



Contents

Preface to the Second Edition *IX*
Preface to the First Edition *XI*

1	What is Monte Carlo? <i>1</i>
1.1	Introduction <i>1</i>
1.2	Topics to be Covered <i>3</i>
1.3	A Short History of Monte Carlo <i>4</i>
	References <i>5</i>
2	A Bit of Probability <i>7</i>
2.1	Random Events <i>7</i>
2.2	Random Variables <i>9</i>
2.2.1	The Binomial Distribution <i>12</i>
2.2.2	The Geometric Distribution <i>13</i>
2.2.3	The Poisson Distribution <i>14</i>
2.3	Continuous Random Variables <i>14</i>
2.4	Expectations of Continuous Random Variables <i>16</i>
2.5	Bivariate Continuous Random Distributions <i>19</i>
2.6	Sums of Random Variables: Monte Carlo Quadrature <i>21</i>
2.7	Distribution of the Mean of a Random Variable: A Fundamental Theorem <i>22</i>
2.8	Distribution of Sums of Independent Random Variables <i>25</i>
2.9	Monte Carlo Integration <i>28</i>
2.10	Monte Carlo Estimators <i>31</i>
	References <i>34</i>
	Further Reading <i>34</i>
	Elementary <i>34</i>
	More Advanced <i>34</i>
3	Sampling Random Variables <i>35</i>
3.1	Transformation of Random Variables <i>36</i>
3.2	Numerical Transformation <i>42</i>
3.3	Sampling Discrete Distributions <i>43</i>

3.4	Composition of Random Variables	47
3.4.1	Sampling the Sum of Two Uniform Random Variables	47
3.4.2	Sampling a Random Variable Raised to a Power	48
3.4.3	Sampling the Distribution $f(z) = z(1 - z)$	50
3.4.4	Sampling the Sum of Several Arbitrary Distributions	50
3.5	Rejection Techniques	53
3.5.1	Sampling a Singular pdf Using Rejection	57
3.5.2	Sampling the Sine and Cosine of an Angle	57
3.5.3	Kahn's Rejection Technique for a Gaussian	59
3.5.4	Marsaglia <i>et al.</i> Method for Sampling a Gaussian	60
3.6	Multivariate Distributions	61
3.6.1	Sampling a Brownian Bridge	62
3.7	The M(RT) ² Algorithm	64
3.8	Application of M(RT) ²	72
3.9	Testing Sampling Methods	74
	References	75
	Further Reading	76
4	Monte Carlo Evaluation of Finite-Dimensional Integrals	77
4.1	Importance Sampling	79
4.2	The Use of Expected Values to Reduce Variance	88
4.3	Correlation Methods for Variance Reduction	91
4.3.1	Antithetic Variates	93
4.3.2	Stratification Methods	95
4.4	Adaptive Monte Carlo Methods	98
4.5	Quasi-Monte Carlo	100
4.5.1	Low-Discrepancy Sequences	101
4.5.2	Error Estimation for Quasi-Monte Carlo Quadrature	103
4.5.3	Applications of Quasi-Monte Carlo	104
4.6	Comparison of Monte Carlo Integration, Quasi-Monte Carlo and Numerical Quadrature	104
	References	105
	Further Reading	106
5	Random Walks, Integral Equations, and Variance Reduction	107
5.1	Properties of Discrete Markov Chains	107
5.1.1	Estimators and Markov Processes	109
5.2	Applications Using Markov Chains	110
5.2.1	Simulated Annealing	111
5.2.2	Genetic Algorithms	112
5.2.3	Poisson Processes and Continuous Time Markov Chains	114
5.2.4	Brownian Motion	122
5.3	Integral Equations	124
5.3.1	Radiation Transport and Random Walks	124
5.3.2	The Boltzmann Equation	126

5.4	Variance Reduction 127
5.4.1	Importance Sampling of Integral Equations 127
	References 129
	Further Reading 130
6	Simulations of Stochastic Systems: Radiation Transport 131
6.1	Radiation Transport as a Stochastic Process 131
6.2	Characterization of the Source 135
6.3	Tracing a Path 136
6.4	Modeling Collision Events 140
6.5	The Boltzmann Equation and Zero Variance Calculations 142
6.5.1	Radiation Impinging on a Slab 144
	References 147
	Further Reading 147
7	Statistical Physics 149
7.1	Classical Systems 149
7.1.1	The Hard Sphere Liquid 151
7.1.2	Molecular Dynamics 153
7.1.3	Kinetic Monte Carlo 154
7.1.4	The Ising Model 155
	References 156
	Further Reading 157
8	Quantum Monte Carlo 159
8.1	Variational Monte Carlo 160
8.2	Green's Function Monte Carlo 161
8.2.1	Monte Carlo Solution of Homogeneous Integral Equations 162
8.2.2	The Schrödinger Equation in Integral Form 163
8.2.3	Green's Functions from Random Walks 165
8.2.4	The Importance Sampling Transformation 167
8.3	Diffusion Monte Carlo 170
8.4	Path Integral Monte Carlo 172
8.5	Quantum Chromodynamics 175
	References 176
	Further Reading 178
9	Pseudorandom Numbers 179
9.1	Major Classes of prn Generators 180
9.1.1	Linear Recurrence Methods 180
9.1.2	Tausworthe or Feedback Shift Register Generators 182
9.1.3	Nonlinear Recursive Generators 183
9.1.4	Combination Generators 184
9.2	Statistical Testing of prng's 185
9.2.1	Theoretical Tests 185

VIII | *Contents*

9.2.2	Empirical Tests	186
9.3	Comparing Two Pseudorandom Number Generators	187
9.3.1	A Multiplicative Congruential Generator Proposed for 32-bit Computers	187
9.3.2	A Bad Random Number Generator	189
9.4	Pseudorandom Number Generation on Parallel Computers	192
9.4.1	Splitting and Leapfrogging	193
9.4.2	Parallel Sequences from Combination Generators	193
9.4.3	Reproducibility and Lehmer Trees	194
9.4.4	SPRNG: A Library of Pseudorandom Number Generators	195
9.5	Summary	195
	References	196

Index 199