

# Contents

<b>1</b>	<b>Basic Physics</b>	<b>1</b>
1.1	Fusion . . . . .	1
1.2	Plasma . . . . .	6
1.3	Coulomb Collisions . . . . .	9
1.4	Electromagnetic Theory . . . . .	15
<b>2</b>	<b>Motion of Charged Particles</b>	<b>21</b>
2.1	Gyromotion and Drifts . . . . .	21
2.1.1	Gyromotion . . . . .	21
2.1.2	$\mathbf{E} \times \mathbf{B}$ Drift . . . . .	24
2.1.3	Grad- $\mathbf{B}$ Drift . . . . .	25
2.1.4	Polarization Drift . . . . .	27
2.1.5	Curvature Drift . . . . .	28
2.2	Constants of the Motion . . . . .	31
2.2.1	Magnetic Moment . . . . .	31
2.2.2	Second Adiabatic Invariant . . . . .	32
2.2.3	Canonical Angular Momentum . . . . .	34
2.3	Diamagnetism* . . . . .	36
<b>3</b>	<b>Magnetic Confinement</b>	<b>41</b>
3.1	Confinement in Mirror Fields . . . . .	41
3.1.1	Simple Mirror . . . . .	41
3.1.2	Tandem Mirrors* . . . . .	46
3.2	Closed Toroidal Confinement Systems . . . . .	49
3.2.1	Confinement . . . . .	49
3.2.2	Flux Surfaces . . . . .	53
3.2.3	Trapped Particles . . . . .	55
3.2.4	Transport Losses . . . . .	59
<b>4</b>	<b>Kinetic Theory</b>	<b>65</b>
4.1	Boltzmann and Vlasov Equations . . . . .	66
4.2	Drift Kinetic Approximation . . . . .	66
4.3	Fokker–Planck Theory of Collisions . . . . .	69
4.4	Plasma Resistivity . . . . .	76
4.5	Coulomb Collisional Energy Transfer . . . . .	78
4.6	Krook Collision Operators . . . . .	82

<b>5</b>	<b>Fluid Theory</b>	<b>85</b>
5.1	Moments Equations . . . . .	85
5.2	One-Fluid Model . . . . .	89
5.3	Magnetohydrodynamic Model . . . . .	93
5.4	Anisotropic Pressure Tensor Model* . . . . .	96
5.5	Strong Field, Transport Time Scale Ordering . . . . .	98
<b>6</b>	<b>Plasma Equilibria</b>	<b>103</b>
6.1	General Properties . . . . .	103
6.2	Axisymmetric Toroidal Equilibria . . . . .	105
6.3	Large Aspect Ratio Tokamak Equilibria . . . . .	111
6.4	Safety Factor . . . . .	116
6.5	Shafranov Shift* . . . . .	120
6.6	Beta . . . . .	123
6.7	Magnetic Field Diffusion and Flux Surface Evolution* . . . . .	125
6.8	Anisotropic Pressure Equilibria* . . . . .	128
<b>7</b>	<b>Waves</b>	<b>131</b>
7.1	Waves in an Unmagnetized Plasma . . . . .	131
7.1.1	Electromagnetic Waves . . . . .	131
7.1.2	Ion Sound Waves . . . . .	133
7.2	Waves in a Uniformly Magnetized Plasma . . . . .	134
7.2.1	Electromagnetic Waves . . . . .	134
7.2.2	Shear Alfvén Wave . . . . .	137
7.3	Langmuir Waves and Landau Damping . . . . .	139
7.4	Vlasov Theory of Plasma Waves* . . . . .	142
7.5	Electrostatic Waves* . . . . .	148
<b>8</b>	<b>Instabilities</b>	<b>155</b>
8.1	Hydromagnetic Instabilities . . . . .	158
8.1.1	MHD Theory . . . . .	159
8.1.2	Chew–Goldberger–Low Theory . . . . .	160
8.1.3	Guiding Center Theory . . . . .	162
8.2	Energy Principle . . . . .	165
8.3	Pinch and Kink Instabilities . . . . .	169
8.4	Interchange (Flute) Instabilities . . . . .	173
8.5	Ballooning Instabilities . . . . .	179
8.6	Drift Wave Instabilities . . . . .	183
8.7	Resistive Tearing Instabilities* . . . . .	186
8.7.1	Slab Model . . . . .	186
8.7.2	MHD Regions . . . . .	187
8.7.3	Resistive Layer . . . . .	189
8.7.4	Magnetic Islands . . . . .	190

8.8	Kinetic Instabilities*	192
8.8.1	Electrostatic Instabilities	192
8.8.2	Collisionless Drift Waves	193
8.8.3	Electron Temperature Gradient Instabilities	195
8.8.4	Ion Temperature Gradient Instabilities	196
8.8.5	Loss–Cone and Drift–Cone Instabilities	197
8.9	Sawtooth Oscillations*	201
<b>9</b>	<b>Neoclassical Transport</b>	<b>205</b>
9.1	Collisional Transport Mechanisms	205
9.1.1	Particle Fluxes	205
9.1.2	Heat Fluxes	207
9.1.3	Momentum Fluxes	208
9.1.4	Friction Force	210
9.1.5	Thermal Force	210
9.2	Classical Transport	212
9.3	Neoclassical Transport – Toroidal Effects in Fluid Theory	215
9.4	Multifluid Transport Formalism*	221
9.5	Closure of Fluid Transport Equations*	224
9.5.1	Kinetic Equations for Ion–Electron Plasma	224
9.5.2	Transport Parameters	228
9.6	Neoclassical Transport – Trapped Particles	231
9.7	Chang–Hinton Ion Thermal Conductivity*	237
9.8	Extended Neoclassical Transport – Fluid Theory*	238
9.8.1	Radial Electric Field	239
9.8.2	Toroidal Rotation	240
9.8.3	Transport Fluxes	240
9.9	Electrical Currents*	242
9.9.1	Bootstrap Current	242
9.9.2	Total Current	243
9.10	Orbit Distortion	244
9.10.1	Toroidal Electric Field – Ware Pinch	244
9.10.2	Potato Orbits	245
9.10.3	Orbit Squeezing	246
9.11	Transport in a Partially Ionized Gas*	247
<b>10</b>	<b>Plasma Rotation*</b>	<b>251</b>
10.1	Neoclassical Viscosity	251
10.1.1	Rate-of-Strain Tensor in Toroidal Geometry	251
10.1.2	Viscous Stress Tensor	252
10.1.3	Toroidal Viscous Force	253
10.1.4	Parallel Viscous Force	257
10.1.5	Neoclassical Viscosity Coefficients	258

10.2	Rotation Calculations . . . . .	260
10.2.1	Poloidal Rotation and Density Asymmetries . . . . .	260
10.2.2	Radial Electric Field and Toroidal Rotation Velocities . . . . .	262
10.3	Momentum Confinement Times . . . . .	264
10.3.1	Theoretical . . . . .	264
10.3.2	Experimental . . . . .	265
<b>11</b>	<b>Turbulent Transport</b>	<b>267</b>
11.1	Electrostatic Drift Waves . . . . .	267
11.1.1	General . . . . .	267
11.1.2	Ion Temperature Gradient Drift Waves . . . . .	270
11.1.3	Quasilinear Transport Analysis . . . . .	270
11.1.4	Saturated Fluctuation Levels . . . . .	272
11.2	Magnetic Fluctuations . . . . .	273
11.3	Candidate Microinstabilities . . . . .	275
11.3.1	Drift Waves and ITG Modes . . . . .	276
11.3.2	Trapped Ion Modes . . . . .	276
11.3.3	Electron Temperature Gradient Modes . . . . .	277
11.3.4	Resistive Ballooning Modes . . . . .	277
11.3.5	Chaotic Magnetic Island Overlap . . . . .	277
11.4	Wave–Wave Interactions* . . . . .	278
11.4.1	Mode Coupling . . . . .	278
11.4.2	Direct Interaction Approximation . . . . .	279
11.5	Drift Wave Eigenmodes* . . . . .	280
11.6	Gyrokinetic and Gyrofluid Simulations . . . . .	282
<b>12</b>	<b>Heating and Current Drive</b>	<b>285</b>
12.1	Inductive . . . . .	285
12.2	Adiabatic Compression* . . . . .	288
12.3	Fast Ions . . . . .	291
12.3.1	Neutral Beam Injection . . . . .	291
12.3.2	Fast Ion Energy Loss . . . . .	293
12.3.3	Fast Ion Distribution . . . . .	296
12.3.4	Neutral Beam Current Drive . . . . .	298
12.3.5	Toroidal Alfvén Instabilities . . . . .	299
12.4	Electromagnetic Waves . . . . .	301
12.4.1	Wave Propagation . . . . .	301
12.4.2	Wave Heating Physics . . . . .	304
12.4.3	Ion Cyclotron Resonance Heating . . . . .	308
12.4.4	Lower Hybrid Resonance Heating . . . . .	309
12.4.5	Electron Cyclotron Resonance Heating . . . . .	310
12.4.6	Current Drive . . . . .	311

<b>13 Plasma–Material Interaction</b>	<b>315</b>
13.1 Sheath . . . . .	315
13.2 Recycling . . . . .	318
13.3 Atomic and Molecular Processes . . . . .	319
13.4 Sputtering . . . . .	324
13.5 Impurity Radiation . . . . .	326
<b>14 Divertors</b>	<b>331</b>
14.1 Configuration, Nomenclature and Physical Processes . . . . .	331
14.2 Simple Divertor Model . . . . .	334
14.2.1 Strip Geometry . . . . .	334
14.2.2 Radial Transport and Widths . . . . .	334
14.2.3 Parallel Transport . . . . .	336
14.2.4 Solution of Plasma Equations . . . . .	337
14.2.5 Two-Point Model . . . . .	338
14.3 Divertor Operating Regimes . . . . .	340
14.3.1 Sheath-Limited Regime . . . . .	340
14.3.2 Detached Regime . . . . .	341
14.3.3 High Recycling Regime . . . . .	341
14.3.4 Parameter Scaling . . . . .	342
14.3.5 Experimental Results . . . . .	343
14.4 Impurity Retention . . . . .	343
14.5 Thermal Instability* . . . . .	346
14.6 2D Fluid Plasma Calculation* . . . . .	349
14.7 Drifts* . . . . .	351
14.7.1 Basic Drifts in the SOL and Divertor . . . . .	351
14.7.2 Poloidal and Radial $\mathbf{E} \times \mathbf{B}$ Drifts . . . . .	352
14.8 Thermoelectric Currents* . . . . .	354
14.8.1 Simple Current Model . . . . .	354
14.8.2 Relaxation of Simplifying Assumptions . . . . .	356
14.9 Detachment . . . . .	358
<b>15 Plasma Edge</b>	<b>361</b>
15.1 H-Mode Edge Transport Barrier . . . . .	361
15.1.1 Relation of Edge Transport and Gradients . . . . .	362
15.1.2 MHD Stability Constraints on Pedestal Gradients . . . . .	364
15.1.3 Representation of MHD Pressure Gradient Constraint . . . . .	368
15.1.4 Pedestal Widths . . . . .	369
15.2 $\mathbf{E} \times \mathbf{B}$ Shear Stabilization of Turbulence . . . . .	371
15.2.1 $\mathbf{E} \times \mathbf{B}$ Shear Stabilization Physics . . . . .	372
15.2.2 Comparison with Experiment . . . . .	374
15.2.3 Possible “Trigger” Mechanism for the L–H Transition . . . . .	374

15.3	Thermal Instabilities . . . . .	376
15.3.1	Temperature Perturbations in the Plasma Edge . . . . .	376
15.3.2	Coupled Two-Dimensional Density–Velocity–Temperature Perturbations . . . . .	379
15.3.3	Spontaneous Edge Transport Barrier Formation . . . . .	384
15.3.4	Consistency with Observed L–H Phenomena . . . . .	389
15.4	MARFEs . . . . .	392
15.5	Radiative Mantle . . . . .	397
15.6	Edge Operation Boundaries . . . . .	398
15.7	Ion Particle Transport in the Edge* . . . . .	398
15.7.1	Generalized “Pinch-Diffusion” Particle Flux Relations . . . . .	399
15.7.2	Density Gradient Scale Length . . . . .	402
15.7.3	Edge Density, Temperature, Electric Field and Rotation Profiles . . . . .	403
<b>16</b>	<b>Neutral Particle Transport*</b>	<b>413</b>
16.1	Fundamentals . . . . .	413
16.1.1	1D Boltzmann Transport Equation . . . . .	413
16.1.2	Legendre Polynomials . . . . .	414
16.1.3	Charge Exchange Model . . . . .	415
16.1.4	Elastic Scattering Model . . . . .	416
16.1.5	Recombination Model . . . . .	419
16.1.6	First Collision Source . . . . .	419
16.2	$P_N$ Transport and Diffusion Theory . . . . .	421
16.2.1	$P_N$ Equations . . . . .	421
16.2.2	Extended Diffusion Theories . . . . .	424
16.3	Multidimensional Neutral Transport . . . . .	428
16.3.1	Formulation of Transport Equation . . . . .	428
16.3.2	Boundary Conditions . . . . .	430
16.3.3	Scalar Flux and Current . . . . .	430
16.3.4	Partial Currents . . . . .	432
16.4	Integral Transport Theory . . . . .	432
16.4.1	Isotropic Point Source . . . . .	433
16.4.2	Isotropic Plane Source . . . . .	434
16.4.3	Anisotropic Plane Source . . . . .	435
16.4.4	Transmission and Probabilities . . . . .	437
16.4.5	Escape Probability . . . . .	437
16.4.6	Inclusion of Isotropic Scattering and Charge Exchange . . . . .	438
16.4.7	Distributed Volumetric Sources in Arbitrary Geometry . . . . .	439
16.4.8	Flux from a Line Isotropic Source . . . . .	439
16.4.9	Bickley Functions . . . . .	440
16.4.10	Probability of Traveling a Distance $\mathbf{r}$ from a Line, Isotropic Source without a Collision . . . . .	441

16.5	Collision Probability Methods . . . . .	442
16.5.1	Reciprocity among Transmission and Collision Probabilities . . .	442
16.5.2	Collision Probabilities for Slab Geometry . . . . .	443
16.5.3	Collision Probabilities in Two-Dimensional Geometry . . . . .	443
16.6	Interface Current Balance Methods . . . . .	445
16.6.1	Formulation . . . . .	445
16.6.2	Transmission and Escape Probabilities . . . . .	445
16.6.3	2D Transmission/Escape Probabilities (TEP) Method . . . . .	447
16.6.4	1D Slab Method . . . . .	452
16.7	Discrete Ordinates Methods . . . . .	453
16.7.1	$P_L$ and $D-P_L$ Ordinates . . . . .	454
16.8	Monte Carlo Methods . . . . .	456
16.8.1	Probability Distribution Functions . . . . .	456
16.8.2	Analog Simulation of Neutral Particle Transport . . . . .	457
16.8.3	Statistical Estimation . . . . .	459
16.9	Navier–Stokes Fluid Model . . . . .	460
<b>17</b>	<b>Power Balance</b> . . . . .	<b>463</b>
17.1	Energy Confinement Time . . . . .	463
17.1.1	Definition . . . . .	463
17.1.2	Experimental Energy Confinement Times . . . . .	464
17.1.3	Empirical Correlations . . . . .	465
17.2	Radiation . . . . .	468
17.2.1	Radiation Fields . . . . .	468
17.2.2	Bremsstrahlung . . . . .	470
17.2.3	Cyclotron Radiation . . . . .	471
17.3	Impurities . . . . .	473
17.4	Burning Plasma Dynamics . . . . .	475
<b>18</b>	<b>Operational Limits</b> . . . . .	<b>479</b>
18.1	Disruptions . . . . .	479
18.1.1	Physics of Disruptions . . . . .	479
18.1.2	Causes of Disruptions . . . . .	481
18.2	Disruption Density Limit . . . . .	481
18.2.1	Radial Temperature Instabilities . . . . .	483
18.2.2	Spatial Averaging . . . . .	485
18.2.3	Coupled Radial Temperature–Density Instabilities . . . . .	487
18.3	Nondisruptive Density Limits . . . . .	490
18.3.1	MARFES . . . . .	490
18.3.2	Confinement Degradation . . . . .	491
18.3.3	Thermal Collapse of Divertor Plasma . . . . .	494
18.4	Empirical Density Limit . . . . .	495

18.5 MHD Instability Limits . . . . .	495
18.5.1 $\beta$ -Limits . . . . .	495
18.5.2 Kink Mode Limits on $q(a)/q(0)$ . . . . .	498
<b>19 Fusion Reactors and Neutron Sources</b>	<b>501</b>
19.1 Plasma Physics and Engineering Constraints . . . . .	501
19.1.1 Confinement . . . . .	501
19.1.2 Density Limit . . . . .	502
19.1.3 <i>Beta</i> Limit . . . . .	503
19.1.4 Kink Stability Limit . . . . .	504
19.1.5 Start-Up Inductive Volt-Seconds . . . . .	504
19.1.6 Noninductive Current Drive . . . . .	505
19.1.7 Bootstrap Current . . . . .	506
19.1.8 Toroidal Field Magnets . . . . .	506
19.1.9 Blanket and Shield . . . . .	507
19.1.10 Plasma Facing Component Heat Fluxes . . . . .	507
19.1.11 Radiation Damage to Plasma Facing Components . . . . .	510
19.2 International Tokamak Program . . . . .	511
19.2.1 Advanced Tokamak . . . . .	514
19.3 Neutron Sources . . . . .	515
<b>Appendices</b>	
A Frequently Used Physical Constants . . . . .	521
B Dimensions and Units . . . . .	523
C Vector Calculus . . . . .	527
D Curvilinear Coordinates . . . . .	529
E Plasma Formulas . . . . .	537
F Further Reading . . . . .	539
G Attributions . . . . .	543
<b>Subject Index</b> . . . . .	<b>549</b>