Contents

Preface *XIII*
Abbreviations and Notations *XIX*

1 The Failure of Classical Physics *1*
1.1 Blackbody Radiation *1*
1.2 Heat Capacity *4*
1.3 The Photoelectric Effect *9*
1.4 Atoms and Their Spectra *12*
1.5 The Double-Slit Experiment *14*
Problem *19*
References *19*

2 The First Steps into the Unknown *21*
2.1 The BBR and Planck's Formula *21*
2.2 Einstein's Light Quanta and BBR *24*
2.2.1 Discussion *27*
2.3 PEE Revisited *30*
2.4 The Third Breakthrough: de Broglie Waves *31*
2.4.1 Exercise *33*
Problems *35*
References *35*

3 Embryonic Quantum Mechanics: Basic Features *37*
3.1 A Glimpse of the New Realm *37*
3.2 Quantum-Mechanical Superposition of States *39*
3.3 What Is Waving There (the Meaning of the Ψ-Function)? *42*
3.4 Observables and Their Operators *47*
3.5 Quantum-Mechanical Indeterminacy *49*
3.6 Indeterminacy and the World *53*
3.7 Quantum Entanglement and Nonlocality *58*
3.8 Quantum-Mechanical Phase Space *62*
3.9 Determinism and Causality in Quantum World *63*

3.9.1 Discussion *63*
Problems *66*
References *66*

4 Playing with the Amplitudes *69*
4.1 Composition of Amplitudes *69*
4.2 Double Slit Revised I *74*
4.3 Double Slit Revised II *77*
4.4 Neutron Scattering in Crystals *78*
4.5 Bosonic and Fermionic States *81*
4.6 Path Integrals *89*
Problems *93*
References *93*

5 Basic Features and Mathematical Structure of QM *95*
5.1 Observables: the Domain of Classical and Quantum Mechanics *95*
5.2 Quantum-Mechanical Operators *97*
5.3 Algebra of Operators *100*
5.4 Eigenvalues and Eigenstates *102*
5.5 Orthogonality of Eigenstates *107*
5.6 The Robertson–Schrödinger Relation *110*
5.7 The Wave Function and Measurements (Discussion) *112*
Problems *116*
References *117*

6 Representations and the Hilbert Space *119*
6.1 Various Faces of a State Function *119*
6.2 Unitary Transformations *121*
6.3 Operators in the Matrix Form *125*
6.4 The Hilbert Space *129*
6.5 Operations in the Hilbert Space *135*
6.6 Nonorthogonal States *142*
Problems *147*
References *148*

7 Angular Momentum *149*
7.1 Orbital and Spin Angular Momenta *149*
7.2 The Eigenstates and Eigenvalues of $\hat{L}$ *151*
7.3 Operator $\hat{L}$ and Its Commutation Properties *154*
7.4 Spin as an Intrinsic Angular Momentum *164*
7.5 Angular Momentum of a Compound System *183*
7.6 Spherical Harmonics *188*
Problems *196*
References *197*

8 The Schrödinger Equation *199*
8.1 The Schrödinger Equation *199*
8.2 State Function and the Continuity Equation *200*
8.3 Separation of Temporal and Spatial Variables: Stationary States *203*
8.4 The Helmholtz Equation and Dispersion Equation for a Free Particle *205*
8.5 Separation of Spatial Variables and the Radial Schrödinger Equation *207*
8.6 Superposition of Degenerate States *209*
8.7 Phase Velocity and Group Velocity *212*
8.8 de Broglie's Waves Revised *218*
8.9 The Schrödinger Equation in an Arbitrary Basis *222*
Problems *226*
References *226*

9 Applications to Simple Systems: One Dimension *227*
9.1 A Quasi-Free Particle *227*
9.2 Potential Threshold *232*
9.3 Tunneling through a Potential Barrier *236*
9.4 Cold Emission *241*
9.5 Potential Well *244*
9.6 Quantum Oscillator *249*
9.7 Oscillator in the *E*-Representation *254*
9.8 The Origin of Energy Bands *257*
9.9 Periodic Structures *260*
Problems *269*
References *271*

10 Three-Dimensional Systems *273*
10.1 A Particle in a 3D Box *273*
10.2 A Free Particle in 3D (Spherical Coordinates) *274*
10.2.1 Discussion *277*
10.3 Some Properties of Solutions in Spherically Symmetric Potential *277*
10.4 Spherical Potential Well *278*
10.5 States in the Coulomb Field and a Hydrogen Atom *281*
10.6 Atomic Currents *287*
10.7 Periodic Table *290*
Problems *293*
References *294*

11 Evolution of Quantum States *295*
11.1 The Time Evolution Operator *295*
11.2 Evolution of Operators *299*

11.3 Spreading of a Gaussian Packet *301*
11.4 The *B*-Factor and Evolution of an Arbitrary State *303*
11.5 The Fraudulent Life of an "Illegal" Spike *306*
11.6 Jinnee Out of the Box *311*
11.7 Inadequacy of Nonrelativistic Approximation in Description of Evolving Discontinuous States *315*
11.7.1 Discussion *316*
11.8 Quasi-Stationary States *317*
11.8.1 Discussion *323*
11.9 3D Barrier and Quasi-Stationary States *324*
11.10 The Theory of Particle Decay *327*
11.11 Particle–Antiparticle Oscillations *331*
11.11.1 Discussion *337*
11.12 A Watched Pot Never Boils (Quantum Zeno Effect) *339*
11.13 A Watched Pot Boils Faster *344*
Problems *350*
References *352*

12 Quantum Ensembles *355*
12.1 Pure Ensembles *355*
12.2 Mixtures *356*
12.3 The Density Operator *358*
12.4 Time Evolution of the Density Operator *366*
12.5 Composite Systems *368*
Problems *376*
References *376*

13 Indeterminacy Revisited *377*
13.1 Indeterminacy Under Scrutiny *377*
13.2 The Heisenberg Inequality Revised *380*
13.3 The Indeterminacy of Angular Momentum *382*
13.4 The Robertson–Schrödinger Relation Revised *384*
13.5 The N–ϕ Indeterminacy *388*
13.6 Dispersed Indeterminacy *390*
Problems *394*
References *395*

14 Quantum Mechanics and Classical Mechanics *397*
14.1 Relationship between Quantum and Classical Mechanics *397*
14.2 QM and Optics *400*
14.3 The Quasi-Classical State Function *401*
14.4 The WKB Approximation *404*
14.5 The Bohr–Sommerfeld Quantization Rules *406*
Problems *409*
References *410*

15 Two-State Systems *411*
15.1 Double Potential Well *411*
15.2 The Ammonium Molecule *415*
15.3 Qubits Introduced *419*
Problem *422*
References *422*

16 Charge in Magnetic Field *423*
16.1 A Charged Particle in EM Field *423*
16.2 The Continuity Equation in EM Field *425*
16.3 Origin of the A-Momentum *427*
16.4 Charge in Magnetic Field *429*
16.5 Spin Precession *432*
16.6 The Aharonov–Bohm Effect *437*
16.6.1 Discussion *441*
16.7 The Zeeman Effect *442*
Problems *444*
References *445*

17 Perturbations *447*
17.1 Stationary Perturbation Theory *447*
17.1.1 Discussion *450*
17.2 Asymptotic Perturbations *455*
17.3 Perturbations and Degeneracy *457*
17.4 Symmetry, Degeneracy, and Perturbations *460*
17.5 The Stark Effect *462*
17.6 Time-Dependent Perturbations *465*
Problems *471*
References *471*

18 Light–Matter Interactions *473*
18.1 Optical Transitions *473*
18.2 Dipole Radiation *474*
18.3 Selection Rules *477*
18.3.1 Oscillator *478*
18.3.2 Hydrogen-Like Atom *478*
Problems *480*
Reference *480*

19 Scattering *481*
19.1 QM Description of Scattering *481*
19.2 Stationary Scattering *487*
19.3 Scattering Matrix and the Optical Theorem *490*
19.4 Diffraction Scattering *494*
19.5 Resonant Scattering *498*

19.6 The Born Approximation *501*
Problems *504*
References *505*

20 Submissive Quantum Mechanics *507*
20.1 The Inverse Problem *507*
20.2 Playing with Quantum States *509*
20.3 Playing with Evolution: Discussion *514*
Problems *522*
References *522*

21 Quantum Statistics *525*
21.1 Bosons and Fermions: The Exclusion Principle *525*
21.1.1 Discussion *531*
21.2 Planck and Einstein Again *540*
21.3 BBR Again *542*
21.4 Lasers and Masers *543*
Problems *545*
References *546*

22 Second Quantization *547*
22.1 Quantum Oscillator Revisited *547*
22.2 Creation and Annihilation Operators: Bosons *548*
22.3 Creation and Annihilation Operators: Fermions *552*
Problems *555*
References *555*

23 Quantum Mechanics and Measurements *557*
23.1 Collapse or Explosion? *557*
23.2 "Schrödinger's Cat" and Classical Limits of QM *563*
23.3 Von Neumann's Measurement Scheme *571*
23.3.1 Discussion *575*
23.4 Quantum Information and Measurements *578*
23.5 Interaction-Free Measurements: Quantum Seeing in the Dark *586*
23.6 QM and the Time Arrow *593*
Problems *595*
References *596*

24 Quantum Nonlocality *599*
24.1 Entangled Superpositions I *599*
24.2 Entangled Superpositions II *601*
24.2.1 Discussion *604*
24.3 Quantum Teleportation *604*
24.4 The "No-Cloning" Theorem *607*

24.5 Hidden Variables and Bell's Theorem *613*
24.6 Bell-State Measurements *619*
24.7 QM and the Failure of FTL Proposals *627*
24.8 Do Lasers Violate the No-Cloning Theorem? *628*
24.9 Imperfect Cloning *636*
24.10 The FLASH Proposal and Quantum Compounds *643*
Problems *649*
References *649*

25 Quantum Measurements and POVMs *651*
25.1 Projection Operator and Its Properties *651*
25.2 Projective Measurements *655*
25.3 POVMs *658*
25.4 POVM as a Generalized Measurement *664*
25.5 POVM Examples *666*
25.6 Discrimination of Two Pure States *670*
25.7 Neumark's Theorem *681*
25.8 How to Implement a Given POVM *686*
25.9 Comparison of States and Mixtures *695*
25.10 Generalized Measurements *697*
Problems *700*
References *701*

26 Quantum Information *703*
26.1 Deterministic Information and Shannon Entropy *703*
26.2 von Neumann Entropy *709*
26.3 Conditional Probability and Bayes's Theorem *711*
26.4 KL Divergence *716*
26.5 Mutual Information *717*
26.6 Rényi Entropy *719*
26.7 Joint and Conditional Renyi Entropy *721*
26.8 Universal Hashing *726*
26.9 The Holevo Bound *731*
26.10 Entropy of Entanglement *733*
Problems *734*
References *735*

27 Quantum Gates *737*
27.1 Truth Tables *737*
27.2 Quantum Logic Gates *741*
27.3 Shor's Algorithm *746*
Problems *751*
References *752*

28 Quantum Key Distribution *753*
28.1 Quantum Key Distribution (QKD) with EPR *753*
28.2 BB84 Protocol *758*
28.3 QKD as Communication Over a Channel *766*
28.4 Postprocessing of the Key *769*
28.5 B92 Protocol *776*
28.6 Experimental Implementation of QKD Schemes *779*
28.7 Advanced Eavesdropping Strategies *788*
Problems *793*
References *793*

Appendix A: Classical Oscillator *795*
Reference *799*

Appendix B: Delta Function *801*
Reference *807*

Appendix C: Representation of Observables by Operators *809*
Appendix D: Elements of Matrix Algebra *813*
Appendix E: Eigenfunctions and Eigenvalues of the Orbital Angular Momentum Operator *817*
Appendix F: Hermite Polynomials *821*
Appendix G: Solutions of the Radial Schrödinger Equation in Free Space *825*
Appendix H: Bound States in the Coulomb Field *827*
Reference *829*

Index *831*