

Index

a

Aharonov–Bohm (AB) effect 116
 Ampere’s law 243
 Anderson–Higgs mechanism 205, 206, 208
 angular momentum quantization 116
 – Berry phase 121
 – electromagnetic (EM) field 124
 – geometrical theory of 119
 – magnetic flux tube 118
 – magnetic monopole 118
 – particle statistics 123–124
 – quantum gauge field theory 124
 – singular gauge potentials 116
 anti-kink solution(s) 162, 163
 auxiliary electromagnetic (EM) field 278
 auxiliary field 273
 auxiliary magnetic flux 278
 auxiliary potential 271
 auxiliary vector field 270

b

back flow 226
 BCS mean-field Hamiltonian 240
 – Bogoliubov transformation 240
 BCS theory 239, 241
 – Hamiltonian formulation 239
 – mean-field equation 241
 – variational wavefunction 241
 Berry phase(s) 107, 277
 – angular momentum quantization 121
 – electron wavefunction 107
 – particle statistics 111–112
 – quantum interference effect 108
 – quantum mechanics 112
 – quantum wavefunction 113
 – quasi-particles 152
 – simple quantum system 107
 – U(1) Gauge theory 112

Bloch’s theorem 209
 N-body system 203
 Bogoliubov transformation 193, 239
 Boltzmann distribution 131
 Boltzmann equation 224, 228, 230
 – distribution function 129
 – Lagrangian formulation 230
 Bose-condensed bosons 211
 – classical solutions 191
 – infinite degeneracy 191
 – single-particle-hole-pair excitation spectrum 211
 Bose condensation 198–199, 201, 205
 – state 199
 – wavefunction 202
 boson fluid 189, 201, 249
 – phonon-like excitation spectrum 201
 – superfluidity 189
 bosonic systems 29
 – Fourier transformation 32
 – quadratic Hamiltonian 31
 – quantum Hamiltonian 32, 33
 – quantum mechanics (QM) 30
 boson(s) 36, 37
 – eigenstate wavefunctions 209
 – excitation spectrum 211, 212
 – fock space 36
 – Hilbert space 37
 – model 220
 – occupancy number 198
 – particle theory 38
 – phase field 275
 – quantum field theory 53
 boundary condition 248
 Bravais lattice vectors 208
 Brillouin zone 255
 broken symmetry 167

c

- canonical quantization scheme 29, 67, 124
 - Coulomb gauge 125
 - Lagrangian for EM field 124
 - scalar fields 29
 - quantum theory 29
- Cauchy–Riemann condition 182
- center of mass (CM) 204
 - velocity 204
- charged superfluids 204–208
 - Higgs mechanism 204
 - superconductivity 204
- charge quantization 264
 - compactness 264
- classical field theory 5, 17, 165
 - basic mathematical tools 5
 - conservation 18
 - energy/momentum 18
 - equations of motion 5
 - Galilean invariance 19, 20
 - Lagrangian formulation 17
 - quadratic field theories 10
 - electromagnetic field 24
- coherence length 247
- confinement phases 267, 269
 - plasma phase transition 267
- conjugate momentum 258
- conservation laws 24
 - energy-momentum conservation 24
 - energy-momentum tensor 25
 - Lagrangian system 18
 - Noether’s theorem 26
 - physical quantity 27
- continuum field theory 24, 34, 55
 - conservation laws 24
 - mathematical tools 34
- continuum phase model 275
- Cooper pairs 233, 248
 - wavefunction 246
- correlation function(s) 95, 167
 - boundary condition 149
 - current-current response function 97
 - density-density response function 98
 - fluctuation-dissipation theorem 101–104
 - Fourier transform 149
 - linear response theory 95
 - Mori approach 148–149
 - response functions 97
 - spin-spin response function 98
 - temperature green’s functions 99
- Coulomb interaction 206, 213, 220, 270
- Coulomb potential 222

d

- density-density response function 196, 229
- different states of vacuum 3
 - continuous physical quantities 3
- Dirac equation 59, 62
 - covariant 59
 - quantization 60
- Dirac fermions 64, 277
 - action 64
 - theory 277
- Dirac Hamiltonian 61, 62
- Dirac matrices 57
- Dyson’s approach 79
 - closed-time-loop Green’s function technique 80
 - Hamiltonian formulation 79
 - one-particle Green’s function 84
 - perturbation expansion 82, 86–89
 - perturbation expansion for the S matrix 81
 - Schrödinger equation 82
 - spectral representation 89–92
 - time-evolution operator at imaginary time 81
- Wick’s theorem 83

e

- effective field theory 129
 - approximation 131
 - Boltzmann equation 129
 - fluid mechanics 129, 132
 - Navier–Stokes equations 129
 - Newton’s law 130
 - Newton’s equation of motion 155
 - particle 130
 - probability distribution function 131
- effective mass 222, 227
- eigenfunction expansion method 6
 - degeneracy 6
- eigenvalue equations 5
- eigen wavefunctions 60
 - amplitude 60
- electric field(s) 259, 266–268
 - definition 259
 - loops 266, 267
- electromagnetic (EM) field 24, 256, 272
 - kinetic energy 24
 - spectrum/wavefunction 126
- electromagnetism 114
 - Gauge theory 114
 - single-particle Hamiltonian 114
- electron-electron interaction 218
- electron Green’s function 281
- electronic excitation spectrum 222
 - collective excitations 222

- single-particle-hole-pair excitations 222
- electrons collective motion 221
- elementary excitation 226
- energy-momentum tensor 25
- energy spectrum 33, 58, 170
- equation of motion 5–10, 164, 173, 273
 - comment on non-linear equations 10
 - eigenfunction expansion method 6
 - Green’s function 7
 - initial condition problem 8
 - Schrödinger equation 8
- excitation spectrum 206, 253

f

- ϕ^4 model 164–166, 168, 169, 172
 - 2D 172
 - multi-kink-anti-kink solution 164
 - hill-like solution 172
- Fermi gas spectrum 211
- Fermi liquid 213, 222–231
 - collective modes 213
- Fermi liquid theory 211, 213, 222–231
 - applications 227–229
 - bosonization description 229
- Fermi momentum 211, 280
- fermion 36, 37
 - Fock space 36
 - gas 280
 - Grassmann field theory 38
 - Grassmann variables 39
 - Green’s function 89, 279
 - Hilbert space 37
 - particle theory 38
 - quantum field theory 38–40
 - spectrum 239, 240
- fermion liquids 211
 - collective excitations 211
 - single-particle excitations 211
- fermion quantum field theory 41
 - Lagrangian formulation 41
- fermionic system(s) 68, 233
 - coherent state 68
- Fermi sea 229, 280
- Fermi surface 224, 225, 230, 283
- Feynman’s path integral approach 79
- field equation 161, 163
 - 1D non-linear 163
- field theory 3
 - Boltzmann equation 129
 - dynamics 5
 - fluid mechanics 129
 - goal 3
 - internal structures 27
 - mathematical analysis 15

- Navier–Stokes equations 129
- partition function 14
- path integral 49
- system 161
- thermodynamics 4
- fluctuation-dissipation theorem 101, 154
 - in hydrodynamic regime 103
- fluid mechanics 129, 132
 - Euler’s equation 133
 - free-energy density 135
 - friction/viscosity 134
 - Galilean transformation 133
 - hydrodynamics limitation 135
 - macroscopic systems 132
 - Newton’s law 133
- fluid system 201
 - normal- and super- fluids 201
 - phonon-like dispersion 201
- flux quantization phenomenon 244–246
- Fock self energies 88
 - geometric series of 89
 - single Green’s function 88
- Fock space 36
 - bosons/fermions 36
 - coherent states 49, 50
- Fourier-transformed field 64
- Fourier transformation 13, 155, 215, 270
- fractional quantum Hall effect (FQHE) 153
- free-energy expression 175, 178

g

- Galilean invariance 20, 112, 113
- Gauge field 264
 - Berry phase 112
 - compactness 264
 - meaning 184–185
- Gauge theory(ies) 107, 114, 183, 255
 - electromagnetism 114
 - Heisenberg equation 115
 - introduction 255
 - model 183
 - single-particle Hamiltonian 114
 - types 255
- Gauge transformation 260
- Gaussian theory 237–239
 - semiclassical 237
- Gauss’ law 259, 264, 267
- Ginsburg–Landau (GL) free energy 179
- Ginsburg–Landau equation 242, 243, 246–248
 - time-dependent 242
- Ginsburg–Landau model 243
- Ginsburg–Landau theory 242, 245

Goldstone theorem 204–207
 – definition 205
 Grassmann exchange rules 39
 Grassmann field theory 40
 – linear transformation 40
 Grassmann number 214
 Grassmann variables 39, 65, 214
 – integration rules 39, 65
 Green’s function 6–8, 85, 89, 168, 196–198, 224–225, 279
 – expansion method 9
 – Fourier transform 89, 99
 – important properties 84
 – one-particle 196, 224, 279
 – perturbation expansion 86
 – quasi-particles 224
 – single-particle 224, 279
 – spectral function for 90, 91, 99
 – time periodic/anti-periodic functions 85

h

Hamiltonian approach 197, 233, 239, 240
 Hamiltonian energy function 4, 160
 harmonic oscillator wavefunctions 31
 harmonic perturbation 98
 Hartree–Fock approximation 93, 94
 Hartree–Fock self energies 87
 Heisenberg model 4, 14, 172, 173, 174, 269
 – 2D 174
 – spin-spin correlation function 174
 Hermitian matrix 5
 Higgs mechanism 204–207
 Hilbert space 12, 14, 36, 39, 148
 – fermions/bosons 37
 – parts 149
 hill-like solution 172
 Hubbard–Stratonovich (HS)
 transformation 213–219, 233, 234
 – electron-pairing 234
 Hubbard–Stratonovich field 215

i

imaginary time 51
 – correlation function 83
 – formulation 277
 – Green’s functions 85
 – Heisenberg time-dependent operator 84
 – partition function 43, 44, 51
 – quantum field theory for bosons 51–52
 – quantum field theory for fermions 52–54
 – time-evolution operator 81
 instanton(s) 159, 165–167
 – 1D classical theories 165
 – introduction 165

internal symmetry 26
 – internal structures 27
 – Noether’s theorem 26

j

Jastrow factor 203

k

Keldysh Green’s function technique 80
 kink solution 162, 163
 – anti-kink solution(s) 165, 166
 – boundary regions 163
 – time-dependent 162
 Klein–Gordon equation 56, 62
 Klein–Gordon model 55
 – energy spectrum 56
 Kosterlitz–Thouless transition 171, 172, 174, 175, 177, 248
 – vortices 248
 Kramers–Kronig relation 225

l

Lagrange multiplier field 14
 Lagrangian formulation 17, 160
 – classical field theory 21
 – electromagnetic field 24
 – fermion quantum field theory 41
 – functional derivative 22
 – lattice field theories 21
 – Maxwell equations 24
 – principle of least action 17
 – space-time symmetric 23
 – systematic exploration 17
 Landau interaction 224
 Landau parameter(s) 222, 227
 Landau’s Fermi liquid theory 213, 223, 229
 Landau’s superfluidity analysis 199–201
 Landau theory 136, 224
 – continuum limit 145
 – effective free energies 143
 – electron-pair wavefunction 144
 – Fourier space continuum limit 145
 – liquid-solid transition 137
 – macroscopic parameters 136
 – order parameters 136–139
 – paramagnetic-ferromagnetic transition 137, 142
 – phase transition 136, 140
 – phenomenological theory 147
 – thermodynamic state 140
 Landau transport equation 229
 Langevin equation of motion 155
 lattice Dirac fermions 61

lattice field theory 32, 34
 – approximate theory 34
 – continuum limit 34
 lattice gauge theory 256, 257, 265, 268
 – instantons 268
 – $U(1)$ 256
 – Z_2 260
 lattice Hamiltonian 63
 Lindhard function 217
 linear response theory 96
 – dynamical changes 96
 – fluctuation-dissipation theorem 104
 local-equilibrium approximation 203
 Lorentz-invariant systems 20, 171
 magnetic field 207
 – time-independent 207
 magnetic flux(es) 183, 246, 259, 260, 269
 – quantum 246
 magnetic monopole 118, 180
 – angular momentum quantization 118, 185–186
 – Berry phase 122
 – Bohr–Sommerfeld quantization 123
 – Dirac quantization condition 122
 – like instantons 268
 – quantization 121
 Maxwell equation(s) 5, 24, 62
 mean-field approximation, *see* saddle-point approximation
 – BCS theory 239
 – Hartee-fock approximation 93, 94
 mean-field equation 240, 242
 mean-field partition function 271
 mean-field theory 177, 270
 Meissner effect 207–208, 244
 Mori approach, *see* Hilbert space

n

Navier–Stokes equation 5
 neutral superfluid 248
 – vortices 248
 Newton’s equation 133, 155
 – particle motion 155
 Newton’s equation of motion 161
 Newtonian mechanics 129
 Noether’s theorem 24, 56
 – momentum/energy conservation 26
 – quantum level 67
 non-interacting electron gas 219
 – density-density response function 219
 – excitation spectrum 211, 212
 non-linear equations 10
 – adiabaticity/counting problem 10
 – evaluation of perturbation series 13

non-linear-sigma model 180, 183

o

off-diagonal long-range order (ODLRO) 202
 one-dimensional scalar fields 164
 – topological index 164
 one-particle quantum wavefunction 244

p

N -particle fermion system 229
 particle-hole excitation(s) 213, 218
 particle-hole pairs 220
 partition function 4, 170, 270, 271, 275, 276
 – classical harmonic oscillators 11
 – computation 4
 – evaluation 11
 – imaginary time 51
 – linear-transformation 12
 – quadratic field theories 10
 – quantum field theory for bosons 51–52
 – quantum field theory for fermions 52–54
 path integral 41
 – classical fields theory 49
 – coherent states 49, 74
 – field theory 41
 – Gaussian integrals for correlation functions 74
 – imaginary time 43
 – one-particle quantum mechanics 42
 – partition function 44
 – quantum field theory application 45, 49
 – quantum mechanics 41
 path integral approach 74–75
 – interacting systems 76–77
 – Wick’s theorem 77–79, 83–84
 path integral formulation 63
 – applications 63
 – Dirac fermions 63
 – Fourier-transformed fields 64
 – Grassmann variable integration rules 65
 – Hamiltonian formulation 63
 path integral quantization 126
 – electromagnetic field 127
 – Faddeev–Popov technique 128
 – Gauge problem 127–128
 – quantum mechanics 127
 Pauli exclusion principle 60, 211
 Pauli matrix 28, 183
 Perturbation expansion 81
 – S matrix 81–83
 perturbation theory 73, 92, 225, 279–282
 – basic assumption of 73

- basic idea 76
 - Dyson’s U-matrix approach 74, 79, 86
 - goal 80
 - Green’s function 86
 - Hartree–Fock approximation 93
 - optimal trial Hamiltonian 92
 - path integral approach 74–75
 - quantum effect of 96
 - second-order 279, 280
 - thermal time-evolution 83
 - trial wavefunction 92
 - variational method 92
 - perturbative solutions 167
 - phase action 193, 206, 277
 - phase transition 136, 140
 - Bravais lattice 142
 - ergodic assumption 137
 - gas-liquid 138
 - Landau theory 136
 - liquid-glass 138
 - paramagnetic-ferromagnetic transition 141
 - phenomenological theory of 139, 147
 - phenomenon/concept of 146
 - quantum system 146
 - solid-liquid transition 141
 - thermodynamic state 140
 - weakly first-order 141
 - phonons 64
 - excitation spectrum 204
 - imaginary time 64
 - plane-wave-like solutions, *see* perturbative solutions
 - plaquette unit square 258
 - plasma phase Hamiltonian 267
 - plasma mode 220
 - plasma phase(s) 263, 268, 269
 - Poisson–Boltzmann equation 178
 - potential energy 163, 250, 265
 - p-wave superconductor 241
- q**
- ϑ -field configuration 173
 - quadratic field theories (QFTs) 10, 33, 53, 159, 171, 202, 213
 - classical variables 53
 - constraints 14
 - continuum limit 13, 55–56
 - Dirac fermions 59–60, 60–63
 - evaluation method 10
 - Fokker–Planck equation 153, 156
 - harmonic oscillators 54
 - Hilbert space 148
 - imaginary-time quantum mechanics 157
 - instantons 159
 - Langevin equations 153
 - linear field theories 11
 - Newton’s equation of motion 151
 - non-linear energy functional 13
 - partition function 10
 - perturbation theory 11
 - phonons 54
 - quantum action 150
 - quasi-particles 152
 - Schrödinger equation 151
 - solitons 159
 - symmetry/conservation laws 67–68
 - topology 159
 - transform energy function 12
 - with non-quadratic terms 33
 - quadratic phase model 253
 - quantum-mechanical wavefunction 30–32, 92, 195, 264
 - quantum mechanics (QM) 30, 163, 168, 169, 208
 - commutation rule 38
 - exchange of particles 35, 111, 123
 - exchange phase 36, 111, 123
 - instantons 168
 - least action 150–152
 - quantum action 150
 - Schrödinger representation 30
 - quantum particle 168
 - quantum rotor Hamiltonian 251
 - quantum rotor model 266
 - quantum system and statistical mechanics 171
 - quantum tunneling processes 159, 168
 - quantum X-Y model 263
 - quasi-particle-hole-pair excitations 229
 - quasi-particles 222, 225
 - charge carried 225
 - current carried 225
 - density 226
 - occupation number 222
- r**
- random-phase approximation (RPA) 218, 219, 222
 - alternative derivation 219
 - excitation spectrum of electron gas 218
 - rotor model 249–251
- s**
- saddle-point approximation 168, 189, 211
 - semiclassical theory for interacting bosons 189

Schrödinger equation 8, 41, 96, 116, 209
 – Dirac criteria 56
 – Green’s function 9
 – key feature of 8
 self-sustained mode, *see* plasma mode
 semiclassical approximation 189, 197–198
 – one-particle QM 189
 sine-Gordon model 163–165
 – multi-kink-anti-kink solution 164
 single-particle wavefunction 203
 singular Gauge potentials 116
 – Aharonov–Bohm (AB) effect 116
 – angular momentum quantization 116
 skyrmion solution(s) 180, 181
 solitary wave 164, 165
 soliton solution 159, 162, 164, 165, 181
 – introduction 159–162
 – quantization 165
 – solitary waves 165
 – stability 162
 – use 159
 Sommerfeld-type expansion 228
 space-time loop, *see* Wilson loop
 space-time symmetric formulation 24
 spectral representation 89
 – correlation functions 89
 – extract general properties of 89
 – fermi momentum 91
 – one-particle green’s function 90
 – spectral function 91
 spin-independent interaction 241
 spin-spin correlation function 174–175
 – low-temperature 174
 Stirling approximation 227
 Stokes’ theorem 116, 245
 String-net condensation state 267
 strong-coupling expansion 251, 266
 strong-coupling phase 253, 265
 – charge confinement 265
 superconductivity 207, 233
 – BCS theory 233
 – insulator transition 249–253
 superconductor 179, 243, 247, 253
 – coherence length 243
 – insulator transitions 253
 – type-I 244
 – type-II 244, 247
 – vortices 179
 superfluid 179, 202
 – collective motion 202
 – duality 275–278
 – state 199
 – two fluid picture 203
 – vortices 179

superfluidity 198–204
 – Bose condensation 198–199
 – superfluid He⁴ 199
 superfluid state 199
 supersolids 208–210
 supersymmetric theories 66
 s-wave superconductors 233, 239, 241
 – BCS theory 233, 239

t

Taylor expansion 140
 thermal time-evolution operator 81
 thermodynamics 4
 – condition 203
 – phases/phase transitions 4, 136, 140
 Thomas–Fermi screening length 221
 tight-binding Hamiltonian 256
 tight-binding model 255
 topological indices 162
 – stability 162
 translational invariance 24, 199
 trial wavefunction approach 229
 two-fluid picture 203

u

U(1) gauge theory 259, 275–278
 U(1) lattice gauge theory 262, 266, 267, 269, 275
 – strong/weak-coupling expansions 262–270
 U(1)/Z₂ lattice gauge theories 255–260
 – introduction 255–262
 unit vector field 184

v

vacuum energies 66
 – free bosons/Dirac fermions 66
 vector field 245
 vortex gases 271
 vortex field 277
 vortex-like topological excitations 246
 vortex line 277
 vortex solution(s) 173, 174, 242
 – importance 174

w

weak-coupling expansions 252, 253, 262, 263
 weak-coupling phase, *see* plasma phase
 Wick’s theorem 77, 78
 – Dyson’s perturbation theory 83
 Wilson loop 272–275
 winding number 169–170

x

X-Ymodel 248, 268

y

Yang-Mills theory 114

z

Z_2 gauge theory 260–262

– model 260

zero viscosity, *see* superfluid state

zeroth-order Hamiltonian 263