{c}\int_V \frac{\mathbf{j}(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|} dV \qquad \mathbf{A}(\mathbf{r}_0) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{j}(\mathbf{r})}{|\mathbf{r}_0 - \mathbf{r}|} dV \tag{A.27}$$

$$\mathbf{h}_{\text{dip}} = \frac{3\mathbf{r}_0(\boldsymbol{\mu}\mathbf{r}_0)}{r_0^5} - \frac{\mathbf{d}}{r_0^3} \qquad \mathbf{h}_{\text{dip}} = \frac{\mu_0}{4\pi}\left[\frac{3\mathbf{r}_0(\boldsymbol{\mu}\mathbf{r}_0)}{r_0^5} - \frac{\boldsymbol{\mu}}{r_0^3}\right] \tag{A.34}$$

$$\begin{aligned} &\operatorname{div} \mathbf{B} = 0 \qquad && \operatorname{div} \mathbf{B} = 0 \\ &\operatorname{rot} \mathbf{H} = 0 \qquad && \operatorname{rot} \mathbf{H} = 0 \end{aligned} \tag{A.47}$$

$$\begin{aligned} &\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \qquad && \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \\ &\mathbf{B} = \mu\mathbf{H} \qquad && \mathbf{B} = \mu\mu_0\mathbf{H} \\ &\mathbf{M} = \chi\mathbf{H} \qquad && \mathbf{M} = \chi\mathbf{H} \end{aligned} \tag{A.48}$$

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1.3 Creating and Measuring Magnetic Fields

Wilfried Andrä and Hannes Nowak

1.3.1 Introduction

The scientific treatment of magnetism in medicine is inseparably connected with both the generation of magnetic fields as well as with their measurement. However, before going into details of this topic it is necessary to provide a definition for the unit of magnetic field strength which will be used throughout the chapters of this book. Unfortunately, there is considerable confusion even in the technical literature on magnetism and, of course, also in the specialist medical literature. Very often, the terms "magnetic field" (H) and "magnetic flux density" or "magnetic induction" (B) are confused, though the meaning of these two terms is quite different, as explained in Section 1.2 where the relationship between H and B is provided in Eq. (1.43).

This mixing of magnetic quantities has, in general, no serious consequences for the reader, however. More important is to have simple rules of conversion in order to transform units of one system into units of another system. The two systems most often used are the "International System of Units" (SI) and the so-called Gaussian system or cgs system. The latter is often used in American publications, whereas in the following chapters of this book the SI system is applied according to the recommendations of the International Organization for Standardization.

In order to convert Gaussian units into SI units, the number of Gaussian units must be multiplied by conversion factors that are listed in Table 1.1. For example, 1 Gauss (G) corresponds to 10^{-4} Tesla (T).

In the following paragraph, typical values of field strength are given in SI units along with the corresponding cgs units in parentheses.

1.3.2 The Generation of Magnetic Fields

Magnetism in medicine has often to deal with magnetic fields that exist naturally. One well-known example is that of the Earth's magnetic field, H_{ea}, the actual am-

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Table 1.1. The relationship between gaussian and SI units.

Quantity	Symbol	Gaussian unit	Conversion factor	SI unit
Magnetic field strength	H	Oerstedt (Oe)	$10^3/4\pi$	A m^{-1}
Magnetization	M	Gauss (G)	10^3	A m^{-1}
Magnetic T polarization	J	Gauss (G)	10^{-4}	Vs m^{-2} = T
Magnetic T induction	B	Gauss (G)	10^{-4}	Vs m^{-2} = T
Susceptibility = M/H	χ	1	4π	1
Using Gaussian units		$B = H + 4\pi M$		
Using SI units		$B = \mu o(H + M)$, with $\mu o = 4\pi 10^{-7}$ V·s A·m^{-1}		

plitude of which is about 40 A m^{-1} (0.5 Oe). In many reports the corresponding flux density $B_{ea} = \mu_0 H_{ea}$ is given ($\mu_0 = 4\pi \times 10^{-7}$ Vs Am^{-1} is the permeability of the free space) with a magnitude of about 5×10^{-5} Vs m^{-2} (0.5 G). Especially in reports dealing with very weak or very strong magnetic fields, the unit Vs m^{-2} is usually replaced by the abbreviation T (= Tesla).

A second type of magnetic field, generated by natural processes, forms the central subject of biomagnetism. The sources of these fields are small electric currents within the human body, giving rise to extremely weak magnetic fields outside the body (this subject is described in more detail in Chapter 2). The corresponding field strength is at maximum of the order of $H_{bio} \approx 10^{-4}$ A m^{-1} (1.25×10^{-6} Oe), and the flux density $B_{bio} \approx 1.25 \times 10^{-10}$ T (1.25×10^{-6} G).

Artificially produced magnetic fields are required in many medical methods. Their magnitudes range approximately from less than 1% of the Earth's magnetic field up to more than 4×10^6 A m^{-1} (5×10^4 Oe). The weak as well as the extremely strong fields are usually generated by electric currents flowing in suitably designed wires. Large-sized solenoids are used in magnetic resonance tomography (MRT). In order to produce the strong constant magnetic fields required for this technique, very high currents must flow for a very long time. This demand can be met by using superconducting wires (see Section 3.2), while in other cases the wire is replaced by copper tubes designed for effective water cooling. The field generated in the center of a long solenoid can easily be estimated using Eq. (1.69):

$$H = iN/L \tag{1.69}$$

where i is the current, N is the number of windings and L denotes the length of the coil. The field on the axis of a solenoid as shown in Figure 1.24 is exactly parallel to the coil axis, and can be calculated using the following equation:

$$H(x)/j = (1/2) \cdot \left[UP \cdot \ln \frac{R_a + \sqrt{R_a^2 + UP^2}}{R_i + \sqrt{R_i^2 + UP^2}} - UM \cdot \ln \frac{R_a + \sqrt{R_a^2 + UM^2}}{R_i + \sqrt{R_i^2 + UM^2}} \right] \tag{1.70}$$

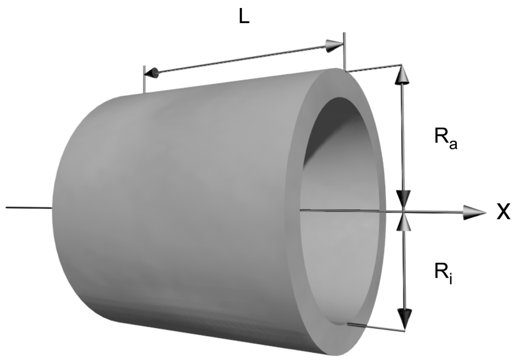

Fig. 1.24. A solenoid for the generation of magnetic fields.

with $UP = (x + L/2)$ and $UM = (x - L/2)$; x is the distance from the coil center; j is given by the Equation $j = i/F$, where i is the total current flowing through the cross-section $F = (R_a - R_i) \cdot L$ of the solenoid. The natures of R_a, R_i, and L are explained in Figure 1.24. The calculation of field strength for points outside the coil axis is more complicated and beyond the scope of this book, but interested readers are referred to specialized literature (e.g., Smythe, 1989). In general, the field can be calculated as a sum of contributions generated by circular currents of different radius and axis position (Landau and Lifschitz, 1967). The final equations are complicated, and normally the calculation of required field strengths is performed by numerical computation. Analytical formulas for long solenoids and locations near the axis have been provided by Jackson (1962). It should be pointed out that off-axis fields are in general oblique to the axis. Coils similar to that shown in Figure 1.24 are used for magnetic stereotaxis as well as for many other applications. In order to achieve different geometrical field distributions, the construction of the current-carrying wires must be correctly chosen. A typical example for such design was provided by Meeker et al. (1996) in a report detailing magnetic stereotaxis. The general considerations on the construction of air-core solenoids, including mechanical problems and problems of cooling, are treated in detail by Zijlstra (1967a).

Fields of medium strength are generated primarily by means of electromagnets, many different types of which have been described in the literature. The constructions are usually designed according to the special application with the aim to achieve the desired field strength (and also a desired field distribution) with a minimum of both electric power consumption and weight. The general advantage of electromagnets is that it is possible to concentrate the field at a certain region with the electric currents flowing in a remote region. However, the ability of magnetic materials to "conduct" magnetic fields in a similar way as copper conducts the electric current is restricted. The principle of an electromagnet is shown in Figure 1.25.

Some examples of electromagnets have been described in Section 1.1. One of these was the so-called "giant magnet" of Dr. Haab, which was designed to remove

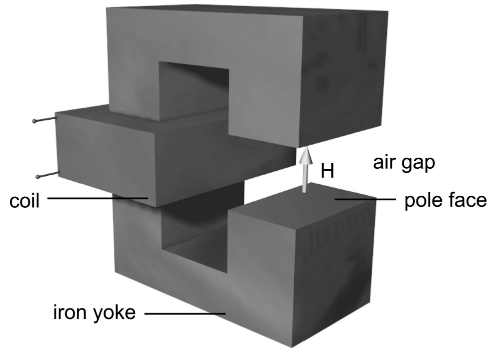

Fig. 1.25. Schematic representation of an electromagnet. The main part of the magnetic flux generated by the coil is conducted through the iron yoke to the air gap.

magnetic objects from the eye (see Fig. 1.4). Clearly, this construction appears quite different to the scheme of Figure 1.25, due mainly to the fact that in Dr. Haab's equipment the yoke is degenerated to a sphere-shaped iron body in order to produce an extremely inhomogeneous magnetic field just at the tip of the magnet. Other examples of rather large electromagnets which are more similar to Figure 1.25 are shown in Figure 3.20 (see Section 3.2). These are designed for MRT, and have flux densities of about 0.3 T and 1 T, respectively. In Figure 3.20 the magnetic core is hidden behind a casing, whereas the air gap with the bed can be distinguished. In principle, the field in the air gap can be calculated, though there is no simple relationship similar to Eq. (1.70). In the case of a comparably narrow air gap the field strength, H_{air}, can be roughly estimated by:

$$H_{\mathrm{air}} = \frac{N \cdot i - H_{\mathrm{fe}} L_{\mathrm{fe}}}{L_{\mathrm{air}}} \tag{1.71}$$

where $N \cdot i$ is the product of the current and the number of windings in the coil. L_{fe} and L_{air} are the path lengths of the field along the yoke and the air gap, respectively. The value of the field strength inside the iron yoke, H_{fe}, however, depends on several parameters, including the magnetic properties of the yoke material as a function of $N \cdot i$ and the actual geometry of the magnet. In most cases the yoke material is not strongly magnetized. Then, if L_{fe} is small compared to L_{air}, the term $H_{\mathrm{fe}} \cdot L_{\mathrm{fe}}$ in Eq. (1.71) may be neglected. More detailed calculations are complicated. In particular, the so-called demagnetizing effects give rise to serious corrections of Eq. (1.71). Demagnetizing relates to the influence of magnetic fields (caused by magnetic poles on the surfaces or inside the yoke) on the magnetization of the yoke material. In this regard, the interested reader is referred to specialist publications on magnetic problems (e.g., Kneller, 1962; Zijlstra, 1967a), wherein the general design of electromagnetic circuits is described.

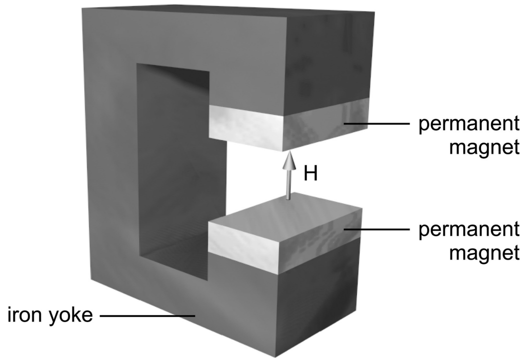

Fig. 1.26. Scheme of a permanent magnet circuit. Without the yoke there would be poles at the back sides of the permanent magnet pieces, giving rise to a stray field opposite to the main field in the air gap.

Cases also exist where permanent magnets are used, and these offer the great advantage of being independent of electric power. On the other hand, the field strength cannot be varied without complicated additional equipment. Permanent magnets might be used preferably in cases where it is not possible to connect to a mains electricity supply, for example during first aid treatment in the case of a road traffic accident (Paliege et al., 1979). More recently, a small permanent magnet was successfully used to remove intraocular ferromagnetic foreign bodies from the eye (Kuhn and Heimann, 1991). The construction of permanent magnet circuits that are properly designed for special applications depends essentially on both the magnetic material as well as the type of application. In this respect, major progress has been made in the development of permanent magnetic materials, and correspondingly the design of equipment has improved considerably during the past few decades. Further details on this subject may be found in specialized books (e.g., McCaig and Clegg, 1987). The scheme of a typical permanent magnet circuit with a yoke for flux closure is shown in Figure 1.26. For configurations similar to this figure, the upper limit of the field strength in the air gap between rectangular pole faces can be estimated by:

$$H_{\text{air}}/M_r = 8 \arctan \frac{L \cdot W}{D\sqrt{L^2 + W^2 + D^2}} \tag{1.72}$$

where D is the width of the air gap, L and W are the edge lengths of the pole face, and M_r is the magnetization of the permanent-magnet pieces.

Recently, several cases have been reported where small permanent magnets (e.g., spheres) were applied for the magnetic monitoring of capsules inside the gastrointestinal tract (see Section 4.2). One special advantage of these so-called markers is the simple mathematical formulae of the magnetic field around these magnets which can (in a good approximation) be described as a dipole field (see Section 1.2).

1.3.3 The Measurement of Magnetic Fields

The application of magnetism to medicine covers a rather large range of fields, and as a consequence a great variety of methods has been developed suitable for the measurement of magnetic field strength. However, within the scope of this book only those methods significant to medical applications will be treated in detail. Each method offers advantages as well as drawbacks which, in general, are more or less pronounced for certain ranges of field strength. Therefore, the instruments used must be chosen according to the field range of the specific application in order to provide optimum sensitivity. Whilst a number of different definitions of the term ''sensitivity'' have been reported in the literature, within the context of this chapter the term ''detection limit'' is preferred. This denotes the lowest value of field strength which can be reliably detected. Of course the meaning of ''reliably'' must also be defined, with one condition being that the signal:noise ratio is greater than 2. A compilation of typical field ranges, together with some selected principles of measurement, is provided in Table 1.2. Many more methods and corresponding instruments – especially for applications in other technical fields – are available, though only few of these have been used for medical techniques. The reason for this is that, in many cases, the detection limit does not meet the corresponding demand, while in other cases the handling may be too complicated. On occasion, potential users may not be familiar with the respective method, and consequently only about five principles of the methods listed in Table 1.2 are usually applied in medicine.

For many years, the primary position with regard to the greatest sensitivity or lowest value of detection limit was held by the SQUID principle; this type of field sensor is described in detail in Section 2.2. More recently, the atomic magnetometer was developed with slightly higher sensitivity (Kominis et al., 2003; Schwindt et al., 2004). In addition, the optical pumped magnetometer could be used to map the human cardiomagnetic field (Bison et al., 2003). This new measuring principle is based on the detection of the so-called Larmor-spin precession of atoms which are excited by optical radiation. The sensitivity of the technique is essentially determined by the relaxation time which passes until the increased energy of the atoms is delivered to the surroundings.

The secondary position with regard to detection limit is held by three principles. The first of these, nuclear precession magnetometry, is used to detect small local variations of an otherwise strong constant magnetic field; details of the basic principle are provided in Section 3.2. The other principles are rotating-coil magnetometry, which is not used widely in medicine, and flux-gate magnetometers, which are used in different fields of application.

Table 1.2. Examples of field ranges and the corresponding measuring principles. The detection limits are given for quasistatic fields.

Field source	$\mu_0 H$ [T]	Measuring principle	Detection limit [T]
Evoked human brain activity	$\leq 10^{-13}$	Atomic magnetometer	$\leq 10^{-15}$
		SQUID	10^{-15}
		Nuclear resonance	10^{-13}
		Optical pumping	
		Torsion magnetometer	
Spontaneous currents in the human brain	10^{-12}	Nuclear precision magnetometer	$\leq 10^{-11}$
Currents of the human heart	$\leq 10^{-10}$	Flux-gate	$\leq 10^{-11}$
		Rotating coil	10^{-11}
		Magnetoresistivity	10^{-10}
Contamination of lunge and stomach	$\leq 10^{-9}$	Hall sensor	10^{-9}
Liver iron	$\leq 10^{-8}$	Magneto-optical sensor	10^{-7}
Magnetic markers	$\leq 10^{-6}$	Magnetotransistor	$\leq 10^{-5}$
Earth's magnetic field	$\geq 10^{-5}$	Magnetodiode	10^{-5}

The operating scheme of a flux-gate magnetometer is shown in Figure 1.27. The unit consists essentially of two magnetic cores (a and b) which are periodically magnetized by an alternating current i in the primary coils (1a and 1b) with a frequency ν. The periodically varying magnetization of the cores induces voltages in the two induction coils (2a and 2b). Without any external field these voltages cancel because they are electrically connected against each other. An external field H, however, leads to a different deformation of the induced voltages as functions of time in the two induction coils, thereby yielding a residual signal with the main contribution of a frequency 2ν. This can be selectively amplified, thus permitting a sensitive measurement of H (Michalowsky, 1993). Very small flux-gate sensors were developed using planar technology (Vincueria et al., 1994), providing the application of a high driving frequency ν. A flux-gate magnetometer with a low detection limit was described by Hinnrichs et al. (2000).

The principle of an anisotropic magnetoresistive (AMR) field sensor (McGuire and Potter, 1975) is illustrated in Figure 1.28. The essential feature is that the electrical resistivity of magnetic materials is influenced by scattering of electrical charge carriers caused by magnetic perturbations. This scattering depends on the

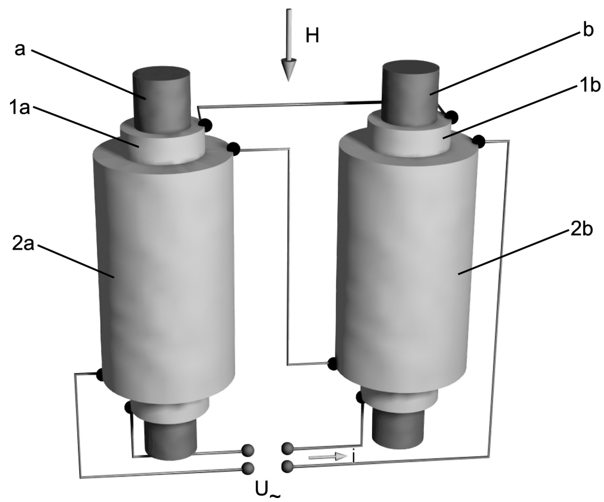

Fig. 1.27. The scheme of a flux-gate magnetometer. See text for details.

angle between the direction of current (i) and magnetization (M). In the case of AMR the magnetization is parallel to an easy axis (e.a.) inside the material, as long as no external field H is acting. Any component of H perpendicular to the easy axis induces a rotation of the magnetization out of this axis, and leads to a variation of the scattering intensity. The special oblique form of the film contacts shown in Figure 1.28 is chosen in order to assure an angle of 45° between current and magnetization for a zero external field. By using this configuration it is also possible to detect the sign of H.

Two new modifications of magnetoresistivity were investigated with the aim of developing even more sensitive field sensors. Giant magnetoresistivity (GMR) is based on the scattering of charge carriers during their transition between separated magnetic films with differently oriented magnetization (Baibich et al., 1988). Whilst AMR yields a relative alteration of the resistivity of the order of some per-

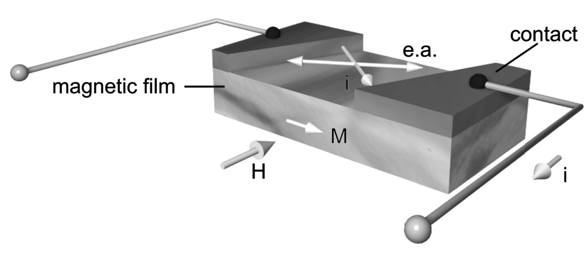

Fig. 1.28. Scheme of an anisotropic magnetoresistive element (AMR). See text for details. The easy axis (e.a.) is usually parallel to the long film edge.

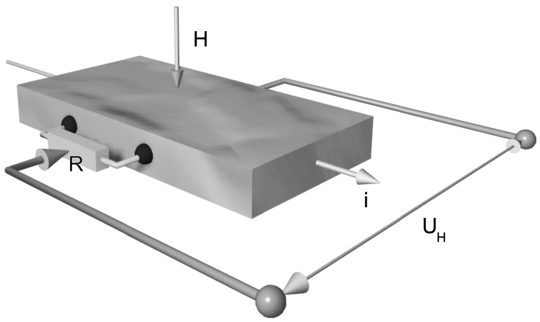

Fig. 1.29. Scheme of a Hall element. See text for details.

cent, GMR is able to change the resistivity by more than 100%. An interesting combination of GMR with a superconducting flux-to-field transformer operating at 77 K was recently described by Pannetier et al. (2004). Another effect is caused by intrinsic magnetic influence on the processes of electrical conductivity in oxide materials (von Helmolt et al., 1993). This effect has been the object of numerous investigations, and produces (relatively) a change in resistivity that is many orders of magnitude higher than that achieved with GMR. This approach is, therefore, referred to as colossal magnoresistivity (CMR). The development of suitable materials and circuits using GMR and CMR is still in progress, and consequently the ultimate detection limit for these two effects cannot yet be estimated. A combination of a superconducting flux-to-field transformer with a low-noise GMR sensor achieved a detection limit of 32×10^{-15} T (Hz)$^{-1/2}$ (Pannetier et al., 2004).

The next principle detailed in Table 1.2 is termed the Hall effect (Zijlstra, 1967b), the basic design of which is shown schematically in Figure 1.29. If a current (i) flowing through a conductor is exposed to an external field H, the charge carriers are deflected in a direction perpendicular to both current and H direction due to the so-called Lorentz force. As a consequence, a voltage U_H is generated which is proportional to H and can be measured by means of contacts across the current direction. In order to identify the correct perpendicular position of the voltage contacts, one contact is split into two and connected by a resistor (R), as shown in Figure 1.29.

Among several other types of magnetometer to have been devised is that of the magneto-optical sensor (see Table 1.2), which is of two basic types. The first type employs either the Faraday- or Kerr-effect, and by using optical fibers one of the most sensitive Faraday sensors achieved a noise level of 10^{-13} T/(Hz)$^{1/2}$ (Deeter, 1996). The second type of magneto-optical sensor transforms the distortion of magnetostrictive materials such as metallic glass, especially in combination with optical fibers, under the influence of a magnetic field. The change in optical length is measured by interference-optical means, for example with a Mach–Zehnder interferometer. Using this approach, a detection limit of 3×10^{-12} T/(Hz)$^{1/2}$ was realized in the low-frequency region (10 Hz) (Dagenais et al., 1988).

1.3.4 Discussion

The various methods used to generate magnetic fields appear to be essentially at their final stages, and few surprising new principles can be expected to be developed in the near future. However, gradual improvements may be introduced with regard to the generation of well-localized fields, either by means of small coils or miniaturized permanent magnets permitting magnetic manipulation in selected target regions.

Whilst methods of field measurement are still under development, the simplicity of operation, stability with respect to both temperature and time, and in some cases also the detection limit may all be improved in the near future. Another important aspect is the miniaturization of field sensors and their arrangement in the form of extended arrays suitable for the rapid measurement of field distributions and their time dependence around the entire human body or in the major organs.

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1.4
Safety Aspects of Magnetic Fields

Jürgen H. Bernhardt and Gunnar Brix

1.4.1
Introduction

In recent years there has been an increasing awareness of the possibility of hazards to health from exposure to sources of nonionizing radiation (NIR) such as industrial low- and high-frequency devices, radar and radio equipment, as well as devices used in medicine. This in turn has led to an interest in recommendations for limiting exposures to NIR (static and low-frequency electric and magnetic fields, radiofrequency fields and microwaves).

The preparation of such recommendations necessitates a critical analysis of existing knowledge on the health effects of nonionizing electromagnetic fields. Risk evaluation and guidance on protection are performed by the International Commission on Non-Ionizing Radiation Protection (ICNIRP; see www.icnirp.org), which represents the opinions of the scientific community at large and ensures liaison with other relevant international organizations such as the International Labour Organization (ILO) and the World Health Organization (WHO).

The aims of this chapter are to summarize the risks of exposure, to assess threshold values for effects injurious to health, and to present basic restrictions and reference levels for limiting exposure at the workplace. The methods of risk evaluation as it is performed by ICNIRP will be described in Section 1.4.2, while the following sections will summarize interaction mechanisms, biological effects, results of epidemiological studies, and recommendations for limiting exposures at the workplace for static magnetic fields (Section 1.4.3), for time-varying magnetic fields (Section 1.4.4), and for radiofrequency fields (Section 1.4.5). Section 1.4.6 describes the protection of patients and volunteers undergoing magnetic resonance (MR) procedures.

1.4.2
Risk Evaluation and Guidance on Protection

The general approach to protection against NIR including electromagnetic fields (EMF) is discussed in more detail in a specific document of the ICNIRP (2002).

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1.4.2.1 Evaluation Process

The evaluation process of the scientific literature performed by ICNIRP consists of three steps:

- Evaluation of single research studies in terms of their relevance to the health effects and the quality of methods used. The evaluations criteria are described in ICNIRP's statement mentioned above, and are used as guidance in this evaluation. This may result in the exclusion of some studies from the health risk assessment, or assigning different weights to studies, depending on their scientific quality.
- For each health effect evaluated, a review of all relevant information is required. At first, this review is normally performed separately for epidemiological, human laboratory, animal and *in-vitro* studies, with further separations as appropriate for the hypothesis.
- Finally, the outcomes of these steps must be combined into an overall evaluation of consistency of human, animal, and *in-vitro* data.

1.4.2.2 Development of Guidance on Protection

Guidance is based solely on scientifically established adverse health effects. Such effects are identified by the health risk assessment. In developing the guidelines, ICNIRP considers direct and indirect, acute and chronic health effects. Different adverse effects can be ranked according to the exposure level at which each becomes relevant. The critical effect is the established adverse health effect relevant at the lowest level of exposure. Protection against the critical effect means that protection is provided against all other adverse effects occurring at higher exposure levels. In principle, the ICNIRP guidelines are set to protect against critical effects, by limiting the related specific biologically effective quantity. The biologically effective quantity reflects the efficacy by which the external exposure causes a certain biological effect. This quantitative relationship between external measurable exposures and the target tissue biologically effective parameter is unique to a single-exposure condition. Reduction factors are included as a measure of caution, to account for quantitative uncertainties in the scientific database and biological variability in response. As a consequence, the guidelines will be set below the thresholds of the critical effects. There is no rigorous scientific basis for establishing reduction factors. They are not intended for compensating uncertainties in measurements performed to check compliance with exposure standards, nor do they incorporate social or political considerations, including precautionary approaches.

Restrictions on the effects of exposure based on established health effects are termed *basic restrictions*. It is the general strategy of ICNIRP to define a basic restriction in terms of the appropriate biologically effective quantity. Depending on frequency, the physical quantities used to specify the basic restrictions on exposure to EMF are current density, specific absorption rate, and power density. Protection against adverse health effects requires that the basic restrictions are not exceeded.

Additionally, reference levels of exposure are provided for comparison with measured values of physical quantities; compliance with reference values given in the guidelines will ensure compliance with basic restrictions. In general, the reference levels are more conservative than the basic restrictions because they have been developed for situations of optimum coupling conditions between the radiation or fields and the exposed person. If measured values are higher than reference levels, it does not necessarily follow that the basic restrictions have been exceeded, but a more detailed analysis is necessary to assess compliance with the basic restrictions. In some circumstances, it may be advisable to distinguish between members of the general public and individuals exposed because of or while performing their work tasks (occupational exposure).

Whereas ICNIRP provides general practical information on measurable levels that are derived from basic restrictions on exposure, it recognizes the need for further technical advice on special exposure situations. This requires physics and engineering expertise to develop practical measures to assess and/or to enable assessment of compliance with exposure guidelines. These measures include guidance on the principles and practice of measurements, design of equipment and/or shielding to reduce exposure, and, where appropriate, setting emission limits for specific types of device (see Section 1.4.6).

The ICNIRP Guidelines (1998) have been adopted by more than 40 countries worldwide. The European Union, for example, has adopted a Directive on the minimum health and safety requirement regarding the exposure of workers to the risks arising from physical agents (electromagnetic fields), which is based largely on the ICNIRP Guidelines (EU, 2004).

1.4.3 Static and Extremely Slowly Time-Varying Magnetic Fields (0 to 1 Hz)

There are numerous sources of environmental static and extremely slowly time-varying magnetic fields – both naturally occurring and man-made – to which humans are exposed. The natural magnetic field consists of one component due to the Earth acting as a permanent magnet, and several other small components which differ in characteristics and are related to such influences as solar activity and atmospheric events. In industry, in some research institutions as well as in medicine, large-magnetic field equipment is used with stray fields in a wide circumference around the equipment.

1.4.3.1 Interaction Mechanisms and Biological Bases for Limiting Exposure

For static and extremely slowly time-varying magnetic fields there are several established physical mechanisms through which the fields interact with human beings or living organisms. These have been reviewed by ICNIRP (1994, 2003), NRPB (2004), Schenck (2005) and Tenforde (1996, 2005). The following categories are the most important.

Magneto-Hydrodynamic Interactions

Static magnetic fields exert forces (called Lorentz forces) on moving electrolytes (ionic charge carriers), giving rise to induced electric fields and currents. To a close approximation, the Lorentz forces exerted on blood flowing through a cylindrical vessel gives rise to a voltage across the vessel. This magnetically induced voltage is commonly referred to as a ''blood flow potential''. Because of the angular dependence, the greatest magneto-hydrodynamic interaction between blood flow and applied magnetic field occurs when the field and flow are orthogonal, under which condition the magnitude of the induced voltage has a maximum value. Another subject of importance in the context of magnetic field safety is the magneto-hydrodynamic slowing of blood flow (see below).

Magneto-Mechanical Interactions

In a homogeneous magnetic field, certain diamagnetic and paramagnetic molecules experience a torque (or force) that tends to orientate them in a way such that the intrinsic magnetic moment is aligned parallel to the external magnetic field. Normally, intense magnetic fields exceeding 5 T are necessary to align molecular aggregates with diamagnetic anisotropy. The lowest thresholds which were reported from studies on retinal rods and sickled erythrocytes were approximately 100 mT.

Magnetic fields also produce a net force on ferromagnetic materials in the body. Some special organisms, in which magnetic particles are present (i.e., bacteria), use this mechanism for orientation within the Earth's magnetic field.

The occurrence of magnetically induced changes in enzyme structure, leading to altered metabolic reaction rates, has also been proposed. However, energy considerations suggest that at magnetic flux densities of less than 10 T, these effects will be negligible in a living person.

Electronic Interactions

Certain chemical reactions involve intermediate electron states, which could be affected by static magnetic fields producing an effect on the transition of an electron from one state to a lower state. Although such effect has the potential to lead to biological consequences, it must be stated that a magnetic field effect on chemical reaction intermediates in biological systems has not yet been demonstrated, under actual physiological conditions. It is likely that the usual lifetime of biologically relevant electron transitions is sufficiently short to ensure that magnetic field interactions exert only a small and perhaps negligible influence on the yield of chemical products. Theoretical analysis suggests that fields up to even 10 T are unlikely to affect chemical interactions.

The biological effects of exposure of animals and cells to static magnetic fields have been investigated from different endpoints, including reproduction and development, cancer, and the nervous system. No consistent effects have been reported using fields below 2 T. The acute responses found during exposure to static fields above about 4 T are consistent with the interaction mechanisms described above (Saunders, 2005). No adverse effects on reproduction and development, or on the growth and development of tumors, have been firmly established. There is, however, little information regarding possible effects of chronic exposure.

With the advent of superconducting magnet technology, patients and volunteers can be routinely exposed to static magnetic fields of 1.5 T and more. Most of the acute effects observed at high-field MR systems are consistent with known mechanisms of interaction. Schenck et al. (1992) reported field-dependent sensations of vertigo, nausea and a metallic taste in the mouth of volunteers exposed to fields of 1.5 or 4 T. These occurred only during movement of the head. Additionally, magnetic phosphenes could be seen during eye movement in a field of at least 2 T. These effects are probably caused by electric currents induced by movement in the field.

Kinouchi et al. (1996) reported that the Lorentz force affecting blood flow generates electric potentials across blood vessels. In practice, "flow" potentials are readily demonstrated in volunteers exposed to static magnetic fields greater than 0.1 T. The largest flow potentials occur across the aorta after ventricular contraction, and appear superimposed on the T-wave of the electrocardiogram (Tenforde, 1992). Kinouchi et al. (1996) calculated that a static field of 5 T would induce maximum current densities around the sinoatrial node of the heart of about 100 mA m^{-2} (500 mV m^{-1} using a tissue conductivity of 0.2 S m^{-1}), which is well below the cardiac excitation threshold. Additionally, a 5–10% reduction in blood flow in the aorta was predicted to occur in static fields of 10–15 T due to magneto-hydrodynamic interactions. Kangarlu et al. (1999), however, found that volunteers exposed to an 8-T field for 1 h showed no change in heart rate or blood pressure either during or after exposure. Chakeres et al. (2003a,b) reported that exposure of 25 healthy volunteers to 8-T fields had no clinically significant effect on heart rate, respiratory rate, blood pressure, finger pulse oxygenation levels, core body temperature, and cognitive function. In order to avoid the movement-induced sensations described above, the volunteers were moved very slowly into the magnet bore. Nevertheless, nine subjects reported sensations of dizziness, while two reported a metallic taste.

1.4.3.2 Epidemiology

Epidemiological studies were performed on workers exposed to static magnetic fields of up to a few mT, and the children of such workers. The International Agency for Research on Cancer (IARC, 2002) has reviewed studies of cancer. Generally, these have not pointed to higher cancer risks, although the number of studies was small, the numbers of cancer cases were limited, and the information on individual exposure levels was poor. Some studies have investigated reproductive outcome for workers involved in the aluminum industry or in MR imaging.

Kanal et al. (1993), for example, did not identify any decreased fertility for either male or female workers. However, no studies of high quality have been carried out with workers occupationally exposed to fields greater than 1 T.

1.4.3.3 Safety Aspects and Exposure Levels

The basic restrictions for static magnetic fields are expressed in terms of the magnetic flux density in units of Tesla. For time-varying magnetic fields the basic quantity is the induced current density, expressed in units of ampere per square meter ($A\ m^{-2}$).

As stated above, biological data for static magnetic field exposure indicate that there are no significant biological effects on people at levels below about 2 T. However, in stronger magnetic fields, vertigo, nausea, a metallic taste and phosphenes can be induced during movement.

The value of 2 T is considered suitable as ceiling value for whole-body occupational exposure with a relaxation of up to 5 T for exposure of the limbs alone (ICNIRP, 1994). However, as very few long-term exposure data are available, it is considered advisable to limit the average exposure for the whole body during the entire working day to 200 mT. In addition to the basic limits, the following cautionary clauses should be taken into account:

- The magnetic field exposure of persons with conductive implants, especially when made of ferromagnetic materials, should not exceed 25 mT averaged over times shorter than 1 s.
- Workers with cardiac pacemakers and electrically active implants should not have access to areas where the magnetic flux density exceeds 0.5 mT.
- Because of existing electromagnetic compatibility problems, or field influence on magnetic data carriers, separate intervention levels may be necessary.

1.4.4 Time-Varying Magnetic Fields (1 Hz to 100 kHz)

The time-varying magnetic fields originating from man-made sources generally have much higher intensities than the naturally occurring fields (ICNIRP, 2003). This is particularly true for sources operating at the power frequencies of 50 or 60 Hz. Other man-made sources are found in research, industry (welding machines, electric furnaces and induction heating) and medicine (MRI).

1.4.4.1 Interaction Mechanisms and Biological Bases for Limiting Exposure

Time-varying magnetic fields exert a force on charged particles such as ions or asymmetrically charged molecules, which results in electric fields and circulating electric currents in tissues in accordance with Faraday's law. At the cellular level, this interaction consists of the induction of voltages across the membranes of cells which, given sufficiently high levels, can stimulate nerve cells to conduct or muscles to contract. These interaction mechanisms occur only where high electrical field strengths of above about 5 V m^{-1} are present (Reilly, 1998).

As the membrane's electrical conductivity is smaller by approximately five orders of magnitude than that of the extracellular fluid, it forms an electrical barrier that mediates interactions of cells with extracellular electric fields. It is, therefore, now assumed that the transduction processes through which induced electric signals influence cellular properties, involve interaction at the level of the cell membrane. A growing body of evidence indicates that induced electric fields and currents circulating in the extracellular medium can alter ion-binding to membrane macromolecules, influence ion transport across the membrane, and modify ligand–receptor interactions at the cell membrane surface. These changes in membrane properties may trigger trans-membrane signaling events. A number of membrane effects and cellular manifestations occur at threshold levels for induced electric fields below 0.1 V m^{-1} (Tenforde, 1996). The most sensitive tissues are those comprising interacting networks of electrically excitable tissue, such as the central and autonomic nervous systems.

The maximal induced electric field strength is proportional to dB/dt (the rate of change of the magnetic flux density), and to a proportionality constant, which depends on the field distribution and direction, the geometry of the body, and the electric characteristics of the tissues (Bernhardt, 1988). When the frequency increases, the magnetically induced electric field and current density increase linearly as a function of frequency. The induced electric field is maximal at the surface of the body and decreases towards the center. Coupling is maximized when the magnetic field is uniform and perpendicular to the frontal cross-section of the body. While geometrically simple models are useful to illustrate fundamental aspects of magnetic field dosimetry, anatomically realistic models of man and high-resolution calculations have been used in recent years to obtain more detailed information (e.g., Dawson et al., 1997; Dimbylow, 1998). Such calculations clearly indicate that the local peak current densities are much higher than the average current densities. In a homogeneous 500-μT field of 50 Hz, the local peak current density may considerably exceed 10 mA m^{-2}, and up to 40 mA m^{-2} in some parts of the body.

The biological effects of low-frequency magnetic fields continue to be studied using a wide variety of exposure conditions, models, and biological endpoints. There is a consensus of the many national and international scientific expert groups, which have comprehensively reviewed the biological effects literature and the biological studies relevant to the assessment of possible adverse health effects of exposure to low-frequency magnetic fields. These include the IARC (2002), ICNIRP (2003) and NRPB (2004).

Studies have been carried out of direct nerve stimulation thresholds in volunteers by intense, pulsed magnetic fields, used in various specialized medical applications such as MRI (see Shellock, 2001) and transcranial magnetic stimulation

(TMS; see also Section 4.4). Threshold rates of change of MRI switched gradient magnetic fields for perception, discomfort and pain resulting from peripheral nerve stimulation has been extensively reviewed by Nyenhuis et al. (2001). Median minimum threshold rates of change of magnetic field during periods of less than 1 ms for perception were generally 15–25 T s^{-1} depending on orientation, and showed large inter-individual differences (Bourland et al., 1999). Cells of the central nervous system (CNS) are considered to be sensitive to induced electric fields that are below the stimulation threshold of nerve axons (*in-vitro* threshold of ca. 4–5 V m^{-1}; Jefferys et al., 2003). Such electric field interactions have been demonstrated in experimental studies using isolated animal brain tissue. However, the CNS *in vivo* is considered to be more sensitive to induced low-frequency electric fields and currents than *in-vitro* preparations, due to the larger number of interacting nerve cells; the data available are consistent with a threshold of 100 mV m^{-1}, which may be constant between a few hertz and a few kHz. Other excitable tissues such as the heart seem less susceptible to the direct effects of induced electric fields, but may be affected indirectly via effects on the CNS (NRPB, 2004).

The retina is considered to be a good model of the sensitivity of CNS tissue to induced electric fields. Retinal function can be affected by exposure to low-frequency magnetic fields and applied electric currents. Field thresholds in the extracellular fluid of the retina for inducing phosphenes have been estimated to lie between about 10 and 60 mV m^{-1} at 20 Hz (NRPB, 2004). However, the extrapolation of such values to other CNS tissues is complex and uncertain. The exact mechanisms underlying phosphene induction and its frequency dependence remain unknown.

1.4.4.2
Epidemiology

There is some epidemiological evidence that exposure to power frequency magnetic fields above 0.4 μT is associated with a small raised risk of leukemia in children (approximately, doubling of the relative risk). The IARC (2002) stated that their findings provided limited evidence for an excess risk in humans exposed at these field levels, and evaluated low-frequency magnetic fields as being "possibly carcinogenic to humans" (Classification 2B). However, in the absence of clear evidence of any carcinogenic effect in adults, or of a plausible explanation from experiments on animals or isolated cells, the ICNIRP (2003) has concluded that the epidemiological evidence is not strong enough to justify a firm conclusion that such fields cause leukemia in children. The IARC also considered the evidence for excess cancer risks of all other kinds, in children and adults, as a result of exposure to extremely low frequency (<300 Hz) electric and magnetic fields, to be inadequate. The findings from studies of health effects other than cancer have generally been inconsistent.

The results of epidemiological studies, either taken individually or as collectively reviewed by expert groups, cannot be used as a basis for the derivation of quantitative restrictions of exposure to low-frequency fields (ICNIRP, 2003; NRPB, 2004).

1.4.4.3
Safety Aspects and Exposure Levels

Scientific established data from which guidance can be developed concerns electric field interactions in the CNS and certain other electrically excitable tissues. A cautious approach has been used to indicate thresholds for adverse health effects. Data on other possible health effects examined lack plausibility, consistency, and coherence.

Threshold tissue electric field strengths of around 100 mV m^{-1} have been identified for effects in the CNS. Comparison of basic restrictions expressed in terms of induced electric field strength with those expressed in terms of induced current density requires computational modeling using tissue- and frequency-dependent values of the electrical conductivity.

For occupational exposure, it is concluded that a restriction of the induced electric field strength in the central, autonomic and enteric nervous systems to less than 100 mV m^{-1} is adequate to protect most adult members of the population. In the frequency range from 4 Hz to 1 kHz, the ICNIRP (1998) decided that occupational exposure should be limited to fields that induce current densities less than 10 mA m^{-2} (corresponding to a tissue field strength of about 50 mV m^{-1} using a tissue conductivity of 0.2 S m^{-1}). Below 4 Hz and above 1 kHz, the basic restriction on induced current density increases progressively, corresponding to the increase in the threshold for nerve stimulation for these frequency range (Fig. 1.30). The basic restrictions for current densities for occupational exposure are presented in Table 1.3. The current densities are given as root-mean-square (rms) values.

Where appropriate, the reference levels are obtained from the basic restrictions by mathematical modeling and by extrapolation from the results of laboratory studies at specific frequencies. They are given for the condition of maximum coupling of the field to the exposed individual, thereby providing maximum protection. The reference levels for occupational exposure (ICNIRP, 1998) are summarized in Table 1.4. The reference levels are intended to be spatially averaged values over the entire body of the exposed individual, but with the important proviso that the basic restrictions on localized exposure are not exceeded. The frequency dependence of the reference levels – shown in Figure 1.31 for the magnetic flux density – is consistent with data on both biological effects and coupling of the field.

1.4.5
Electromagnetic Fields (100 kHz to 300 GHz)

1.4.5.1
Interaction Mechanisms and Biological Bases for Limiting Exposure

There are well-documented bioeffects linked to excess temperature elevation. Such effects have been observed from exposure to radiofrequency (RF) electromagnetic fields resulting from whole-body or local heating. An important first step in assess-

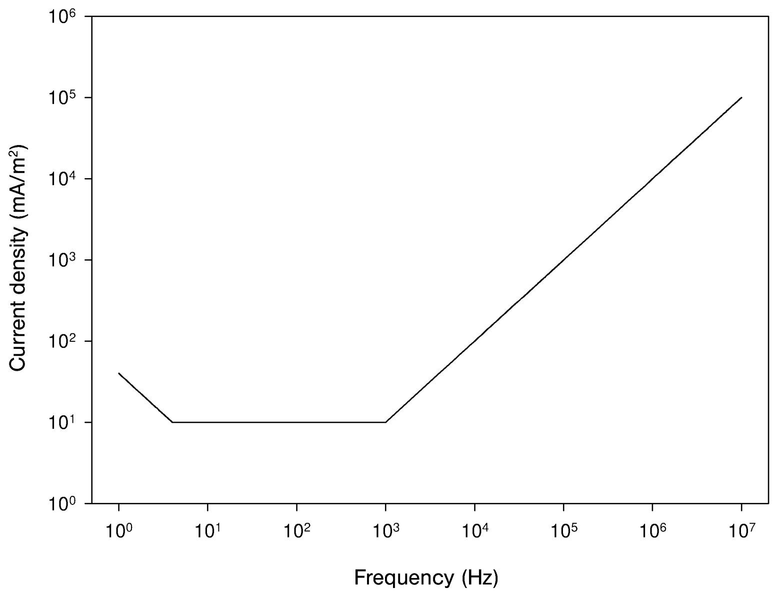

Fig. 1.30. Basic restriction for the current density for head and trunk for occupational exposure for frequencies between 1 Hz and 10 MHz. The frequency dependence reflects the frequency dependence of the thresholds of nerve- and muscle stimulation, including a safety factor. (From ICNIRP, 1998).

ing the RF exposure health risk is to define the level of energy absorption and the resulting possible temperature elevation over the entire frequency range. This database is fundamental to the establishment of exposure limits (ICNIRP, 1998). In addition, consideration must be given to long-term or chronic exposures of workers to low-level electromagnetic fields.

It is convenient to divide the RF range into different spectral regions according to the predominant or more significant mechanisms of energy absorption:

- In the sub-resonance range (0.1–10 MHz), exposure of the human body to electromagnetic fields can result in high rates of energy deposition in the hand, wrist and ankle due to current flow through small effective cross-sectional areas. In this frequency region, the biological response of humans arises not only from tissue heating but also from the stimulation of excitable tissues, such as nerve and muscles, via induced currents. Thus, the thermal mechanism dominates at higher frequencies, while induced currents become also important at lower frequencies. Therefore, in the frequency range of 0.1 to a few MHz, the significant dosimetric quantities for establishing basic exposure limits are both the internal current density and the absorbed energy.
- In the resonance range (10–300 MHz), the human body can be thought of as an absorbing antenna. A maximum absorption is reached, for plane wave exposure,

Table 1.3. Basic restriction for occupational exposure for time-varying magnetic fields for frequencies up to 300 GHz. (From ICNIRP, 1998).

Frequency range	Current-density for head and trunk [mA m^{-2}][a]	Whole-body average SAR [W kg^{-1}]	Localized SAR (head and trunk) [W kg^{-1}]	Localized SAR (limbs) [W kg^{-1}]	Power density [W m^{-2}]
Up to 1 Hz	40				
1–4 Hz	$40/f$	–			
4 Hz to 1 kHz	10		–		
1–100 kHz	$f/100$				
100 kHz to 10 MHz	$f/100$	0.4	10	20	
10 MHz to 10 GHz		0.4	10	20	
10–300 GHz					50

[a] Root mean square (rms) values; f indicates frequency (in Hz). All specific absorption rate (SAR) values are to be averaged over any 6-min period. Localized SAR averaging mass is any 10 g of contiguous tissue; the maximum SAR so obtained should be the value used for the estimation of exposure. There are several other clauses to be considered (see ICNIRP, 1998).

Table 1.4. Reference levels for occupational exposure to time-varying magnetic fields (unperturbed rms values).

Frequency range	Magnetic field strength H [A m^{-1}]	Magnetic flux density B [μT]	Equivalent plane wave power density S_{eq} [W m^{-2}]
Up to 1 Hz	1.63×10^5	2×10^5	
1–8 Hz	$1.63 \times 10^5/f^2$	$2 \times 10^5/f^2$	
8–25 Hz	$2 \times 10^4/f$	$2.5 \times 10^4/f$	
0.025–0.82 kHz	$20/f$	$25/f$	
0.82–65 kHz	24.4	30.7	
0.065–10 MHz	$1.6/f$	$2.0/f$	
10–400 MHz	0.16	0.2	10
400–2000 MHz	$0.008\sqrt{f}$	$0.01\sqrt{f}$	$f/40$
2–300 GHz	0.36	0.45	50

f as indicated in the frequency range column. There are several clauses to be considered (see ICNIRP, 1998).

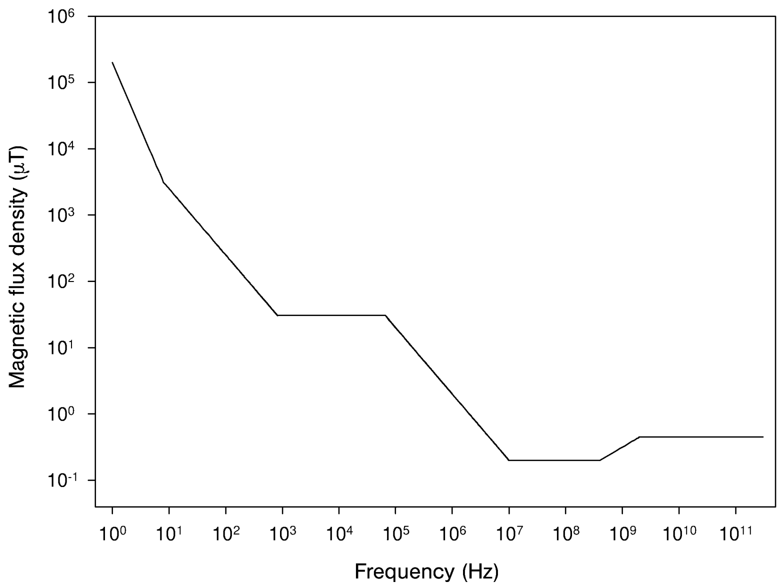

Fig. 1.31. Reference level for the magnetic flux density for occupational exposure. The frequency dependence of the reference magnetic flux density level is consistent with data on both biological effects and coupling of the field. (From ICNIRP, 1998).

in the frequency range of 30–80 MHz. At higher frequencies the wavelength becomes small compared to the body size, and so-called "hot spots" of absorption may occur in smaller parts of the body such as the head, though the total absorption will be reduced.

- Above 300 MHz, localized energy absorption can occur due to dimensional resonance phenomena or quasi-optical focusing of the incident fields. As the frequency is further increased, the depth of penetration of the incident electromagnetic energy will be reduced until most of the energy is absorbed close to the body surface at 300 GHz.

The time rate of electromagnetic energy absorption by a unit of mass of a biological system is defined as the *specific absorption rate* (SAR), the unit of which is watt per kilogram (W kg^{-1}). The SAR may be spatially averaged over the total mass of an exposed body or its parts, and may be temporally averaged over a given time of exposure or over a single pulse or modulation period of the radiation. The SAR is the significant dosimetric quantity for establishing basic exposure restrictions in the frequency range of a few MHz to a few GHz.

A review of the bioeffects literature indicates that heat-related disorders should not occur in the majority of healthy adults, provided that core temperature does not rise above 38 °C (corresponding to a temperature rise of 1 °C above baseline).

In this case it is also likely to prevent adverse effects on the performance of the most cognitive tasks. High rates of physical activity and/or warm, humid environments will reduce the additional RF heat loads that most adults can tolerate without exceeding 38 °C. An RF heat load of 0.4 W kg^{-1} averaged over the whole body should be sufficiently low that these other factors can be ignored.

It should be mentioned, however, that the individual sensitivity to heat-related stress varies among the general population. Additionally, adults taking drugs that have direct effects on the control of body temperature, or on metabolism or heat production of the body, may also be considered at greater risk.

Adverse effects on the testis should not occur, provided that temperature increases are less than 1 °C. Other tissues, such as kidney, liver and muscle, seem less sensitive. Temperature rises in the brain, retina, and spinal cord to above 38 °C, of the other tissues of the neck and trunk to above 39 °C, and of the tissues of the limbs to above 40 °C, may result in localized heat-induced damage. The lenses of the eye are particularly sensitive due to the limited ability of the eye to dissipate heat. The ability to dissipate heat from locally heated tissues depends on their temperature in relation to their surroundings and rate of blood flow through the tissue. People with cardiovascular disease, which will reduce the blood circulation, may be at increased sensitivity by RF electromagnetic fields compared with people with normal cardiovascular responses.

There are relatively few dosimetric studies linking localized temperature increases and SAR in most parts of the body. Studies indicate a range of localized temperature increases of 0.05 to 0.12 °C in the brain from a localized SAR of 1 W kg^{-1}. A number of studies have suggested that low-level RF fields may induce different subtle biological responses, particularly possible effects of pulsed fields on brain function and on changes in heat shock protein expression (NRPB, 2004). Further studies are necessary to examine these effects. However, none of these possible effects is considered sufficient to derive basic restrictions for human exposure.

1.4.5.2
Epidemiology

A large number of occupational studies have been conducted over several decades, particularly on cancer, cardiovascular disease, adverse reproductive outcome, and cataract, in relation to RF exposure. More recently, studies have been conducted on residential exposure, mainly from radio and television transmitters, and especially focusing on leukemia. There have also been studies of mobile telephone users, particularly on brain tumors and less often on other cancers and on symptoms. To date, the results of these studies have provided no consistent or convincing evidence of any causal relationship between RF exposure and adverse health effects. However, the studies have too many deficiencies to rule out an association. A key concern across all studies is the quality of assessment of RF exposure. A comprehensive review of epidemiologic studies regarding the effects of RF fields on human health has been produced by Ahlbom et al. (2004).

1.4.5.3
Safety Aspects and Exposure Limits

The basic restrictions for frequencies up to 300 GHz are presented in Table 1.3. The basic restriction for the current density is plotted in Figure 1.30 for the relevant frequency range. Reference levels for occupational exposure are presented in Table 1.4. As an example, reference levels for the magnetic flux density are plotted in Figure 1.31 for the entire frequency range. In addition, occupationally exposed workers with metallic implants and pacemaker wearers are groups at particular risk, and may not be protected by the prescribed limits.

1.4.6
Protection of Patients and Volunteers Undergoing MR Procedures

With the significant level of growth in the number of patients examined using MR technology and the rapid development of MR hardware, the consideration of possible risks and health effects associated with the use of diagnostic MR devices is gaining increasingly in importance. In Germany, for example, the annual frequency of MR examinations increased between 1996 and 2003 from 22 to 64 examinations per 1000 inhabitants.

As will be described in detail in Chapter 3, three types of magnetic fields are employed in MR imaging and spectroscopy:

- a high static magnetic field generating a macroscopic nuclear magnetization;
- rapidly alternating magnetic gradient fields for spatial encoding of the MR signal; and
- RF electromagnetic fields for excitation and preparation of the spin system.

The biophysical interaction mechanisms and biological effects of these fields are discussed in Sections 1.4.3 to 1.4.5. Supplementary information and an exhaustive bibliography concerning safety aspects of clinical MR procedures can be found in the recent literature (e.g., Ordidge et al., 2000; Shellock, 2001). The following section provides a brief summary of exposure limits and precautions to be taken in order to minimize health hazards and risks to patients and volunteers undergoing MR procedures according to:

- the technical product standard IEC 60601-2-33 issued by the International Electrotechnical Commission in 2002 (IEC, 2002); and
- the safety recommendation issued by the International Commission on Non-Ionizing Radiation Protection in 2004 (ICNIRP, 2004).

In order to reflect the uncertainty over identified deleterious effects and, moreover, to offer the necessary flexibility for the development and clinical evaluation of new MR technologies, both the IEC standard and the ICNIRP recommendation give exposure limits for three different modes of operation:

- *Normal operating mode*: Routine MR examinations that did not cause physiological stress to patients.
- *Controlled operating mode*: Specific MR examinations outside the normal operating range where discomfort and/or physiological stress may occur in some patients. A clinical decision must be taken to balance such effects against foreseen benefits; exposure must be carried out under medical supervision.
- *Experimental operating mode*: Experimental MR procedures with exposure levels outside the controlled operating range. In view of the potential risks for patients and volunteers, special ethical approval and adequate medical supervision is required.

1.4.6.1
Static Magnetic Fields

The possible health effects that might result from acute exposure to static magnetic fields were reviewed in Section 1.4.3. The basic actions are physical effects (translation and orientation), electrodynamic forces on moving electrolytes, and effects on electron spin states of chemical reaction intermediates.

Until now, most MR examinations have been made using static magnetic fields up to 3 T, although whole-body MR systems with static magnetic fields up to 8 T are already used in clinical tests. The literature does not indicate any serious adverse health effects from the exposure of healthy human subjects up to 8 T. However, it should be noted that, to date, there have been no epidemiological studies performed to assess possible long-term health effects in patients or volunteers. The greatest potential hazard comes from metallic, in particular ferromagnetic materials (such as scissors, coins, pins, oxygen cylinders) that are accelerated in the inhomogeneous magnetic field in the periphery of an MR system and quickly become dangerous projectiles. This risk can only be minimized by a strict and careful management of both patients and staff (Medical Devices Agency, 2002).

Because exposure to magnetic fields above 2 T can produce nausea and vertigo, it is recommended that examinations above this static magnetic flux density be conducted in the controlled operating mode under medical supervision. The recommended upper limit for this operating mode is 4 T, due to the limited data concerning possible effects above this static field strength. For MR examinations performed in the experimental operating mode, there is no upper limit for the magnetic flux density.

1.4.6.2
Time-Varying Magnetic Gradient Fields

The rapidly switched magnetic gradient fields used in MRI for spatial encoding induce electric fields in the human body in accordance with Faraday's law which, if of sufficient magnitude, can produce nerve and muscle stimulation (see Section 1.4.4). The induced electric field is proportional to dB/dt, the time rate of change of the magnetic field. From a safety standpoint, the primary concern with regard to time-varying magnetic fields is cardiac fibrillation, because it is a life-threatening condition. In contrast, peripheral nerve stimulation is of practical concern because uncomfortable or intolerable stimulations would interfere with the examination (e.g., patient movements) or would even result in a termination of the examination.

Recommendations on limiting patient and volunteer exposure to time-varying magnetic fields are based primarily on the extensive investigations on peripheral nerve stimulation in humans performed at Purdue University (Schaefer et al., 2000; Nyenhuis et al., 2001). In the reported studies, data were obtained for the perception threshold, the threshold for uncomfortable stimulation, and the threshold for intolerable stimulation during exposure to gradient fields. The results indicate that the lowest percentile for intolerable stimulation is approximately 20% above the median perception threshold for peripheral nerve stimulation, which can be parameterized by the following empirical relationship:

$$\frac{dB}{dt} = 20 \cdot \left(1 + \frac{0.36}{\tau}\right) \quad (\text{in Ts}^{-1}) \tag{1.73}$$

In Eq. (1.73), τ is the effective stimulus duration (in ms) defined as the duration of the period of monotonic increasing or decreasing gradient.

The maximum recommended exposure level for time-varying magnetic fields is set equal to a dB/dt of 80% of the median perception threshold given in the relationship above for normal operation, and 100% of the median for controlled operation. As shown in Figure 1.32, the threshold for cardiac stimulation (Reilly, 1998) is well above the median perception threshold for peripheral nerve stimulation, except at very long pulse durations which are, however, not relevant for MR procedures.

1.4.6.3 Radiofrequency Electromagnetic Fields

Time-varying electromagnetic fields with frequencies above 10 MHz (RF fields), that are used in MR studies to excite and prepare the spin system, deposit energy in the human body that is mainly converted to heat. The parameter relevant for the evaluation of biological effects of RF fields is the increase in tissue temperature, which is dependent not only on localized power absorption and the duration of RF exposure, but also on heat transfer and the activation of thermoregulatory mechanisms leading to thermal equalization within the body.

As reviewed in Section 1.4.5, no adverse health effects are expected if the increase in body-core temperature does not exceed 1 °C. In the case of infants, pregnant women, elderly, and persons with cardiocirculatory impairment, however, it is desirable to limit body-core temperature increases to 0.5 °C. Additionally, local temperatures under exposure to the head, trunk, and/or extremities should be limited to the values given in Table 1.5.

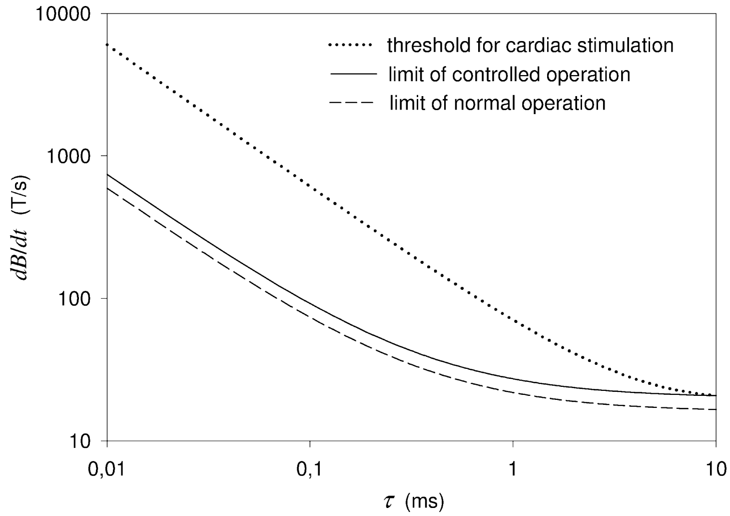

Fig. 1.32. Threshold for cardiac stimulation (Reilly, 1998) and limits for normal and controlled operation of a magnetic resonance device. Data are expressed as *dB/dt* as a function of the effective stimulus duration τ. The limit for the controlled operation mode is given by the median perception threshold for peripheral nerve stimulation.

Since temperature changes in the various organs and tissues of the body during an MR procedure are difficult to measure in clinical routine, RF exposure of patients is usually characterized by means of the SAR (in W kg^{-1}) As only parts of the patient's body are exposed simultaneously during an MR procedure, not only the whole-body SAR but also partial-body SARs for the head, the trunk, and the extremities should be estimated on the basis of suitable patient models (e.g., Brix et al., 2001). Based on the published experimental studies concerning temperature rise and theoretical simulations, the SAR levels summarized in Table 1.4 should not be exceeded in order to limit temperature rise to the values given in Table 1.5.

Table 1.5. Basic restrictions for body temperature rise and partial-body temperatures for volunteers and patients undergoing MR procedures.

Operating mode	**Rise of body-core temperature [°C]**	**Spatially localized temperature limits**		
		Head [°C]	**Trunk [°C]**	**Extremities [°C]**
Normal	0.5	38	39	40
Controlled	1	38	39	40
Experimental	>1	>38	>39	>40

Table 1.6. SAR levels valid for volunteers and patients undergoing MR procedures at environmental temperatures below 24 °C.

Operating mode	Averaging time: 6 min					
	Whole-body SAR [W kg^{-1}]	Partial-body SAR [W kg^{-1}]		Local SAR (averaged over 10 g tissue) [W kg^{-1}]		
	Body region					
	Whole-body	Any, except head	Head	Head	Trunk	Extremities
Normal	2	2–10[a]	3	10	10	20
Controlled	4	4–10[a]	3	10	10	20
Experimental	>4	>(4–10)[a]	>3	10	>10	>20
Short-term SAR	The SAR limit over any 10-s period shall not exceed three times the corresponding average SAR limit.					

[a] Partial-body *SARs* scale dynamically with the ratio *r* between the patient mass exposed and the total patient mass:
– normal operating mode: $SAR = (10 - 8 \cdot r)$ W kg^{-1}
– controlled operating mode: $SAR = (10 - 6 \cdot r)$ W kg^{-1}

With respect to the application of the SAR levels defined in Table 1.6, the following points should be taken into account:

- Partial-body SARs scale dynamically with the ratio r between the patient mass exposed and the total patient mass. For $r \to 1$, they converge against the corresponding whole-body values, for $r \to 0$ against the localized SAR level of 10 W kg^{-1} defined for occupational exposure of head and trunk (ICNIRP, 1998; cf. Table 1.3).
- The recommended SAR limits do not relate to an individual MR sequence, but rather to running SAR averages computed over each 6-min period, which is assumed to be a typical thermal equilibration time of smaller masses of tissue (Brix et al., 2002).

1.4.6.4 Contraindications

Pregnant Patients

Pregnant patients undergoing MR examinations are exposed to the combined magnetic and electromagnetic fields used in MR imaging. The few studies on pregnancy outcome in humans following MR examinations have not revealed any adverse effects, but are very limited because of the small numbers of patients involved and difficulties in the interpretation of the results. It is thus advised that

MR procedures may be used in pregnant patients only after critical risk/benefit analysis, in particular during the first trimester, to investigate important clinical problems or to manage potential complications for the patient or fetus. Moreover, it is recommended that exposure duration should be reduced to the minimum and that the exposure levels of the normal operation mode are not exceeded.

Special Safety Issues and Contraindications

MR examinations of patients who have electrically, magnetically, or mechanically activated implants (e.g., cardiac pacemakers and defibrillators, cochlear implants, electronic drug infusion pumps), as well of patients with passive implants or other objects of ferromagnetic or unknown material (e.g., aneurysm and hemostatic clips, orthopedic implants, pellets, and bullets), is contraindicated. Lists of implants and materials tested for safety or compatibility in association with MR systems have been published and updated (e.g., Shellock, 2005; www.MRIsafety.com).

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