# 1
# Rocket Fundamentals

Many people have had, and still have, misconceptions about the basic principle of a rocket. Here is a comment of the publisher of the renowned *New York Times* from 1921 about the pioneer of US astronautics, Robert Goddard, who at that time was carrying out the first experiments with liquid propulsion engines:

> "Professor Goddard … does not know the relation of action to reaction, and of the need to have something better than a vacuum against which to react – to say that would be absurd. Of course he only seems to lack the knowledge ladled out daily in high schools."

The publisher's doubts whether rocket propulsion in vacuum could work is based on our daily experience that you can only move forwards by pushing backwards against an object or medium. Rowing is based on the same principle. You use the blades of the oars to push against the water. But this example already shows that the medium you push against, which is water, does not have to be at rest, it may move backwards. So basically it would suffice to fill a blade with water and push against it by very quickly guiding the water backwards with the movement of the oars. Of course, the forward thrust of the boat gained thereby is much lower compared with rowing with the oars in the water, as the large displacement resistance in the water means that you push against a far bigger mass of water. But the principle is the same. Instead of pushing water backwards with a blade, you could also use a pile of stones in the rear of your boat, and hurl them backwards as fast as possible. With this you would push ahead against the accelerating stone. And this is the basis of the propulsion principle of a rocket: it pushes against the gases it ejects backwards with full brunt. So, with the propellant, the rocket carries the mass against which it pushes to move forwards, and this is why it also works in vacuum.

This repulsion principle, which is called the "rocket principle" in astronautics, is based on the physical principle of conservation of momentum. It states that the total (linear) momentum of a system remains constant with time. If, at initial time $t_0$ the boat (rocket) with mass $m_1$ plus stone (propellant) with mass $m_2$ had velocity $v_0$, implying that the initial total momentum

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was $\boldsymbol{p}_0(t_0) = (m_1 + m_2)\boldsymbol{v}_0$, then, at some time $t_+ > t_0$, on hurling the stone (propellant) away with velocity $\boldsymbol{v}_2$ the boat will have velocity $\boldsymbol{v}_1$ (neglecting water friction) and the total momentum $\boldsymbol{p}\,(t_+) = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$ must be the same. That is

$$\boldsymbol{p}\,(t_0) = \boldsymbol{p}\,(t_+) \qquad \textbf{principle of the conservation of (linear) momentum}$$

from which follows

$$(m_1 + m_2) \cdot \boldsymbol{v}_0 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

**Note:** *The principle of conservation of momentum is only valid for the vector form of the momentum equation, which is quite often ignored. A bomb that is ignited generates a huge amount of momentum out of nothing, which apparently would invalidate an absolute value form of the momentum equation. But if you add up the vectorial momenta of the bomb's fragments, it becomes obvious that the vectorial linear momentum has been conserved.*

Given $m_1$, $m_2$, $\boldsymbol{v}_0$ and velocity $\boldsymbol{v}_2$ of the stone (propellant) expelled, one is able to calculate from this equation the increased boat (rocket) velocity $\boldsymbol{v}_1$. Doing so, this equation affirms our daily experience that hurling the stone backwards increases the speed of the boat, while doing it forwards decreases its speed.

## 1.1 The Rocket Principle

With a rocket, the situation is a bit more complicated, as it does not eject one stone after another, but it emits a continuous gas jet. It can be shown (see Ruppe (1966, p. 24ff)) that ejecting the same amount of mass continuously rather than in chunks maximizes the achievable thrust. In order to describe the gain of rocket speed by the continuous mass ejection stream adequately in mathematical and physical terms, we have to consider the ejected mass and time steps as infinitesimally small and in an external rest frame, a so-called inertial (unaccelerated, see Section 13.1) reference system. This is depicted in Fig. 1.1, where in an inertial reference system with its origin at the center of the Earth a rocket with mass $m$ in space experiences no external forces.

At a given time $t$ the rocket may have velocity $\boldsymbol{v}$ and momentum $\boldsymbol{p}(t) = m\boldsymbol{v}$. By ejecting the propellant mass $dm_p > 0$ with **effective exhaust velocity** $\boldsymbol{v}_*$ – the meaning of which will become clear in the next section – and hence with propellant momentum $\boldsymbol{p}_p\,(t + dt) = (\boldsymbol{v} + \boldsymbol{v}_*) \cdot dm_p$, it will lose part of its mass $dm = -dm_p < 0$ and hence gain rocket speed $d\boldsymbol{v}$ by acquiring momentum $\boldsymbol{p}_r\,(t + dt) = (m + dm)\,(\boldsymbol{v} + d\boldsymbol{v})$.

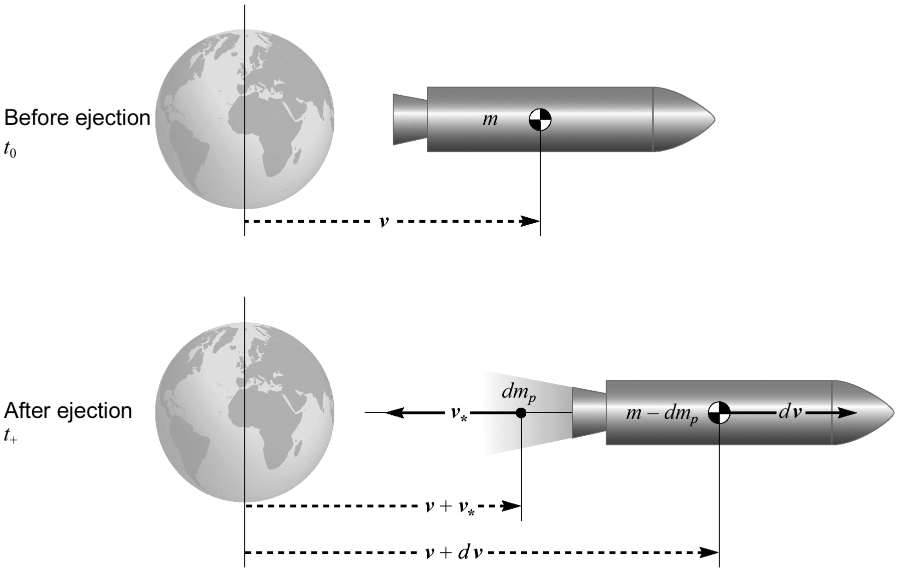


**Figure 1.1** A rocket in force-free space before (above) and after (below) it ejected a propellant mass $dm_p$ with effective exhaust velocity $v_*$, thereby gaining speed $dv$. Velocities relative to the external inertial system (Earth) are dashed, and those with regard to the rocket are solid.

**Note:** *In the literature $dm > 0$ often denotes the positive mass flow rate of the propellant, and m the mass of the rocket. This is inconsistent, and leads to an erroneous mathematical description of the relationships, because if m is the mass of the rocket, logically dm has to be the mass change of the rocket, and thus it has to be negative. This is why in this book we will always discriminate between rocket mass and propulsion mass using the consistent description $dm = -dm_p < 0$ implying $\dot{m} = -\dot{m}_p < 0$ for their flows.*

For this line of events we can apply the principle of conservation of momentum as follows:

$$\boldsymbol{p}(t) = \boldsymbol{p}\,(t + dt) = \boldsymbol{p}_p\,(t + dt) + \boldsymbol{p}_r\,(t + dt)$$

From this follows

$$\begin{aligned} m\boldsymbol{v} &= -dm\,(\boldsymbol{v} + \boldsymbol{v}_*) + (m + dm)\,(\boldsymbol{v} + d\boldsymbol{v}) \\ &= m\boldsymbol{v} - dm \cdot \boldsymbol{v}_* + m \cdot d\boldsymbol{v} + dm \cdot d\boldsymbol{v} \end{aligned}$$

As the double differential $dm \cdot d\boldsymbol{v}$ mathematically vanishes with respect to the single differentials $dm$ and $d\boldsymbol{v}$, we get with division by $dt$:

$$m\dot{\boldsymbol{v}} = \dot{m}\boldsymbol{v}_* \qquad (1.1.1)$$

According to Newton's second law (Eq. (7.1.12), $\boldsymbol{F} = m\dot{\boldsymbol{v}}$, the term on the left side corresponds to a force due to the repulsion of the propellant, which we correspondingly indicate by

$$\boldsymbol{F}_* = \dot{m}\boldsymbol{v}_* \tag{1.1.2}$$

with $\dot{m} = -\dot{m}_p < 0$. This means that the thrust of a rocket is determined by the product of propellant mass flow rate and exhaust velocity. Observe that due to $\dot{m} < 0$ $\boldsymbol{F}_*$ is exactly in opposite direction to the exhaust velocity $\boldsymbol{v}_*$ (But depending on the steering angle of the engine, $\boldsymbol{v}_*$ and hence $\boldsymbol{F}_*$ does not necessarily have to be in line of the flight direction $\boldsymbol{v}$.). Therefore with regard to the absolute values we can write

$$\boxed{F_* = -\dot{m}v_* = \dot{m}_p v_*} \qquad \textbf{propellant force (thrust)} \tag{1.1.3}$$

Equation (1.1.2), or (1.1.3) respectively, is of vital importance for astronautics, as it describes basic physical facts, just like every other physical relationship, relating just three parameters, such as $W = F \cdot s$ or $U = R \cdot I$. This is its statement: thrust is the product of exhaust velocity times mass flow rate. Only the two properties together make up a powerful thruster. The crux of the propellant is not its "energy content" (in fact the energy to accelerate the propellant might be provided externally, which is the case with ion propulsions), but the fact that it possesses mass, which is ejected backwards, and thus accelerates the rocket forwards by means of conservation of momentum. The higher the mass flow rate, the larger the thrust. If "a lot of thrust" is an issue, for instance during launch, when the thrust has to overcome the gravitational pull of the Earth, and since the exhaust speed of engines is limited, you need thrusters with a huge mass flow rate. The more the better. Each of the five first-stage engines of a Saturn V rocket had a mass flow rate of about 2.5 metric tons per second, in total 12.5 tons per second, to achieve the required thrust of 33 000 N (corresponds to 3400 tons of thrust). This tremendous mass flow rate is exactly why, for launch, chemical thrusters are matchless up to now, and they will certainly continue to be so for quite some time.

## 1.2 Rocket Thrust

### 1.2.1 Pressure Becomes Thrust

If the masses $dm_p$ were stones, and if we hurled them backwards, then the thrust would just be the repulsion of the stones. But generally we hurl gases with the engine. Gases are a loose accumulation of molecules, which, depending on temperature, display internal molecular motion, and thus generate pressure. On the other hand, the rocket at launch moves in an atmosphere whose gas molecules exert an external pressure. In order to understand the impact of the propellant gas pressure and external ambient pressure on the engine's thrust, let's have a look at the pressure conditions in an engine (see Fig. 1.2).

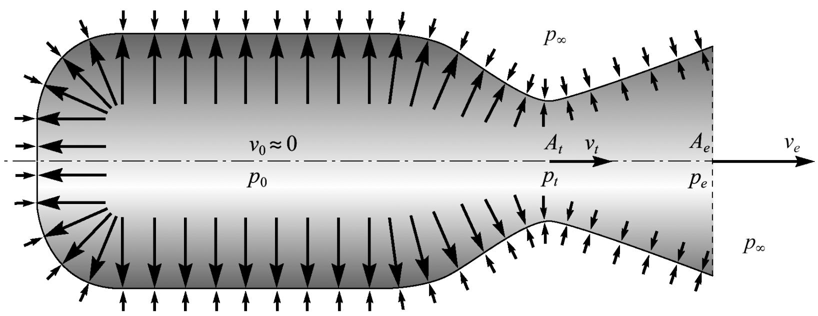


**Figure 1.2** Pressure conditions inside and outside an engine chamber.

Inside the combustion chamber, and depending on the location within the chamber, we assume a variable pressure $p_{int}$, which exerts the force $d\boldsymbol{F}_{int} = p_{int} \cdot d\boldsymbol{A}$ on a wall segment $d\boldsymbol{A}$. In the area surrounding the chamber we assume an equal external ambient pressure $p_\infty$. The propellant force $\boldsymbol{F}_*$ generated by the chamber must be the sum of all effective forces acting on the entire engine wall with surface $S$

$$\boldsymbol{F}_* = \iint_S d\boldsymbol{F}_{eff} = \iint_S (p_{int} - p_\infty) \cdot d\boldsymbol{A} \tag{1.2.1}$$

The surface vector can be split into two components: a r̲adial component $\boldsymbol{u}_r$ and an ax̲ial component $\boldsymbol{u}_x$ (Fig. 1.3),

$$d\boldsymbol{A} = d\boldsymbol{A}_r + d\boldsymbol{A}_x = (\sin\theta \cdot \boldsymbol{u}_r + \cos\theta \cdot \boldsymbol{u}_x) \cdot dA$$

where the wall angle $\theta$ is the angle between surface normal and chamber axis. If the *combustion chamber is axially symmetric*, then we have

$$\iint_S (p_{int} - p_\infty) \cdot d\boldsymbol{A}_r = 0$$

and therefore we only get axial contributions

$$\boldsymbol{F}_* = \iint_S (p_{int} - p_\infty) \cdot d\boldsymbol{A}_x = \boldsymbol{u}_x \iint_S (p_{int} - p_\infty) \cos\theta \cdot dA \tag{1.2.2}$$

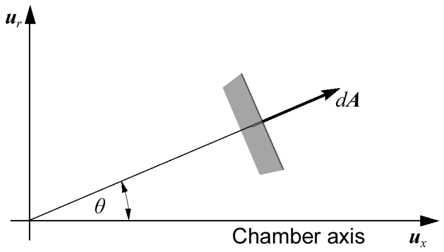


**Figure 1.3** Definition of the wall angle with regard to the chamber axis.

Maintaining the internal pressure conditions, and thus without a change in thrust, we now deform the combustion chamber, so that we get a rectangular combustion chamber (see Fig. 1.4). Now that all wall angles are only $\theta = 0°$, $90°$, $180°$, $270°$ the following is valid

$$F_* = -\iint_{A_\phi} (p_{int} - p_\infty)\,(-1) \cdot dA - \iint_{A_\phi - A_t} (p_{int} - p_\infty) \cdot dA \tag{1.2.3}$$

where $F_*$ now expresses the propellant force of the combustion chamber in forward direction, the direction in which the total force is effectively pushing.

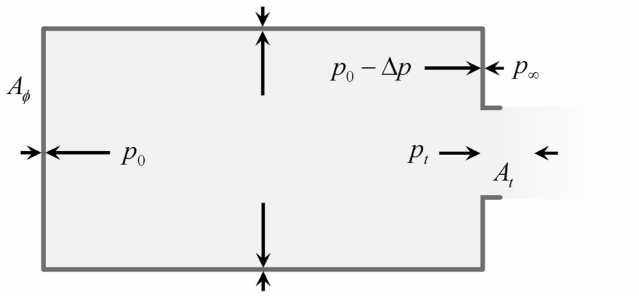


**Figure 1.4** Pressure conditions in the idealized combustion chamber.

As there is no wall at the throat with the surface $A_t$, no force can be exerted on it, and thus on the chamber's back side the integral is limited to the surface $A_\phi - A_t$. The maximum combustion chamber pressure $p_{int} = p_0$ is on the front side of the chamber, where the gas is about at rest. Because the gas flow increases in the direction of the throat where it exits the chamber, the pressure at the rear of the chamber is reduced by a certain amount $\Delta p$: $p_{int} = p_0 - \Delta p(r)$, and due to the axial symmetry of the chamber this pressure drop is also axially symmetrical, so that at the throat $p_{int} = p_0 - \Delta p(r) = p_t$ applies.

So Eq. (1.2.3) reads as follows:

$$F_* = (p_0 - p_\infty)\, A_\phi - \iint\limits_{A_\phi - A_t} (p_0 - p_\infty) \cdot dA + \iint\limits_{A_\phi - A_t} \Delta p \cdot dA$$

As

$$\iint\limits_{A_\phi - A_t} (p_0 - p_\infty) \cdot dA = (p_0 - p_\infty) \left(A_\phi - A_t\right)$$

and

$$\iint\limits_{A_\phi - A_t} \Delta p \cdot dA = \iint\limits_{A_\phi} \Delta p \cdot dA - \iint\limits_{A_t} \Delta p \cdot dA = \iint\limits_{A_\phi} \Delta p \cdot dA - (p_0 - p_t)\, A_t$$

we get

$$F_* = (p_t - p_\infty)\, A_t + \iint\limits_{A_\phi} \Delta p \cdot dA \tag{1.2.4}$$

Let's have a closer look at the integral of the last equation. It describes a force which results from the pressure reduction along the rear combustion chamber wall. This pressure reduction is due to the propellant flow through the throat. This mass flow, of course, does not generate a sudden pressure drop at the rear wall, but rather a pressure gradient along the chamber axis, i.e.

$$\iint\limits_{A_\phi} \Delta p \cdot d\boldsymbol{A} \to - \iiint\limits_{chamber} \boldsymbol{\nabla} p \cdot dV$$

The pressure gradient corresponds to an acceleration field $d\boldsymbol{v}/dt$ of the mass flow. According to the Euler equation of hydrodynamics, they are intimately connected with each other via the mass density $\rho$:

$$-\boldsymbol{\nabla} p = \rho \frac{d\boldsymbol{v}}{dt} \qquad \textbf{Euler equation}$$

This equation expresses Newton's law in hydrodynamics. If we apply the Euler equation to the volume integral, we obtain

$$\iiint\limits_{chamber} \boldsymbol{\nabla} p \cdot dV = - \iiint\limits_{chamber} \frac{d\boldsymbol{v}}{dt} \frac{dm_p}{dV} dV = - \int\limits_0^{\boldsymbol{v}_t} \dot{m}_p \cdot d\boldsymbol{v}$$

The velocity integral now ranges from the velocity at the front part of the chamber, where the pressure gradient (and hence the drift velocity of the propellant) is zero, to its throat, where the velocity takes on the exit value $\boldsymbol{v}_t$.

According to the continuity equation (Eq. (1.2.9)), the mass flow rate $\dot{m}_p$ is invariant along the combustion chamber and also in the subsequent nozzle, and thus it is constant. So we find

$$\iint\limits_{A_\phi} \Delta p \cdot d\boldsymbol{A} = -\iiint\limits_{chamber} \boldsymbol{\nabla} p \cdot dV = \dot{m}_p \int\limits_0^{\boldsymbol{v}_t} d\boldsymbol{v} = \dot{m}_p \boldsymbol{v}_t \tag{1.2.5}$$

If we apply this result to Eq. (1.2.4), we get

$$F_* = \dot{m}_p v_t + (p_t - p_\infty) A_t$$

So far our considerations have been independent of the exact form of the combustion chamber, as long as it is axially symmetric. So we can consider the nozzle to be also a part of the combustion chamber. Then all the parameters considered so far at the throat of the combustion chamber are also valid for the nozzle exit, i.e.

$$\boxed{F_* = \dot{m}_p v_e + (p_e - p_\infty) A_e =: F_e + F_p} \tag{1.2.6}$$

We recover its vectorial form by the direction information in Eq. (1.2.2)

$$\boldsymbol{F}_* = -\boldsymbol{u}_e \left[\dot{m}_p v_e + (p_e - p_\infty) A_e\right] \tag{1.2.7}$$

where $\boldsymbol{u}_e$ is the unit vector of the exit surface in the direction of the exhaust jet and $v_e$ the **exhaust velocity**. The first term on the right side of Eq. (1.2.6) is called **momentum thrust** $F_e$, and the second term is called **pressure thrust** $F_p$. The first name is well chosen, because if you integrate expression $\dot{m}_p v_e$ with regard to time, you get the momentum $m_p v_e$, which is merely the recoil momentum of the ejected propellant. The second term is formally not quite correct, as according to Eq. (1.2.5), the momentum thrust is also generated by a pressure on the chamber because of its internal pressure gradient. At the end it's all pressure which accelerates the engine, and with it the rocket.

### Effective exhaust velocity

If we compare Eq. (1.2.6) with Eq. (1.1.3), we can see that the effective exhaust velocity is made up of two contributions:

$$v_* = v_e + (p_e - p_\infty)\frac{A_e}{\dot{m}_p} \qquad \textbf{effective exhaust velocity} \tag{1.2.8}$$

The expression "effective exhaust velocity" makes it clear that it is not only about exhaust velocity $v_e$, but modified by the pressure thrust. However, for a real thruster the pressure thrust indeed is only a small contribution. For an ideally adapted nozzle with $p_e \approx p_\infty$ (Section 4.1.6) it even is negligibly small.

### 1.2.2
### Momentum Thrust and Pressure Thrust

Ultimately, if it is only pressure that drives a rocket, how does this fit together with the rocket principle discussed in Section 1.1, which was based on repulsion and not on pressure? And what is the physical meaning of "pressure thrust"? You often find the statement that pressure thrust occurs when the pressure at the exit (be it nozzle or chamber exit) hits the external pressure. The pressure difference at this point times the surface is supposed to be the pressure thrust. Though the result is right, the explanation is not. First, the exit pressure does not abruptly meet the external pressure. There is rather a smooth pressure transition from the exit pressure to the external pressure covering in principle an infinite volume behind the engine. Second, even if such a pressure difference could be traced back mathematically to a specific surface, this would not cause a thrust, because, as we will see later, the gas in the nozzle expands backwards with supersonic speed, and such a gas cannot have a causal effect on the engine to exert a thrust on it.

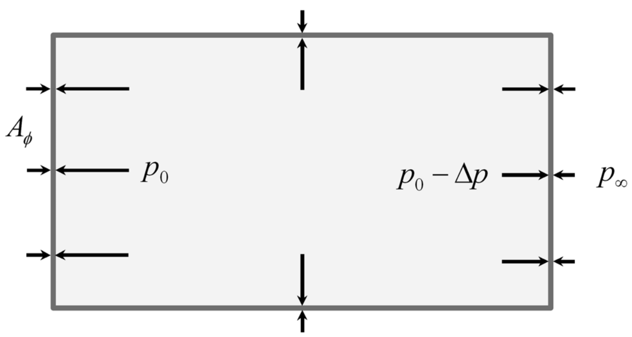


**Figure 1.5** Pressure conditions of the idealized combustion chamber if it could be, hypothetically, fully closed.

For a true explanation let's imagine for a moment, and purely hypothetically, a fully closed combustion chamber (see Fig. 1.5) with the same pressure conditions as in the idealized combustion chamber with mass flow rate (see Fig. 1.4). The surface force on the front side would be $F_{front} = (p_0 - p_\infty)A_\phi$ on the front side, and $F_{rear} = (p_0 - \Delta p - p_\infty)A_\phi$ on the rear side. Hence the net forward thrust would be $F_* = F_{front} - F_{rear} = \Delta p \cdot A_\phi$. Because the wall angle on the rear side is $0°$ and because of Eq. (1.2.5), this translates into $F_* = \Delta p \cdot A_\phi = \dot{m}_p v_t$. Therefore, we can say the following:

The **momentum thrust** $F_e$ physically results from the fact that, in a hypothetically closed engine chamber, due to the mass flow rate $\dot{m}_p$ there is a bigger chamber pressure on the front side compared to the by $\Delta p$ smaller pressure on the back side. This causes a net pressure force $\Delta p \cdot A_\phi$.

Ultimately it is the Euler equation, which relates the mass flow rate $\dot{m}_p$ with the pressure differences in the pressure chamber. In order to have the hypothetical gas flow indeed flowing, we need to make a hole with area $A_t$ into the rear side (see Fig. 1.4). Once this is done, the counterthrust at the rear side decreases by

$$\Delta F_{rear} = -(p_0 - \Delta p - p_\infty)A_t = -(p_t - p_\infty)A_t$$

which in turn increases the forward thrust by the same amount. But this is just the pressure thrust. Therefore:

The **pressure thrust** $F_p$ is the additional thrust which originates from the absence of the counter-pressure force at the exit opening of the engine.

If the exit pressure happens to be equal to the external pressure, then the external pressure behaves like a wall, the pressure thrust vanishes, and we have an ideally adapted nozzle (see Section 4.1.6).

### 1.2.3 Continuity Equation

The momentum thrust can also be described in a different mathematical form. Let's have a general look at the behavior of propellant gas perfusing an engine. A propellant mass $dm_p$ perfuses a given cross section of the engine with area $A$ with velocity $v$ (see Fig. 1.6). During the time interval $dt$, the volume of amount $dV = A \cdot ds = Av \cdot dt$ will have passed through it. Therefore

$$dm_p = \rho \cdot dV = \rho Av \cdot dt$$

where $\rho$ is the mass density, which we assume to be constant. From this we derive the mass flow rate equation

$$\boxed{\dot{m}_p = \rho v A} \qquad \textbf{continuity equation} \text{ (conservation of mass)} \qquad (1.2.9)$$

A constant mass density simply means that nowhere within the volume new mass is generated or disappears. This is exactly what the word "continuity"

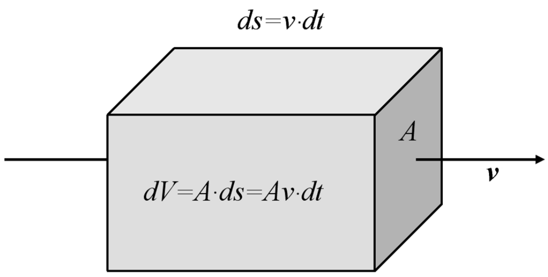


**Figure 1.6** The volume $dV$ which a mass flow with velocity $v$ passes in time $dt$.

means. We could also call it "conservation of mass". So the conservation of mass directly implies Eq. (1.2.9).

At the engine exit, the continuity equation reads $\dot{m}_p = \rho_e v_e A_e$. Applying this to Eq. (1.2.6) yields

$$F_e = \dot{m}_p v_e = \rho_e A_e v_e^2 \tag{1.2.10}$$

This equation begs the question whether the momentum thrust is linearly or quadratically dependent on $v_e$. The answer depends on the engine in question. Depending on the type (e.g. electric or chemical engine) of engine, a change of its design in general will vary all the parameters $v_e$ and $\dot{m}_p$, $\rho_e$, $A_e$ in a specific way. This is why the demanding goal of engine design is to tune all the engine parameters, including $v_e$, such that the total thrust is maximized. Hence it is not only $v_e$ alone which is decisive for the momentum thrust of an engine, but it is necessary to adjust all the relevant engine parameters in a coordinated way.

## 1.3 Rocket Performance

The mechanical power of an exhaust jet, the so called **jet power**, is defined as the change of the kinetic energy of the ejected gas (jet energy) per time unit, i.e.

$$P_e := \frac{dE_e}{dt} = \frac{d}{dt}\left(\frac{1}{2}m_p v_e^2\right) = \frac{1}{2}\dot{m}_p v_e^2 = \frac{1}{2}F_e v_e \qquad \textbf{jet power} \tag{1.3.1}$$

It describes the time rate of expenditure of the jet energy. The thrust power of an engine is the thrust energy generated per time unit, i.e.

$$P_* := \frac{dE_*}{dt} = \frac{d}{dt}\left(\frac{1}{2}m_p v_*^2\right) = \frac{1}{2}\dot{m}_p v_*^2 = \frac{1}{2}F_* v_* \qquad \textbf{thrust power} \tag{1.3.2}$$

where the latter parts in both equations occur because of Eqs. (1.1.3) and (1.2.6). The power transmitted to a spacecraft (S/C) with velocity $v$ is simply calculated according to classic physics by the product of force times velocity, i.e.

$$P_{S/C} = F_* \cdot v \qquad \textbf{transmitted spacecraft power} \tag{1.3.3}$$

Note that the forces (here $F_e$ and $F_*$) are independent of the chosen reference system, whereas the velocities $v_e$ and $v_*$ are only meant with respect to the rocket. So *jet and thrust power are properties with respect to the rocket*, while the *transmitted spacecraft power is valid in the rocket system and the external inertial reference system* because $v$ is the same in both of them. Note, however, that $v$ depends on the chosen external reference system.

The so-called tot̲al impulse $I_{tot}$ of an engine is the integral product of total thrust and propulsion duration

$$\begin{aligned} I_{tot} &:= \int_0^t F_* dt = v_* \int_0^t \dot{m}_p \cdot dt \\ &= m_p v_* \quad @\ v_*(t) = const \qquad \textbf{total impulse} \end{aligned} \tag{1.3.4}$$

The latter is only valid as long as the effective exhaust velocity is constant. This is, in its strict sense, not the case during launch, where the external pressure (and hence the effective exhaust velocity) varies due to the pressure thrust.

The total impulse can be used to define the very important (weight-)specific impulse which characterizes the general performance and therefore is a figure of merit of an engine. The **weight-specific impulse** is defined as "*the achievable total impulse with respect to a given propellant weight* $m_p g_0$", i.e. with Eq. (1.3.4)

$$\boxed{I_{sp} := \frac{I_{tot}}{m_p g_0} = \frac{v_*}{g_0}} \quad @\ v_*(t) = const \quad \textbf{(weight-)specific impulse} \tag{1.3.5}$$

By this definition the specific impulse has the curious, but simple, dimension "second." Typical values are 300–400 seconds for chemical propulsions, 300–1500 seconds for electrothermal propulsions (Resistojet, Arcjet), and approximately 2000–6000 seconds for electrostatic (ion engines) and electromagnetic engines (see Fig. 1.7).

In Europe, in particular at ESA, the **mass-specific impulse** with definition "$I_{sp}$ = *the achievable total impulse with respect to a given propellant mass* $m_p$" is more common. This leads to the simple identity $I_{sp} = v_*$. However, the definition "$I_{sp}$ = weight-specific impulse" is more established worldwide, which is why we also will use it throughout this book. In either case you should keep in mind that quite generally:

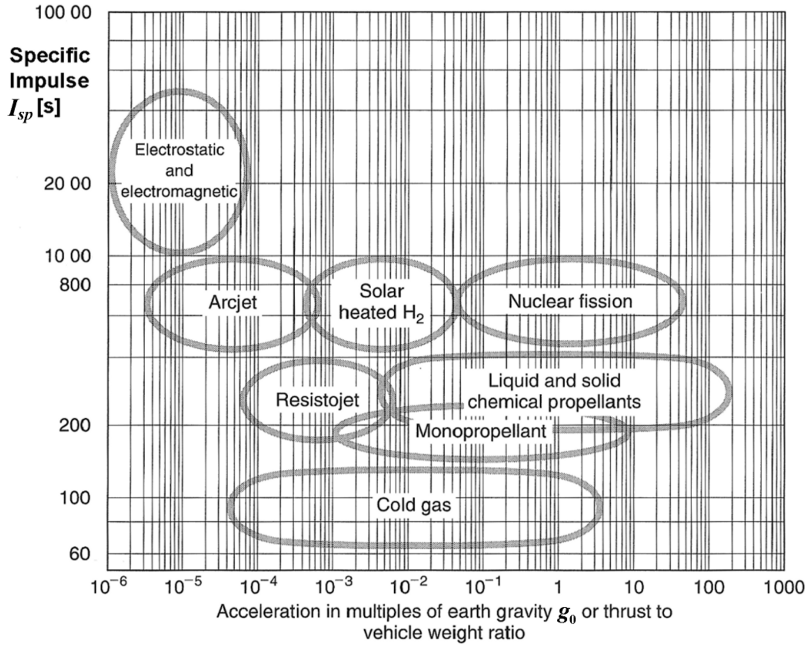


**Figure 1.7** Specific impulse and specific thrust of different propulsion systems.

> The specific impulse is an important figure of merit of an engine, and is in essence the effective exhaust velocity.

## 1.4 Rocket Equation of Motion

Apart from its own thrust, also external forces determine the trajectory of a rocket. They are typically summarized to one external force $F_{ext}$

$$\boldsymbol{F}_{ext} := \boldsymbol{F}_G + \boldsymbol{F}_D + \boldsymbol{F}_L \ \ldots \tag{1.4.1}$$

with $\boldsymbol{F}_G$ = gravitational force, $\boldsymbol{F}_D$ = aerodynamic drag, and $\boldsymbol{F}_L$ = aerodynamic lift (see Fig. 1.8). For each of these external forces, the rocket can be considered as a point on which the external force acts. This point has a unique location with regard to the geometry of the rocket, and it is in general different for every type of force. The masses of the rocket can be treated as lumped together at the center of mass where the gravitational force applies. The aerodynamic drag and lift forces virtually apply at the so-called center of pressure. And possible magnetic fields have still another imaginary point of impact. If the

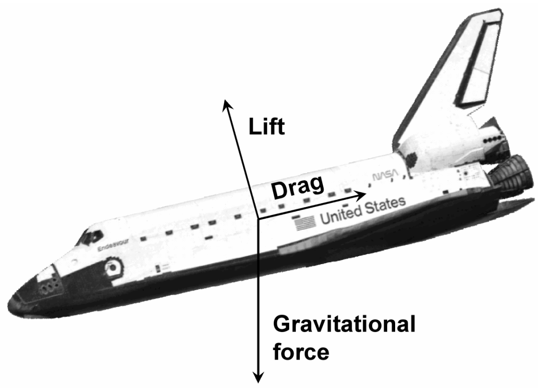


**Figure 1.8** Effects of different external forces on a spacecraft.

latter do not coincide with the center of mass, which in general is the case, the distance in between results in torques due to the inertial forces acting effectively at the center of mass. Here, we disregard the resulting complex rotational movements, and we just assume that all the points of impact coincide, or that the torques are compensated by thrusters.

Newton's second law, Eq. (7.1.12), gives us an answer to the question of how the rocket will move under the influence of all the forces $\boldsymbol{F}_i$ including the propellant force:

$$m\dot{\boldsymbol{v}} = \sum_{all\ i} \boldsymbol{F}_i$$

We therefore find the following equation of motion for the rocket:

$$m\dot{\boldsymbol{v}} = \boldsymbol{F}_* + \boldsymbol{F}_{ext}$$

and with Eq. (1.1.2), we finally obtain

$$\boxed{m\dot{\boldsymbol{v}} = \dot{m}\boldsymbol{v}_* + \boldsymbol{F}_{ext}} \qquad \textbf{rocket equation of motion} \qquad (1.4.2)$$

This is the key differential equation for the motion of the rocket. In principle the speed can be obtained by a single integration step and its position by a double integration. Note that this equation applies not only to rockets but also to any type of spacecraft during launch, reentry or when flying in space with or without propulsion.

## Problems

### Problem 1.1 Balloon Propulsion

Consider a balloon, which is propelled by exhausting its air with density $\rho = 1.29\ \mathrm{g\,dm^{-3}}$. The balloon has a volume of $2\ \mathrm{dm^3}$, the exit (throat) diameter is $A_t = 0.5\ \mathrm{cm^2}$. Let's assume the balloon exhausts the gas with constant mass flow rate within 2 s. Show that the momentum thrust is $F_e = 0.026$ N and the pressure thrust is $F_p = 0.013$ N and hence that the momentum thrust is roughly twice as big as the pressure thrust.

*Hint*: Observe that the exhaust velocity at the throat does not reach the speed of sound. Make use of Bernoulli's equation $p + \frac{1}{2}\rho v^2 = const$.

### Problem 1.2 Nozzle Exit Area of an SSME

The thrust of a Space Shuttle main engine (SSME) at 100% power level is $1.817 \times 10^6$ N at sea level and $2.278 \times 10^6$ N in vacuum. By using only this information, derive that the nozzle exit area is $A_e = 4.55\ \mathrm{m^2}$.