

## Contents

**Preface** XV

**List of Contributors** XIX

**Prologue** XXIII

*Wolfgang Arendt, Delio Mugnolo and Wolfgang Schleich*

<b>1</b>	<b>Weyl's Law</b>	<b>1</b>
	<i>Wolfgang Arendt, Robin Nittka, Wolfgang Peter, Frank Steiner</i>	
1.1	Introduction	1
1.2	A Brief History of Weyl's Law	2
1.2.1	Weyl's Seminal Work in 1911–1915	2
1.2.2	The Conjecture of Sommerfeld (1910)	5
1.2.3	The Conjecture of Lorentz (1910)	7
1.2.4	Black Body Radiation: From Kirchhoff to Wien's Law	8
1.2.5	Black Body Radiation: Rayleigh's Law	10
1.2.6	Black Body Radiation: Planck's Law and the Classical Limit	12
1.2.7	Black Body Radiation: The Rayleigh–Einstein–Jeans Law	14
1.2.8	From Acoustics to Weyl's Law and Kac's Question	18
1.3	Weyl's Law with Remainder Term. I	19
1.3.1	The Laplacian on the Flat Torus $\mathbb{T}^2$	19
1.3.2	The Classical Circle Problem of Gauss	20
1.3.3	The Formula of Hardy–Landau–Voronoi	21
1.3.4	The Trace Formula on the Torus $\mathbb{T}^2$ and the Leading Weyl Term	22
1.3.5	Spectral Geometry: Interpretation of the Trace Formula on the Torus $\mathbb{T}^2$ in Terms of Periodic Orbits	24
1.3.6	The Trace of the Heat Kernel on $d$ -Dimensional Tori and Weyl's Law	25
1.3.7	Going Beyond Weyl's Law: One can Hear the Periodic Orbits of the Geodesic Flow on the Torus $\mathbb{T}^2$	27
1.3.8	The Spectral Zeta Function on the Torus $\mathbb{T}^2$	28
1.3.9	An Explicit Formula for the Remainder Term in Weyl's Law on the Torus $\mathbb{T}^2$ and for the Circle Problem	29

1.3.10	The Value Distribution of the Remainder Term in the Circle Problem	32
1.3.11	A Conjecture on the Value Distribution of the Remainder Term in Weyl's Law for Integrable and Chaotic Systems	34
1.4	Weyl's Law with Remainder Term. II	38
1.4.1	The Laplace–Beltrami Operator on $d$ -Dimensional Compact Riemann Manifolds $\mathcal{M}^d$ and the Pre-Trace Formula	38
1.4.2	The Sum Rule for the Automorphic Eigenfunctions on $\mathcal{M}^d$	39
1.4.3	Weyl's Law on $\mathcal{M}^d$ and its Generalization by Carleman	40
1.4.4	The Selberg Trace Formula and Weyl's Law	42
1.4.5	The Trace of the Heat Kernel on $\mathcal{M}^2$	44
1.4.6	The Trace of the Resolvent on $\mathcal{M}^2$ and Selberg's Zeta Function	45
1.4.7	The Functional Equation for Selberg's Zeta Function $Z(s)$	48
1.4.8	An Explicit Formula for the Remainder Term in Weyl's Law on $\mathcal{M}^2$ and the Hilbert–Polya Conjecture on the Riemann Zeros	49
1.4.9	The Prime Number Theorem vs. the Prime Geodesic Theorem on $\mathcal{M}^2$	51
1.5	Generalizations of Weyl's Law	52
1.5.1	Weyl's Law for Robin Boundary Conditions	52
1.5.2	Weyl's Law for Unbounded Quantum Billiards	53
1.6	A Proof of Weyl's Formula	54
1.7	Can One Hear the Shape of a Drum?	59
1.8	Does Diffusion Determine the Domain?	63
	References	64

## 2 Solutions of Systems of Linear Ordinary Differential Equations 73

*Werner Balser, Claudia Röscheisen, Frank Steiner, Eric Sträng*

2.1	Introduction	73
2.2	The Exponential Ansatz of Magnus	76
2.3	The Feynman–Dyson Series, and More General Perturbation Techniques	78
2.4	Power Series Methods	80
2.4.1	Regular Points	80
2.4.2	Singularities of the First Kind	81
2.4.3	Singularities of Second Kind	82
2.5	Multi-Summability of Formal Power Series	84
2.5.1	Asymptotic Power Series Expansions	84
2.5.2	Gevrey Asymptotics	85
2.5.3	Asymptotic Existence Theorems	85
2.5.4	$k$ -Summability	86
2.5.5	Multi-Summability	89
2.5.6	Applications to PDE	90
2.5.7	Perturbed Ordinary Differential Equations	91
2.6	Periodic ODE	92
2.6.1	Floquet–Lyapunov Theorem and Floquet Theory	92

2.6.2	The Mathieu Equation	93
2.6.3	The Whittaker–Hill Formula	93
2.6.4	Calculating the Determinant	94
2.6.5	Applications to PDE	94
	References	95
<b>3</b>	<b>A Scalar–Tensor Theory of Gravity with a Higgs Potential</b>	<b>99</b>
	<i>Nils Manuel Bezares-Roder, Frank Steiner</i>	
3.1	Introduction	99
3.1.1	General Relativity and the Standard Model of Particle Physics	99
3.1.2	Alternative Theories of Gravity and Historical Overview	111
3.2	Scalar-Tensor Theory with Higgs Potential	115
3.2.1	Lagrange Density and Models	115
3.2.2	The Field Equations	118
3.2.3	Field Equations After Symmetry Breakdown	119
3.2.4	Outlook	124
	References	131
<b>4</b>	<b>Relating Simulation and Modeling of Neural Networks</b>	<b>137</b>
	<i>Stefano Cardanobile, Heiner Markert, Delio Mugnolo, Günther Palm, Friedhelm Schwenker</i>	
4.1	Introduction	137
4.2	Voltage-Based Models	138
4.3	Changing Paradigm – From Biological Networks of Neurons to Artificial Neural Networks	142
4.4	Numerical Simulation of Neural Networks	143
4.5	Population-Based Simulation of Large Spiking Networks	148
4.6	Synaptic Plasticity and Developing Neural Networks	152
	References	153
<b>5</b>	<b>Boolean Networks for Modeling Gene Regulation</b>	<b>157</b>
	<i>Christian Wawra, Michael Köhl, Hans A. Kestler</i>	
5.1	Introduction	157
5.2	Biological Background	158
5.3	Aims of Modeling	160
5.4	Modeling Techniques	161
5.5	Modeling GRNs with Boolean Networks	162
5.6	Dynamic Behavior of Large Random Networks	165
5.7	Inference of Gene Regulatory Networks from Real Data	169
5.7.1	Problem Definition	170
5.7.2	Identifying Algorithms	170
5.7.3	Noisy Data and the Data First Approach	171
5.7.4	An Information Theoretical Approach	174
5.7.5	Using the Chi-Square Test to Find Relationships Among Genes	175
5.8	Conclusion	175

References 177

## 6 Symmetries in Quantum Graphs 181

*Jens Bolte, Stefano Cardanobile, Delio Mugnolo, Robin Nittka*

- 6.1 Symmetries 181
- 6.2 Quantum Graphs 185
- 6.3 Energy Methods for Schrödinger Equations 186
- 6.4 Symmetries in Quantum Graphs 190
- 6.5 Schrödinger Equation with Potentials 192
- 6.6 Concluding Remarks and Open Problems 193
- References 195

## 7 Distributed Architecture for Speech-Controlled Systems Based on Associative Memories 197

*Zöhre Kara Kayikci, Dmitry Zaykovskiy, Heiner Markert, Wolfgang Minker,  
Günther Palm*

- 7.1 Introduction 197
- 7.2 System Architecture 199
- 7.3 Feature Extraction on Mobile Devices 202
  - 7.3.1 ETSI DSR Front-End 202
    - 7.3.1.1 Feature Extraction 202
    - 7.3.1.2 Feature Compression 203
  - 7.3.2 Implementation of the Front-End on Mobile Phones 204
    - 7.3.2.1 Multi-Threading 204
    - 7.3.2.2 Fixed-Point Arithmetic 204
    - 7.3.2.3 Processing Time on Real Devices 205
- 7.4 Speech Recognition Systems Based on Associative Memory 205
  - 7.4.1 Features to Subword Units Conversion using HMMs 206
    - 7.4.1.1 Acoustic Models 206
    - 7.4.1.2 Language Model and Dictionary 207
  - 7.4.2 Subword Units to Words Conversion  
using Neural Associative Memory 207
    - 7.4.2.1 Neural Associative Memories 207
    - 7.4.2.2 The Neural Associative Memory-Based Architecture  
for Word Recognition 209
    - 7.4.2.3 The Functionality of the Architecture 210
    - 7.4.2.4 Learning of New Words 211
- 7.5 Words to Semantics Conversion using Associative Memory 211
  - 7.5.1 Spoken Word Memory 212
  - 7.5.2 Language Parser 213
  - 7.5.3 Ambiguities 214
  - 7.5.4 Learning of New Objects 215
- 7.6 Sample System/Experimental Results 215
- 7.7 Conclusion 216
- References 217

<b>8</b>	<b>Machine Learning for Categorization of Speech Utterances</b>	219
	<i>Amparo Albalade, David Suendermann, Roberto Pieraccini, Wolfgang Minker</i>	
8.1	Introduction	219
8.2	An Overview of Pattern Recognition	222
8.3	Utterance Classification as a Text-Classification Problem	224
8.4	Utterance Corpus Description	225
8.5	Utterance Preprocessing	226
8.6	Feature Extraction Based on Term Clustering	227
8.6.1	Term Vector of Lexical Co-occurrences	228
8.6.2	Hard Term Clustering	229
8.6.2.1	Disambiguation	230
8.6.3	Fuzzy Term Clustering	231
8.6.4	Pole-Based Overlapping Clustering	231
8.6.4.1	PoBOC with Fuzzy $C$ -medoids	232
8.6.4.2	Utterance Feature Vector	232
8.6.5	Utterance Categorization	232
8.7	Supervised Methods for Utterance Categorization	233
8.7.1	Naïve Bayes Classifier	233
8.7.2	Vector Model with Term Weighting for Utterance Classification	233
8.7.2.1	Term Frequency	234
8.7.2.2	IDF, RIDF and ISCF Scores	234
8.8	Evaluation Methods and Results	235
8.8.1	Classification with One Labeled Utterance and Feature Extraction	235
8.8.2	Classification Based on $F$ Samples per Category	236
8.9	Summary and Conclusion	239
	References	239
<b>9</b>	<b>Semi-Supervised Clustering in Functional Genomics</b>	243
	<i>Johann M. Kraus, Günther Palm, Friedhelm Schwenker, Hans A. Kestler</i>	
9.1	Introduction	243
9.2	Biological Background	244
9.2.1	Functional Genomics	244
9.2.2	DNA Microarray Technology	244
9.3	Cluster Analysis	245
9.3.1	Clustering Microarray Data	246
9.3.2	Cluster Methods	248
9.3.2.1	Hierarchical Clustering	248
9.3.2.2	Partitional Clustering	250
9.3.2.3	Incremental Updates	252
9.3.2.4	Model-Based Clustering	253
9.3.2.5	Spectral Clustering and Other Graph-Based Methods	254
9.3.2.6	Biclustering	255
9.3.3	Cluster Validation	257

9.4	Semi-Supervised Clustering	259
9.4.1	Modeling Background Knowledge	261
9.4.2	Results	262
9.4.3	Challenges	263
9.5	Summary	264
	References	266

## 10 Image Processing and Feature Extraction from a Perspective of Computer Vision and Physical Cosmology 273

*Holger Stefan Janzer, Florian Raudies, Heiko Neumann, Frank Steiner*

10.1	Introduction	273
10.1.1	Overall View	274
10.2	Background from Computer Vision	275
10.3	Background from Physical Cosmology	275
10.4	Image Formation and Characterization	277
10.4.1	Generation and Representation of 2D Images	278
10.4.1.1	Perspective Projection	278
10.4.1.2	Stereographic Projection	279
10.4.1.3	Mollweide Projection	279
10.4.2	Image Properties	280
10.4.2.1	Quantization Property	280
10.4.2.2	Spatial Resolution Property	281
10.4.2.3	Scale Space Property	281
10.4.3	Basic Image Characteristics	282
10.4.3.1	Histogram	282
10.4.3.2	Covariance and Correlation	282
10.4.3.3	Joint Histogram	283
10.4.3.4	Co-occurrence	284
10.4.3.5	Fourier Transformation of One Observation of an RF	284
10.4.3.6	Fourier Transformation of a Common Distribution Function	285
10.4.3.7	Two-Point Correlation Function and Power Spectrum	285
10.4.4	Image Registration	287
10.4.4.1	Data Term	288
10.4.4.2	Smoothness Term	289
10.4.4.3	Constraint Term	289
10.5	Methods of Image Processing	289
10.5.1	Filtering Process	290
10.5.2	Linear and Space-Invariant Filters	290
10.5.2.1	Gaussian	290
10.5.2.2	First-Order Derivative	291
10.5.2.3	Second-Order Derivative and Laplacian of Gaussian (LoG)	292
10.5.2.4	Gabor Filter	292
10.5.2.5	Gabor Filter Bank	293
10.5.3	Morphological Filtering	294
10.5.4	Extraction of Image Structures	295

10.6	Invariant Features of Images	296
10.6.1	Statistical Moments and Fourier Descriptors	297
10.6.1.1	Statistical Joint Central Moments	297
10.6.1.2	Fourier Descriptors	297
10.6.2	Stereography and Topology	298
10.6.2.1	Stereography	298
10.6.2.2	Topology	299
10.6.3	Minkowski Functionals and Minkowski Valuations	301
10.6.3.1	Stochastic Geometry	301
10.6.3.2	Integral Geometry	302
10.6.3.3	Applications	303
10.7	Concluding Remarks	307
	References	309
<b>11</b>	<b>Boosting Ensembles of Weak Classifiers in High Dimensional Input Spaces</b>	<b>311</b>
	<i>Ludwig Lausser, Friedhelm Schwenker, Hans A. Kestler</i>	
11.1	Introduction	311
11.2	Hypothesis Boosting Problem	312
11.3	Learn	313
11.4	Boosting by Majority	315
11.5	AdaBoost	316
11.5.1	Training Sample Error	317
11.5.2	Generalization Error	318
11.5.3	AdaBoost on Noisy Data	319
11.6	BrownBoost	320
11.7	AdaBoost for Feature Selection	323
11.8	Conclusion	325
	References	331
<b>12</b>	<b>The Sampling Theorem in Theory and Practice</b>	<b>333</b>
	<i>Wolfgang Arendt, Michal Chovanec, Jürgen Lindner, Robin Nittka</i>	
12.1	Introduction and History	333
12.2	The Sampling Theorem in Applications	334
12.2.1	Applications in Theory	334
12.2.2	Applications in Practice	335
12.2.3	Special Case: Applications in the Field of Information Transmission	336
12.3	Mathematical Formulation of the Sampling Theorem	339
12.3.1	Notation	339
12.3.2	The Sampling Theorem	340
12.3.3	Efficient Proof	341
12.3.3.1	Dirichlet's Theorem	341
12.3.3.2	A First Attempt of a Proof	342
12.3.3.3	The Efficient Proof	343
12.3.4	Conventional Proof	344

- 12.3.4.1 Tempered Distributions 344
- 12.3.4.2 Fourier Transformation 345
- 12.3.4.3 Inversion Theorem 346
- 12.3.4.4 Examples 347
- 12.3.4.5 Convolution 348
- 12.3.4.6 The Conventional Proof 350
- 12.3.4.7 A Convolution Theorem for a Specific Function 352
- References 353

**13 Coding and Decoding of Algebraic–Geometric Codes 355**

*Martin Bossert, Werner Lütkebohmert, Jörg Marhenke*

- 13.1 Introduction 355
- 13.2 Introduction to Linear Codes 355
- 13.3 Introduction to Forward Error Correction 357
  - 13.3.1 Binary Symmetric Channel, BSC 358
  - 13.3.2 Additive White Gaussian Noise Channel, AWGN 358
  - 13.3.3 Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Decoding 359
  - 13.3.4 Hard- and Soft-Decision Decoding 360
  - 13.3.5 Bounded Distance Decoding 360
- 13.4 Algebraic–Geometric Codes 360
- 13.5 Computation of Riemann–Roch Spaces 363
- 13.6 Decoding up to Half the Minimum Distance 365
- 13.7 Interpolation-Based Decoding 368
- 13.8 Power Decoding of Low Rate Reed–Solomon Codes 372
- 13.9 Interpolation-Based Soft-Decision Decoding 373
- 13.10 Soft-Decision Decoding with the Dorsch Algorithm 376
- References 377

**14 Investigation of Input–Output Gain in Dynamical Systems for Neural Information Processing 379**

*Stefano Cardanobile, Michael Cohen, Silvia Corchs, Delio Mugnolo, Heiko Neumann*

- 14.1 Overview 379
- 14.2 Introduction 379
- 14.3 The Basic Unit: Analytical Study of the Dipole 383
  - 14.3.1 Well-Posedness Results 384
  - 14.3.2 Linearization 385
- 14.4 The Basic Unit: Numerical Analysis of the Dipole 388
- 14.5 Model of a Recurrent Network 389
- 14.6 Discussion and Conclusions 391
- References 392

**15 Factorization 395**

*Rüdiger Mack, Wolfgang P. Schleich, Daniel Haase, Helmut Maier*

- 15.1 Introduction 395

15.1.1	Central Ideas	395
15.1.2	Outline of the Article	396
15.2	How to Factor Numbers	398
15.2.1	Prime Numbers, a Primality Test and a Naive Approach to Factorization	399
15.2.2	A More Sophisticated Algorithm	400
15.2.3	Problems with this Algorithm and Probability of Success	400
15.3	How to Find the Period of a Function: The Magic Role of Entanglement	401
15.3.1	Encoding in Quantum Systems	401
15.3.2	Mapping of Periodicity	403
15.3.3	Projection	404
15.3.4	Phase State	404
15.3.5	Subtleties	405
15.4	Analogy with Atom Optics	406
15.4.1	Scattering Atoms off a Standing Wave	406
15.4.2	Method of Stationary Phase	407
15.4.3	Interference in Phase Space	408
15.5	Exponential Growth of Hilbert Space as a Resource of Exponential Speedup	409
15.6	Conclusions	410
15.A	Modular Arithmetic	411
15.A.1	Basic Idea	411
15.A.2	Modular Multiplication	412
15.A.3	Modular Exponentiation	412
15.A.3.1	Recurrence Relation	413
15.A.3.2	Periodicity	413
15.B	Chinese Remainder Theorem	414
15.B.1	Residues Represented as a Matrix	414
15.B.2	Modular Multiplication	416
15.B.3	Period of Function from Chinese Remainder Theorem	416
15.C	Euler's Function	417
15.C.1	Definition	417
15.C.2	Multiplicative Property	418
15.C.3	A Compact Formula for $\phi$	418
15.D	Euclidean Algorithm	420
15.E	Primitive Root	420
15.E.1	Definition	421
15.E.2	Periods for Prime Numbers	421
15.F	Probability for Lucky Choice	423
15.F.1	Expression for the Period	423
15.F.2	Analysis of Different Cases	424
15.G	Elements of Atom Optics	425
15.G.1	Quantum State of Motion in Raman–Nath Approximation	426
15.G.2	Momentum Distribution	426

15.G.3	Discreteness of Momentum due to Interference	427
15.H	Factorization with a Gauss Sum due to its Periodicity	429
	References	430
<b>16</b>	<b>Isomorphism and Factorization – Classical and Quantum Algorithms</b>	<b>433</b>
	<i>Sebastian Dörn, Daniel Haase, Jacobo Torán, Fabian Wagner</i>	
16.1	Introduction	433
16.2	Factorization of Integers: Classical Algorithms	434
16.3	Graph Isomorphism: Classical Algorithms	435
16.4	Quantum Algorithms for Integer Factorization	436
16.4.1	The Quantum Fourier Transform and Period Finding	437
16.4.2	Generalization of the Period-Finding Algorithm	439
16.5	Quantum Approach to Graph Isomorphism	443
16.5.1	The Hidden-Subgroup Problem and Graph Isomorphism	443
16.5.2	The Quantum Query Model and Graph Isomorphism	444
16.5.3	Quantum Walks and the Fix-Automorphism Problem	445
16.6	Reductions of Integer Factorization and Graph Isomorphism to Ring Isomorphism	447
16.6.1	Factoring Integers and Finding Ring Automorphisms	448
16.6.2	Graph Isomorphism and Ring Isomorphism	449
	References	451
<b>17</b>	<b>QuickSort from an Information Theoretic View</b>	<b>455</b>
	<i>Beatrice List, Markus Maucher, Uwe Schöning, Rainer Schuler</i>	
17.1	Introduction	455
17.1.1	Recursion for the Expected Number of Comparisons	457
17.2	An Upper Bound	458
17.3	A Lower Bound	459
17.4	The $\delta$ -Random Source	462
17.5	Conclusion	464
	References	464
	Further Reading	464
<b>Index</b>		<b>465</b>