

Contents

Preface	<i>IX</i>
Acknowledgments	<i>XI</i>
1 Introduction	<i>1</i>
1.1 Overview of Nonlinear Wave Phenomena	<i>1</i>
1.2 Nonlinear Waves and Electronic Transport in Materials	<i>3</i>
1.3 Structural Outline of the Book	<i>7</i>
2 Dynamical Systems, Bifurcations, and the Chapman–Enskog Method	<i>9</i>
2.1 Introduction	<i>9</i>
2.2 Review of Dynamical Systems Concepts	<i>9</i>
2.2.1 Attractors	<i>10</i>
2.2.1.1 Steady States – Fixed Points	<i>10</i>
2.2.1.2 Limit Cycles	<i>12</i>
2.2.1.3 Chaotic Attractors	<i>14</i>
2.2.2 Bifurcations – Basic Definitions and Types	<i>18</i>
2.2.2.1 Saddle-Node Bifurcation (Local)	<i>19</i>
2.2.2.2 Transcritical and Pitchfork Bifurcations (Local)	<i>20</i>
2.2.2.3 Hopf Bifurcation (Local)	<i>21</i>
2.2.2.4 Degenerate Hopf and Takens–Bogdanov Bifurcations (Local, Co-dimension 2)	<i>23</i>
2.2.2.5 Heteroclinic and Homoclinic Connections as Examples of Nonlocal Bifurcations	<i>26</i>
2.3 Analysis of the Hopf Bifurcation: An Introduction to the Chapman–Enskog Method	<i>28</i>
2.3.1 Multiple Scales and Chapman–Enskog Methods	<i>28</i>
2.3.2 General Formulation of the Hopf Problem Using CEM	<i>28</i>
2.3.2.1 An Example from Physiology	<i>32</i>
2.3.3 Utility of the CEM for Higher Order Bifurcations	<i>34</i>
2.3.3.1 Degenerate Simple Eigenvalue	<i>34</i>
2.3.3.2 Degenerate Hopf Bifurcation	<i>37</i>

3	Excitable Media I: Continuum Systems	43
3.1	Introduction	43
3.2	Basic Excitability – the FitzHugh–Nagumo System	43
3.3	Matched Asymptotics: Excitability and Oscillations	47
3.4	The Scalar Bistable Equation; Wave Pulses as Heteroclinic Connections	51
3.4.1	Wave Fronts Near $w = w_0$ and a Formula for dc/dw	54
3.4.2	Wave Fronts for a Cubic Source	56
3.4.3	Linear Stability of the Wave Fronts	57
3.5	Traveling Waves of the FitzHugh–Nagumo System	58
3.5.1	Wave Fronts	58
3.5.2	Pulses of the FHN System	59
3.5.3	Wave Trains	62
4	Excitable Media II: Discrete Systems	65
4.1	Introduction	65
4.2	The Spatially Discrete Nagumo Equation	66
4.2.1	Depinning Transition of Wave Fronts	69
4.2.2	Construction of the Wave Front Profile Near the Depinning Transition	70
4.2.3	Wave Front Velocity Far from the Depinning Transition	75
4.3	Asymptotic Construction of Pulses	76
4.4	Numerically Calculated Pulses	79
4.5	Propagation Failure	83
4.6	Pulse Generation at a Boundary	85
4.7	Concluding Remarks	87
5	Electronic Transport in Condensed Matter: From Quantum Kinetics to Drift-diffusion Models	89
5.1	Introduction	89
5.1.1	Wigner Function for Non-interacting Particles in an External Potential	90
5.1.2	Classical Limit	91
5.1.3	Boltzmann Transport Equation and BGK Collision Model	92
5.1.4	Parabolic Scaling	94
5.1.5	Derivation of a Drift-Diffusion Equation	97
5.1.5.1	Method of Multiple Scales	97
5.1.5.2	Chapman–Enskog Method	99
5.1.5.3	Einstein Relation	100
5.2	Superlattices	100
5.2.1	Kinetic Theory Description of a Superlattice with a Single Populated Miniband	102
5.2.1.1	Wigner Equation	103
5.2.1.2	Equivalent form of the Quantum Kinetic Equation	109
5.2.2	Derivation of Reduced Equations for n and F	110
5.2.2.1	Nondimensional Wigner Equation	111
5.2.2.2	Derivation of a Reduced System	113
5.3	Concluding Remarks	119

6	Electric Field Domains in Bulk Semiconductors I: the Gunn Effect	125
6.1	Introduction	125
6.2	\mathcal{N} -shaped Current-Field Characteristics and Kroemer's Model	126
6.2.1	Intervalley Transfer Mechanism	127
6.2.2	Kroemer's Drift-Diffusion Model	130
6.2.3	Boundary Conditions	131
6.2.4	Nondimensionalization	132
6.3	Stationary Solutions and Their Linear Stability in the Limit $L \gg 1$	136
6.3.1	Stationary States and Their Linear Stability under <i>Current Bias</i>	136
6.3.2	Construction of the Stationary Solution and of $\Phi(J)$ under Voltage Bias	137
6.3.3	Linear Stability of the Stationary Solution under Voltage Bias	140
6.4	Onset of the Gunn Effect	147
6.4.1	The Linear Inhomogeneous Problem and Secular Terms	147
6.4.2	Hopf Bifurcation	148
6.4.3	Amplitude Equation for $\ln L \gg 1$	152
6.5	Asymptotics of the Gunn Effect for Long Samples and N-shaped Electron Velocity	154
6.5.1	General Formulation of Asymptotics for $\delta = O(1)$	155
6.5.1.1	A Single Dipole Wave	157
6.5.1.2	Several Dipole Waves	159
6.5.1.3	Shedding Waves at the Cathode	159
6.5.1.4	Overall Gunn Oscillation	160
6.5.2	Explicit Formulation of Asymptotics for $\delta \rightarrow 0+$	161
6.6	Asymptotics of the Gunn Effect for Long Samples and Saturating Electron Velocity	163
6.6.1	A Single Dipole Wave	163
6.6.2	The Dipole Wave Arrives at the Anode	166
6.6.3	Coexistence of Two Dipole Waves	167
6.6.4	Explicit Formulation of Asymptotics for $\delta \rightarrow 0+$	168
6.6.4.1	One Pulse Far from the Contacts	168
6.6.4.2	The Pulse Reaches the Anode	169
6.6.4.3	Coexistence of Two Pulses	169
6.7	References on the 1D Gunn Effect and Closing Remarks	170
7	Electric Field Domains in Bulk Semiconductors II: Trap-mediated Instabilities	175
7.1	Introduction	175
7.2	Drift-Diffusion Transport Model for Trap-Mediated System	177
7.3	Nondimensional Form and the Reduced Model	180
7.4	Steady States, J - E Curves, and Steady Wave Solutions on the Infinite Line under Current Bias	182
7.5	Nonlinear Wave Solutions in Finite Samples under Voltage Bias	188

7.6	Multiple Shedding of Wavefronts in Extrinsic Material	191
7.6.1	Numerical Results of Wavefront Shedding	192
7.6.2	Asymptotic Model for Wavefront Shedding	196
8	Nonlinear Dynamics in Semiconductor Superlattices	203
8.1	Introduction	203
8.2	Spatially Discrete Model for the Doped Weakly Coupled SL	208
8.2.1	Tunneling Current Density	210
8.2.2	Boundary Conditions	212
8.2.3	Photoexcitation in an Undoped SL	213
8.2.4	Continuous Drift-Diffusion Model for a Strongly Coupled SL	213
8.3	Nondimensionalization of the Discrete Drift-Diffusion Model	214
8.4	Wave Fronts and Stationary States under Current Bias	215
8.4.1	Pinning	217
8.4.1.1	Pinning of Wave Fronts with a Single Active Well	219
8.4.1.2	Continuum Limit	223
8.4.1.3	Role of Diffusivity	225
8.5	Static Field Domains in Voltage-Biased SLs	226
8.6	Relocation of EFDs	229
8.7	Self-Sustained Oscillations of the Current	237
8.7.1	Asymptotic Theory	238
8.7.2	Dependence of the Oscillations on Control Parameters	242
8.7.2.1	Doping Density	242
8.7.2.2	Temperature	244
8.7.2.3	Effect of Other Parameters on Self-Oscillations	246
8.8	Spin Transport in Dilute Magnetic Semiconductor Superlattices	246
9	Nonlinear Wave Methods for Related Systems in the Physical World	255
9.1	Introduction	255
9.1.1	NNDC, SNDC, and ZNDC	255
9.2	Superlattice Transport Model with Both Vertical and Lateral Dynamics	257
9.3	Semi-Insulating GaAs	260
9.4	Multidimensional Gunn Effect	263
9.5	Fluctuations in Gunn Diodes	264
9.6	Dynamics of Dislocations in Mechanical Systems; Nanoarrays	265
	Index	273