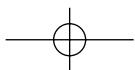


Contents

Preface XIII

1	Principles of Mathematical Modeling	1
1.1	A Complex World Needs Models	1
1.2	Systems, Models, Simulations	3
1.2.1	Teleological Nature of Modeling and Simulation	4
1.2.2	Modeling and Simulation Scheme	4
1.2.3	Simulation	7
1.2.4	System	7
1.2.5	Conceptual and Physical Models	8
1.3	Mathematics as a Natural Modeling Language	9
1.3.1	Input–Output Systems	9
1.3.2	General Form of Experimental Data	10
1.3.3	Distinguished Role of Numerical Data	10
1.4	Definition of Mathematical Models	11
1.5	Examples and Some More Definitions	13
1.5.1	State Variables and System Parameters	15
1.5.2	Using Computer Algebra Software	18
1.5.3	The Problem Solving Scheme	19
1.5.4	Strategies to Set up Simple Models	20
1.5.4.1	Mixture Problem	24
1.5.4.2	Tank Labeling Problem	27
1.5.5	Linear Programming	30
1.5.6	Modeling a Black Box System	31
1.6	Even More Definitions	34
1.6.1	Phenomenological and Mechanistic Models	34
1.6.2	Stationary and Instationary models	38
1.6.3	Distributed and Lumped models	38
1.7	Classification of Mathematical Models	39
1.7.1	From Black to White Box Models	40
1.7.2	SQM Space Classification: S Axis	41
1.7.3	SQM Space Classification: Q Axis	42



1.7.4	SQM Space Classification: M Axis	43
1.8	Everything Looks Like a Nail?	45
2	Phenomenological Models	47
2.1	Elementary Statistics	48
2.1.1	Descriptive Statistics	48
2.1.1.1	Using <i>Calc</i>	49
2.1.1.2	Using the <i>R Commander</i>	51
2.1.2	Random Processes and Probability	52
2.1.2.1	Random Variables	53
2.1.2.2	Probability	53
2.1.2.3	Densities and Distributions	55
2.1.2.4	The Uniform Distribution	57
2.1.2.5	The Normal Distribution	57
2.1.2.6	Expected Value and Standard Deviation	58
2.1.2.7	More on Distributions	60
2.1.3	Inferential Statistics	60
2.1.3.1	Is Crop A's Yield Really Higher?	61
2.1.3.2	Structure of a Hypothesis Test	61
2.1.3.3	The <i>t</i> test	62
2.1.3.4	Testing Regression Parameters	63
2.1.3.5	Analysis of Variance	63
2.2	Linear Regression	65
2.2.1	The Linear Regression Problem	65
2.2.2	Solution Using Software	66
2.2.3	The Coefficient of Determination	68
2.2.4	Interpretation of the Regression Coefficients	70
2.2.5	Understanding <i>LinRegEx1.r</i>	70
2.2.6	Nonlinear Linear Regression	72
2.3	Multiple Linear Regression	74
2.3.1	The Multiple Linear Regression Problem	74
2.3.2	Solution Using Software	76
2.3.3	Cross-Validation	78
2.4	Nonlinear Regression	80
2.4.1	The Nonlinear Regression Problem	80
2.4.2	Solution Using Software	81
2.4.3	Multiple Nonlinear Regression	83
2.4.4	Implicit and Vector-Valued Problems	86
2.5	Neural Networks	87
2.5.1	General Idea	87
2.5.2	Feed-Forward Neural Networks	89
2.5.3	Solution Using Software	91
2.5.4	Interpretation of the Results	92
2.5.5	Generalization and Overfitting	95
2.5.6	Several Inputs Example	97

2.6	Design of Experiments	99
2.6.1	Completely Randomized Design	100
2.6.2	Randomized Complete Block Design	103
2.6.3	Latin Square and More Advanced Designs	104
2.6.4	Factorial Designs	106
2.6.5	Optimal Sample Size	108
2.7	Other Phenomenological Modeling Approaches	109
2.7.1	Soft Computing	109
2.7.1.1	Fuzzy Model of a Washing Machine	110
2.7.2	Discrete Event Simulation	111
2.7.3	Signal Processing	113
3	Mechanistic Models I: ODEs	117
3.1	Distinguished Role of Differential Equations	117
3.2	Introductory Examples	118
3.2.1	Archaeology Analogy	118
3.2.2	Body Temperature	120
3.2.2.1	Phenomenological Model	120
3.2.2.2	Application	121
3.2.3	Alarm Clock	122
3.2.3.1	Need for a Mechanistic Model	122
3.2.3.2	Applying the Modeling and Simulation Scheme	123
3.2.3.3	Setting Up the Equations	125
3.2.3.4	Comparing Model and Data	126
3.2.3.5	Validation Fails – What Now?	127
3.2.3.6	A Different Way to Explain the Temperature Memory	128
3.2.3.7	Limitations of the Model	129
3.3	General Idea of ODE's	130
3.3.1	Intrinsic Meaning of π	130
3.3.2	e^x Solves an ODE	130
3.3.3	Infinitely Many Degrees of Freedom	131
3.3.4	Intrinsic Meaning of the Exponential Function	132
3.3.5	ODEs as a Function Generator	134
3.4	Setting Up ODE Models	135
3.4.1	Body Temperature Example	135
3.4.1.1	Formulation of an ODE Model	135
3.4.1.2	ODE Reveals the Mechanism	136
3.4.1.3	ODE's Connect Data and Theory	137
3.4.1.4	Three Ways to Set up ODEs	138
3.4.2	Alarm Clock Example	139
3.4.2.1	A System of Two ODEs	139
3.4.2.2	Parameter Values Based on A priori Information	140
3.4.2.3	Result of a Hand-fit	141
3.4.2.4	A Look into the Black Box	142
3.5	Some Theory You Should Know	143

3.5.1	Basic Concepts	143
3.5.2	First-order ODEs	145
3.5.3	Autonomous, Implicit, and Explicit ODEs	146
3.5.4	The Initial Value Problem	146
3.5.5	Boundary Value Problems	147
3.5.6	Example of Nonuniqueness	149
3.5.7	ODE Systems	150
3.5.8	Linear versus Nonlinear	152
3.6	Solution of ODE's: Overview	153
3.6.1	Toward the Limits of Your Patience	153
3.6.2	Closed Form versus Numerical Solutions	154
3.7	Closed Form Solutions	156
3.7.1	Right-hand Side Independent of the Independent Variable	156
3.7.1.1	General and Particular Solutions	156
3.7.1.2	Solution by Integration	157
3.7.1.3	Using Computer Algebra Software	158
3.7.1.4	Imposing Initial Conditions	160
3.7.2	Separation of Variables	161
3.7.2.1	Application to the Body Temperature Model	164
3.7.2.2	Solution Using <i>Maxima</i> and <i>Mathematica</i>	165
3.7.3	Variation of Constants	166
3.7.3.1	Application to the Body Temperature Model	167
3.7.3.2	Using Computer Algebra Software	169
3.7.3.3	Application to the Alarm Clock Model	170
3.7.3.4	Interpretation of the Result	171
3.7.4	Dust Particles in the ODE Universe	173
3.8	Numerical Solutions	174
3.8.1	Algorithms	175
3.8.1.1	The Euler Method	175
3.8.1.2	Example Application	176
3.8.1.3	Order of Convergence	178
3.8.1.4	Stiffness	179
3.8.2	Solving ODE's Using <i>Maxima</i>	180
3.8.2.1	Heuristic Error Control	181
3.8.2.2	ODE Systems	182
3.8.3	Solving ODEs Using <i>R</i>	184
3.8.3.1	Defining the ODE	184
3.8.3.2	Defining Model and Program Control Parameters	186
3.8.3.3	Local Error Control in <i>lsoda</i>	186
3.8.3.4	Effect of the Local Error Tolerances	187
3.8.3.5	A Rule of Thumb to Set the Tolerances	188
3.8.3.6	The Call of <i>lsoda</i>	189
3.8.3.7	Example Applications	190
3.9	Fitting ODE's to Data	194
3.9.1	Parameter Estimation in the Alarm Clock Model	194

3.9.1.1	Coupling lsoda with nls	195
3.9.1.2	Estimating One Parameter	197
3.9.1.3	Estimating Two Parameters	198
3.9.1.4	Estimating Initial Values	199
3.9.1.5	Sensitivity of the Parameter Estimates	200
3.9.2	The General Parameter Estimation Problem	201
3.9.2.1	One State Variable Characterized by Data	202
3.9.2.2	Several State Variables Characterized by Data	203
3.9.3	Indirect Measurements Using Parameter Estimation	204
3.10	More Examples	205
3.10.1	Predator–Prey Interaction	205
3.10.1.1	Lotka–Volterra Model	205
3.10.1.2	General Dynamical Behavior	207
3.10.1.3	Nondimensionalization	208
3.10.1.4	Phase Plane Plots	209
3.10.2	Wine Fermentation	211
3.10.2.1	Setting Up a Mathematical Model	212
3.10.2.2	Yeast	213
3.10.2.3	Ethanol and Sugar	215
3.10.2.4	Nitrogen	216
3.10.2.5	Using a Hand-fit to Estimate N_0	217
3.10.2.6	Parameter Estimation	219
3.10.2.7	Problems with Nonautonomous Models	220
3.10.2.8	Converting Data into a Function	222
3.10.2.9	Using Weighting Factors	222
3.10.3	Pharmacokinetics	223
3.10.4	Plant Growth	226
4	Mechanistic Models II: PDEs	229
4.1	Introduction	229
4.1.1	Limitations of ODE Models	229
4.1.2	Overview: Strange Animals, Sounds, and Smells	230
4.1.3	Two Problems You Should Be Able to Solve	231
4.2	The Heat Equation	233
4.2.1	Fourier’s Law	234
4.2.2	Conservation of Energy	235
4.2.3	Heat Equation = Fourier’s Law + Energy Conservation	236
4.2.4	Heat Equation in Multidimensions	238
4.2.5	Anisotropic Case	238
4.2.6	Understanding Off-diagonal Conductivities	239
4.3	Some Theory You Should Know	241
4.3.1	Partial Differential Equations	241
4.3.1.1	First-order PDEs	242
4.3.1.2	Second-order PDEs	243
4.3.1.3	Linear versus Nonlinear	243

x | *Contents*

4.3.1.4	Elliptic, Parabolic, and Hyperbolic Equations	244
4.3.2	Initial and Boundary Conditions	245
4.3.2.1	Well Posedness	246
4.3.2.2	A Rule of Thumb	246
4.3.2.3	Dirichlet and Neumann Conditions	247
4.3.3	Symmetry and Dimensionality	248
4.3.3.1	1D Example	249
4.3.3.2	2D Example	251
4.3.3.3	3D Example	252
4.3.3.4	Rotational Symmetry	252
4.3.3.5	Mirror Symmetry	253
4.3.3.6	Symmetry and Periodic Boundary Conditions	253
4.4	Closed Form Solutions	254
4.4.1	Problem 1	255
4.4.2	Separation of Variables	255
4.4.3	A Particular Solution for Validation	257
4.5	Numerical Solution of PDE's	257
4.6	The Finite Difference Method	258
4.6.1	Replacing Derivatives with Finite Differences	258
4.6.2	Formulating an Algorithm	259
4.6.3	Implementation in R	260
4.6.4	Error and Stability Issues	262
4.6.5	Explicit and Implicit Schemes	263
4.6.6	Computing Electrostatic Potentials	264
4.6.7	Iterative Methods for the Linear Equations	264
4.6.8	Billions of Unknowns	265
4.7	The Finite-Element Method	266
4.7.1	Weak Formulation of PDEs	267
4.7.2	Approximation of the Weak Formulation	269
4.7.3	Appropriate Choice of the Basis Functions	270
4.7.4	Generalization to Multidimensions	271
4.7.5	Summary of the Main Steps	272
4.8	Finite-element Software	274
4.9	A Sample Session Using Salome-Meca	276
4.9.1	Geometry Definition Step	277
4.9.1.1	Organization of the GUI	277
4.9.1.2	Constructing the Geometrical Primitives	278
4.9.1.3	Excising the Sphere	279
4.9.1.4	Defining the Boundaries	281
4.9.2	Mesh Generation Step	281
4.9.3	Problem Definition and Solution Step	283
4.9.4	Postprocessing Step	285
4.10	A Look Beyond the Heat Equation	286
4.10.1	Diffusion and Convection	288
4.10.2	Flow in Porous Media	290

4.10.2.1	Impregnation Processes	291
4.10.2.2	Two-phase Flow	293
4.10.2.3	Water Retention and Relative Permeability	293
4.10.2.4	Asparagus Drip Irrigation	295
4.10.2.5	Multiphase Flow and Poroelasticity	296
4.10.3	Computational Fluid Dynamics (CFD)	296
4.10.3.1	Navier–Stokes Equations	296
4.10.3.2	Backward Facing Step Problem	298
4.10.3.3	Solution Using Code-Saturne	299
4.10.3.4	Postprocessing Using Salome-Meca	301
4.10.3.5	Coupled Problems	302
4.10.4	Structural Mechanics	303
4.10.4.1	Linear Static Elasticity	303
4.10.4.2	Example: Eye Tonometry	306
4.11	Other Mechanistic Modeling Approaches	309
4.11.1	Difference Equations	309
4.11.2	Cellular Automata	310
4.11.3	Optimal Control Problems	312
4.11.4	Differential-algebraic Problems	314
4.11.5	Inverse Problems	314

A	CAELinux and the Book Software	317
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B	R (Programming Language and Software Environment)	321
B.1	Using <i>R</i> in a <i>Konsole</i> Window	321
B.1.1	Batch Mode	321
B.1.2	Command Mode	322
B.2	<i>R Commander</i>	322

C	Maxima	323
C.1	Using <i>Maxima</i> in a <i>Konsole</i> Window	323
C.1.1	Batch Mode	323
C.1.2	Command Mode	323
C.2	<i>wxMaxima</i>	324
	References	325

Index	335
--------------	-----

