

Contents

Preface XV

Notation XIX

1	Magnets for Accelerators	1
1.1	The Large Hadron Collider	2
1.2	A Magnet Metamorphosis	7
1.3	Superconductor Technology	16
1.3.1	Critical Current Density of Superconductors	16
1.3.2	Strands	19
1.3.3	Cables	22
1.4	The LHC Dipole Coldmass	27
1.5	Superfluid Helium Physics and Cryogenic Engineering	29
1.6	Cryostat Design and Cryogenic Temperature Levels	32
1.7	Vacuum Technology	33
1.8	Powering and Electrical Quality Assurance	35
1.9	Electromagnetic Design Challenges	38
1.9.1	The CERN Field Computation Program ROXIE	42
1.9.2	Analytical and Numerical Field Computation	44
	References	46
2	Algebraic Structures and Vector Fields	49
2.1	Mappings	49
2.2	Groups, Rings, and Fields	50
2.3	Vector Space	51
2.3.1	Linear Independence and Basis	53
2.4	Linear Transformations	54
2.5	Affine Space	56
2.5.1	Coordinates	58
2.6	Inner Product Space	60

2.6.1	Metric Space	62
2.6.2	Orthonormal Bases	63
2.6.3	The Erhard Schmidt Orthogonalization	64
2.7	Orientation	66
2.8	A Glimpse on Topological Concepts	68
2.8.1	Homotopy	69
2.8.2	The Boundary Operator	70
2.9	Exterior Products	72
2.10	Identities of Vector Algebra	75
2.11	Vector Fields	75
2.12	Phase Portraits	77
2.13	The Physical Dimension System	80
	References	84
3	Classical Vector Analysis	85
3.1	Space Curves	86
3.1.1	The Frenet Frame of Space Curves	88
3.2	The Directional Derivative	93
3.3	Gradient, Divergence, and Curl	94
3.4	Identities of Vector Analysis	96
3.5	Surfaces in E_3	96
3.6	The Differential	98
3.7	Differential Operators on Scalar and Vector Fields in $\mathbf{r}$ and $\mathbf{r}'$	102
3.8	The Path Integral of a Vector Field	103
3.9	Coordinate-Free Definitions of the Differential Operators	104
3.10	Integral Theorems	106
3.10.1	The Kelvin–Stokes Theorem	106
3.10.2	Green’s Theorem in the Plane	107
3.10.3	The Gauss–Ostrogradski Divergence Theorem	108
3.10.4	A Variant of the Gauss Theorem	108
3.10.5	Green’s First Identity	109
3.10.6	Green’s Second Identity (Green’s Theorem)	110
3.10.7	Vector Form of Green’s Theorem	110
3.10.8	Generalization of the Integration-by-Parts Rule	110
3.10.9	The Stratton Theorems	111
3.11	Curvilinear Coordinates	111
3.11.1	Components of a Vector Field	113
3.11.2	Contravariant Coefficients	114
3.11.3	Covariant Coefficients	115
3.12	Integration on Space Elements	115
3.13	Orthogonal Coordinate Systems	117
3.13.1	Differential Operators	119

3.13.2	Cylindrical Coordinates	121
3.13.3	Spherical Coordinates	122
3.14	The Lemmata of Poincaré	125
3.15	De Rham Cohomology	126
3.16	Fourier Series	129
	References	136
4	Maxwell's Equations and Boundary Value Problems in Magnetostatics	137
4.1	Maxwell's Equations	138
4.1.1	The Global Form	138
4.1.2	The Integral Form	139
4.1.3	The Local Form	141
4.1.4	Maxwell's Original Set of Equations	142
4.2	Kirchhoff's Laws	143
4.3	Conversion of Energy in Electromagnetic Fields	143
4.4	Constitutive Equations	144
4.5	Boundary and Interface Conditions	146
4.6	Magnetic Material	151
4.6.1	Ferromagnetism	152
4.6.2	Measurement of Hysteresis Curves	155
4.6.3	Magnetic Anisotropy in Laminated Iron Yokes	159
4.6.4	Magnetostriction	160
4.6.5	Permanent Magnets	161
4.6.6	Magnetization Currents and Fictitious Magnetic Charges	163
4.7	Classification Diagrams for Electromagnetism	165
4.8	Field Lines	167
4.8.1	Classification of Electromagnetic Field Problems	167
4.9	Boundary Value Problems 1: Magnetostatic	171
4.9.1	Scalar-Potential Formulations	171
4.9.2	Vector-Potential Formulations	173
4.9.3	The Scalar Laplace Equation in 2D	179
4.10	Boundary Value Problems 2: Magnetic Diffusion Problems	180
	References	184
5	Fields and Potentials of Line-Currents	187
5.1	Green Functions	188
5.2	Potentials on Bounded Domains	189
5.3	Properties of Harmonic Fields	191
5.4	The Biot–Savart Law	193
5.5	Field of a Straight Line-Current Segment	197
5.6	Field of a Ring Current	200

5.7	The Magnetic Dipole Moment	203
5.8	The Magnetic Double Layer	205
5.8.1	The Solid Angle	206
5.8.2	Approximating the Solid Angle of a Current Loop	208
5.9	The Image-Current Method	209
5.9.1	Plane Boundaries	211
5.9.2	Circular Boundaries	213
5.10	Stored Energy in a Magnetostatic Field	216
5.10.1	Self and Mutual Inductance	218
5.10.2	The Geometric Mean Distance	220
5.10.3	Magnetic Flux	222
5.11	Magnetic Energy in Nonlinear Circuits	224
5.11.1	Differential Inductance	224
5.12	Magnetic Forces and the Maxwell Stress Tensor	227
5.13	Fields and Potentials of Magnetization Currents	230
5.14	Magnetic Levitation	232
	References	235
6	Field Harmonics	237
6.1	Circular Harmonics	238
6.1.1	Determining the Multipole Coefficients	240
6.1.2	Magnetic Shielding; Permeable Cylindrical Shell in a Uniform Field	253
6.1.3	Integrated Multipoles in Accelerator Magnets	255
6.2	Spherical Harmonics	257
6.2.1	Legendre Series Expression for the Vector Potential	262
6.2.2	Determining the Zonal Harmonics	263
6.3	Separation in Cartesian Coordinates	265
	References	268
7	Iron-Dominated Magnets	269
7.1	C-Shaped Dipole	270
7.2	Quadrupole	271
7.3	Ohmic Losses in Dipole and Quadrupole Coils	272
7.4	Magnetic Circuit with Varying Yoke Width	272
7.5	Branched Circuits	274
7.6	Ideal Pole Shapes of Iron-Dominated Magnets	275
7.6.1	Shimming	277
7.7	Rogowski Profiles	278
7.8	Combined-Function Magnets	281
7.9	Permanent Magnet Excitation	282
7.10	Cooling of Normal-Conducting Magnets	287
	References	291

8	Coil-Dominated Magnets	293
8.1	Accelerator Magnets	294
8.1.1	Generation of Pure Multipole Fields	295
8.1.2	Sensitivity to Coil-Block Positioning Errors	305
8.1.3	Force Distribution	305
8.1.4	Margins in the LHC Main Dipole	306
8.2	Combined-Function Magnets and the Unipolar Current Dipole	309
8.3	Rectangular Block-Coil Structures	310
8.4	Field Enhancement in Coil Ends of Accelerator Magnets	311
8.5	Magnetic Force Distribution in the LHC Dipole Coil Ends	312
8.6	Nested Helices	314
8.7	Solenoids	315
8.7.1	Helmholtz and Maxwell Coils	315
8.7.2	Fabry Factors	317
8.7.3	Off-Axis Fields	321
8.7.4	Zonal Harmonics	324
	References	325
9	Complex Analysis Methods for Magnet Design	327
9.1	The Field of Complex Numbers	328
9.2	Holomorphic Functions and the Cauchy–Riemann Equations	329
9.3	Power Series	331
9.4	The Complex Form of the Discrete Fourier Transform	333
9.5	Complex Potentials	335
9.6	Conformal Mappings	336
9.7	Complex Representation of Field Quality in Accelerator Magnets	338
9.7.1	Feed-Down	338
9.7.2	Reference Frame Rotation	342
9.7.3	Reflection about the Vertical Axis	343
9.8	Complex Integration	344
9.8.1	Cauchy’s Theorem and the Integral Formula	345
9.8.2	Properties of Holomorphic Functions	346
9.8.3	The Residual Theorem	348
9.9	The Field and Potential of a Line Current	349
9.9.1	Series Expansion of the Line-Current Field	350
9.9.2	Circular Sector Windings	351
9.10	Multipoles Generated by a Magnetic Dipole Moment	351
9.11	Beth’s Current-Sheet Theorem	352
9.12	Electromagnetic Forces on the Dipole Coil	354
9.13	The Field of a Polygonal Conductor	356
9.14	Magnetic Flux Density Inside Elliptical Conductors	358
	References	362

10	Field Diffusion	363
10.1	Time Constants and Penetration Depths	363
10.2	The Laplace Transform	365
10.3	Conductive Slab in a Time-Transient Applied Field	370
10.3.1	The Step-Excitation Function	371
10.3.2	Linear Ramp of the Applied Field	373
10.3.3	Sinusoidal Excitation	375
10.4	Eddy Currents in the LHC Cold Bore and Beam Screen	376
	References	382
11	Elementary Beam Optics and Field Requirements	383
11.1	The Equations of Charged Particle Motion in a Magnetic Field	383
11.2	Magnetic Rigidity and the Bending Magnets	387
11.3	The Linear Equations of Motion	389
11.4	Weak Focusing	390
11.5	Thin-Lens Approximations	392
11.6	Transfer Matrices	393
11.7	Strong Focusing and the FODO Cell	395
11.8	The Beta Function, Tune, and Transverse Resonances	397
11.9	Off-Momentum Particles	407
11.9.1	Dispersion	408
11.9.2	Chromaticity	410
11.10	Field Error Specifications	412
	References	413
12	Reference Frames and Magnet Polarities	415
12.1	Magnet Polarity Conventions	416
12.1.1	Spool-Piece Correctors	418
12.1.2	Twin-Aperture and Two-in-One Magnets	418
12.2	Reference Frames	420
12.3	Multipole Expansions	421
12.3.1	The Magnet Frame	421
12.3.2	The Local Reference Frame of Beam 1	423
12.3.3	Definition of Field Errors in the Accelerator Design Program MAD	424
12.3.4	Transformation between the Magnet and the Beam 1 Frames	424
12.4	Orbit Correctors	426
12.5	Position of the Connection Terminals	426
12.6	Turned Magnets and Magnet Assemblies	427
12.7	Electrical Circuits in the LHC Machine	429
	References	432

13	Finite-Element Formulations	433
13.1	One-Dimensional Finite-Element Analysis	434
13.1.1	Quadratic Elements	439
13.2	FEM with the Vector-Potential (Curl–Curl) Formulation	441
13.2.1	The Weak Form in 3D	443
13.2.2	The Weak Form in 2D	444
13.3	Complementary Formulations	445
13.3.1	FEM with Reduced Vector-Potential Formulation	445
13.3.2	FEM, Employing the Vector Poisson Equation	449
13.3.3	The A- ϕ Formulation for Eddy-Current Problems	451
	References	453
14	Discretization	455
14.1	Quadrilateral Mesh Generation	456
14.1.1	Parametric Modeling	457
14.1.2	Topology Decomposition	458
14.1.3	Domain Decomposition	459
14.1.4	Meshing of Simple Domains	460
14.1.5	Smoothing	461
14.1.6	Remeshing and Morphing	462
14.2	Finite-Element Shape Functions	465
14.2.1	The Linear Triangular Element, 2D	466
14.2.2	Barycentric Coordinates	469
14.2.3	Local Coordinates	470
14.2.4	Mapped Elements	471
14.2.5	Generation of the Shape Functions	474
14.2.6	Transformation of Differential Operators	476
	References	479
15	Coupling of Boundary and Finite Elements	481
15.1	The Boundary-Element Method	482
15.1.1	The Node Collocation Method	486
15.2	BEM–FEM Coupling	487
15.3	BEM–FEM Coupling using the Total Scalar-Potential	489
15.4	The M(B) Iteration	491
15.5	Applications	492
15.5.1	2D Calculations	492
15.5.2	Saturation Effects in the Iron Yoke	495
15.5.3	3D Calculations	496
	References	502

16	Superconductor Magnetization	503
16.1	Superconductor Magnetization	507
16.2	Critical Surface Modeling	509
16.3	The Critical State Model	513
16.4	The Ellipse on a Cylinder Model	516
16.5	Nested Intersecting Circles and Ellipses	519
16.6	Hysteresis Modeling	521
16.7	Magnet Field Errors due to the Superconducting Filament Magnetization	527
16.8	The M(B) Iteration	530
16.9	Software Implementation	532
16.10	Applications to Magnet Design	532
16.10.1	Compensation of Multipole Field Errors	534
16.11	Nested Magnets	537
	References	540
17	Interstrand Coupling Currents	543
17.1	Analysis of Linear Networks	544
17.1.1	The Linear U(I) Relation in a Branch	545
17.1.2	The Topology of Networks	547
17.1.3	The Branch/Node Incidence Matrix and the Node-Potential Method	548
17.1.4	The Mesh Matrix and the Mesh-Current Method	551
17.1.5	Transient Field Analysis	553
17.2	A Network Model for the Interstrand Coupling Currents	555
17.3	Steady-State Calculations	557
17.3.1	Spectral Analysis of the Solution	559
17.4	Time-Transient Analysis	560
17.4.1	Spectral Analysis of the Solution	561
17.5	The M(B) Iteration Scheme for ISCCs	562
17.6	Approximation for the Interstrand Coupling Currents	563
17.7	Interfilament Coupling Currents	564
17.8	Applications to Magnet Design	566
17.8.1	Field Advance	566
17.8.2	Rapid Cycling Magnets	567
	References	572
18	Quench Simulation	575
18.1	The Heat Balance Equation	577
18.2	Electrical Network Models of Superconductors	580
18.3	Current Sharing	582
18.4	Winding Schemes and Equivalent Electrical Circuit Diagrams	584

18.5	Quench Detection	585
18.6	Magnet Protection	586
18.7	Numerical Quench Simulation	589
18.7.1	The Thermal Model	591
18.7.2	External Electrical Circuits	594
18.8	The Time-Stepping Algorithm	595
18.9	Applications	596
18.9.1	Validating the Model	598
18.9.2	Fast Ramping Magnets	601
	References	607
19	Differential Geometry Applied to Coil-End Design	609
19.1	Constant-Perimeter Coil Ends	612
19.2	Differential Geometry of the Strip Surfaces	615
19.2.1	The Frenet–Serret Equations for Strips	616
19.2.2	The Generators of Strips	618
19.3	Discrete Theory of the Strip Surface	621
19.4	Optimization of the Strip Surface	627
19.5	Coil-End Transformations	630
19.6	Corrector Magnet Coil End with Ribbon Cables	631
19.7	End-Spacer Manufacturing	633
19.8	Splice Configurations	634
	References	636
20	Mathematical Optimization Techniques	637
20.1	Mathematical Formulation of the Optimization Problem	639
20.2	Optimality Criteria for Unconstrained Problems	641
20.3	Karush–Kuhn–Tucker Conditions	642
20.4	Pareto Optimality	644
20.5	Methods for Decision Making	646
20.5.1	Goal Programming	646
20.5.2	The Pareto-Strength Algorithm	650
20.5.3	Constraint Formulation and Sensitivity Analysis	651
20.5.4	Payoff Tables	653
20.6	Box Constraints	654
20.7	Treatment of Nonlinear Constraints	655
20.8	Deterministic Optimization Algorithms	655
20.8.1	Line Search	656
20.8.2	Multidimensional Search Methods	659
20.8.3	Gradient Methods	660
20.9	Genetic Optimization Algorithms	667
20.9.1	Parameter Representation	669

20.9.2	Gray Coding	669
20.9.3	Genetic Operators	671
20.9.4	Convergence	678
20.10	Applications	681
20.10.1	Conceptual Coil Design with Genetic Algorithms	683
20.10.2	Deterministic Optimization of Coil Cross Sections	685
20.10.3	Yoke Design as a Material-Distribution Problem	688
20.10.4	Shape Optimization of the Iron Yoke	689
20.10.5	Payoff Tables for Dipole Designs	691
20.10.6	Lagrange Multiplier Estimation	693
20.10.7	Manufacturing Tolerances	694
20.10.8	Tuning Range	696
20.10.9	Tracing of Manufacturing Errors	697
	References	700

Appendix

A	Material Property Data for Quench Simulations	703
A.1	Mass Density	703
A.2	Electrical Resistivity	705
A.3	Thermal Conductivity	708
A.4	Heat Capacity	711
	References	715
B	The LHC Magnet Zoo	717
B.1	Superconducting Magnets	717
B.2	Normal-Conducting Magnets	726
	References	729
C	Ramping the LHC Dipoles	731
D	SI (MKSA) Units	735
E	Glossary	737
	Index	745