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Introduction

In 1913, the Nobel Prize in Physics was awarded to Heike Kamerlingh Onnes "for his investigations on the properties of matter at low temperatures which led, *inter alia*, to the production of liquid helium." His crowning achievement was the liquefaction of helium in 1908, which pioneered a new era in low-temperature physics and enabled him to discover superconductivity [3] in 1911.

The theory of superconductivity was developed by Bardeen, Cooper, and Schrieffer (BCS) in 1957 [4]. "In the competitive world of theoretical physics, the BCS theory was the triumphant solution of a long-standing riddle. Between 1911 and 1957, all the best theorists in the world ... had tried and failed to explain superconductivity." [5]. The path to the development of the theory of superconductivity was cleared by the pioneering work of Bardeen and Pines [6], who examined the superconducting isotope effect, took into account the electron–phonon interactions, and determined that electrons could overcome the Coulomb repulsion and attract each other. This weak attraction between the electrons inside the superconducting material is the key to explaining the condensation of electrons and their superconductivity.

In traditional superconductors, the electron–electron attraction, which translates into a small but noticeable reduction of the total energy, occurs between two electrons having opposite wavevectors and opposite spins. At low temperatures, because of the electron–electron attraction, the electrons form a condensate, which is a collective bound state having zero entropy and a reduced total energy, compared with the zero-temperature Fermi distribution energy. This reduction of energy is achieved by allowing, even at zero temperature, for some fraction of the electrons in the superconducting material to have momenta larger than the Fermi momentum, thus maximizing their interaction potential.

Starting from the BCS theory, it was rigorously derived [7] that the condensate can be described by a collective wavefunction, also called superconducting order parameter. The order parameter provides the most complete description of the ensemble of superconducting electrons and depends on three spatial coordinates only, as

$$\psi(\boldsymbol{r}, t) = \left(\frac{n_{\mathrm{cond}}(\boldsymbol{r}, t)}{2}\right)^{1/2} \exp[i\phi(\boldsymbol{r}, t)] \tag{1.1}$$

Superconductivity in Nanowires, First Edition. Alexey Bezryadin.

The normalization of the wavefunction is chosen such that the square of the absolute value of the wavefunction equals half the local density of the condensed electrons $n_{\mathrm{cond}}(\boldsymbol{r}, t)$.[1)] Like the single-particle wavefunction, the collective condensate wavefunction is a complex function of the radius-vector $\boldsymbol{r}$ and time t. The fact that the effective wavefunction of many electrons only depends on one radius-vector $\boldsymbol{r}$ reflects how all electrons behave "coherently," that is, as a single particle.

If the gradient of the phase of the order parameter is not equal to zero (i.e., if $\nabla\phi(\boldsymbol{r}, t) \neq 0$), then the condensate "flows," that is, it carries a nonzero electrical current, called "supercurrent." (Remember that the gradient operator ∇ is a vector having three components given by the spatial partial derivative operators $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$, where x, y, and z are the Cartesian coordinates.) In some sense, the BCS condensate acts as a huge "macromolecule" of electrons. As with actual molecules, one needs to perform a positive work on the condensate to free an electron from such a huge electronic macromolecule. This work is called the superconducting energy gap Δ. The energy of the condensate is reduced due to the attractive interactions between the electrons. The most important property of the BCS condensate is that it can flow through the lattice of positively charged ions without friction. This happens because slowing the entire bound state of all electrons is much more difficult than slowing down single unbound electrons, which exist in normal (i.e., nonsuperconducting) metals.

It is frequently stated that electrons in a superconductor form "Cooper pairs" (CP) or "BCS pairs," which, by virtue of being bosons, are able to condense at low temperatures, thus forming a superfluid bosonic state. Qualitative statements of this sort are difficult to prove or disprove in physics because they do not carry any precise meaning, unless accompanied by corresponding formulas or graphs. The view that a superconducting state is a condensate of bosons (CPs) should be considered incorrect though because there are no bound electronic pairs in a conventional superconductor. The BCS state is a collective condensed state of a macroscopic number of fermions (electrons), not bosons. All condensed electrons participate equally in the condensate. The term "condensate" represents all electrons participating in a collective ground state, in which all the electrons behave quantum-coherently. Even if a localization phenomenon occurs in disordered superconducting films and wires, it is still more appropriate to speak about localized condensate "droplets" or condensate "lakes" than about localized Cooper pairs because two electrons do not form a superconducting, BCS-condensed state.

The Cooper pair density is usually defined as $n_{\mathrm{CP}} \sim N_0\Delta$, where N_0 is the density of states at the Fermi level and Δ is the energy gap. Here, we use a different convention and define the number of electronic pairs (superpairs) in the condensate as n_{s}. The two quantities, n_{s} and n_{CP}, differ strongly. To see this, consider that in clean superconductors (i.e., not having impurities or defects) at zero temperature, $n_{\mathrm{s}} = n/2$, where n is the total density of electrons. Therefore, $n_{\mathrm{CP}}/n_{\mathrm{s}} \sim \Delta/E_{\mathrm{F}} \sim 0.0001$.

1) In that respect, the superconducting order parameter is different from the wavefunction of a single electron because the squared absolute value of the single-electron wavefunction equals the probability density of finding the electron at the specified location.

The physical reality of the density of superconducting electrons, which in our notations equals $2n_s$, is asserted by the measurements of the depth of the magnetic field penetration. According to the Meissner effect, superconductors expel magnetic field. Yet the expulsion is not perfect. The field penetrates to a certain depth, called the clean-limit penetration depth, or the London penetration depth, which is expressed as $\lambda_L^2 = mc^2/8\pi n_s e^2$. At zero temperature, the depth is defined through the total density of electrons in the condensate, which, at $T = 0$, equals the total electronic density n. Thus, at $T = 0$, one gets $\lambda_L^2 = mc^2/4\pi ne^2$.

The reason it is more convenient to use the number of electronic pairs in the condensate is the superconducting parity effect. In a series of beautiful experiments, Tuominen and coworkers [8] showed that the number of electrons participating in the BCS condensed state is an even number. If the total number of electrons in a superconducting island is odd, one of the electrons gets expelled from the condensate, causing a significant energy (Δ) increase of the whole system. This "uncondensed" or "unpaired" electron is located, energetically, above the energy gap of the superconductor. These experiments, while revealing the parity effect, were done in a setting resembling a single-electron tunneling transistor [9]. Its advantage is that in such a device, the number of electrons on a metallic Coulomb "island," which is just some small, micrometer-scale, metallic disc, can be controlled precisely, using the gate electrode of the transistor [9].

The frictionless flow of the BCS condensate requires an explanation, or at least a discussion. A frictionless flow is equivalent to a current flow, with zero voltage applied, which continues indefinitely if the system is not perturbed. Such a stable persistent current is called a supercurrent. The existence of a frictionless supercurrent may be justified as follows. First, note that the velocity of the condensate, also called the "superfluid velocity" $\boldsymbol{v}_s$, is proportional to the phase gradient of its wavefunction, [1], namely,

$$\boldsymbol{v}_s = \left(\frac{\hbar}{2m}\right)\nabla\phi(\boldsymbol{r}, t) \tag{1.2}$$

where $\hbar = 1.054 \times 10^{-34}$ J s is the reduced Planck's constant, and $m = 9.109 \times 10^{-31}$ kg is the electronic mass. This formula is correct only when the magnetic field is zero everywhere, so the vector potential is put to zero.[2)] For now, we assume that *the vector potential is zero* everywhere. Then, the electrical current density carried by the condensate, called the "supercurrent density," can be expressed as

$$\boldsymbol{j}_s = 2en_s\boldsymbol{v}_s = \left(\frac{\hbar e}{m}\right)n_s\nabla\phi(\boldsymbol{r}, t) \tag{1.3}$$

2) To simplify the discussion, we assume that the magnetic field is negligible everywhere, so the corresponding magnetic vector potential can be chosen as zero, $\boldsymbol{A} = 0$. If the magnetic field is present, the superfluid velocity is proportional to a linear combination of the phase gradient and the vector potential.

where $e = -|e|$ is the electronic charge, and $n_s = |\psi|^2 = n_{cond}(\boldsymbol{r}, t)/2$ is the mean density of the electrons pairs participating in the BCS condensate.[3] Thus, it is clear that for the supercurrent to remain steady in time (i.e., to have $\partial \boldsymbol{j}_s/\partial t = 0$), it is sufficient to have a constant phase gradient of the corresponding wavefunction (i.e., $\partial \nabla \phi(\boldsymbol{r}, t)/\partial t = 0$) and a constant density of the condensate (i.e., $\partial n_s/\partial t = 0$).

Let us now argue that these two quantities remain fixed in time if no voltage is applied. Assume that at $t = 0$, both n_s and the phase gradient are greater than zero and constant in space, that is, there is a uniform supercurrent flow. Then, the wavefunction can be written as $\psi = \sqrt{n_s}\exp(i\boldsymbol{k}\boldsymbol{r})$. The phase then is $\phi(\boldsymbol{r}) = \boldsymbol{k}\cdot\boldsymbol{r}$, where the vector $\boldsymbol{k}$ is called the wavevector of the wavefunction. The corresponding superfluid velocity is $v_s = \hbar\boldsymbol{k}/2m$. To show that the resistance of the superconductor is zero, we will argue that the superfluid velocity, the phase gradient, and n_s do not change with time if the electric field is zero.[4]

First, let us consider the superfluid density n_s. Its value is set by the requirement that the corresponding thermodynamic potential is minimized. For example, if the volume and the temperature are fixed and the electric field in the superconductor is zero, then the corresponding thermodynamic potential is Helmholtz free energy, $F = U - TS$, where U is the internal energy, T is the temperature and S is the entropy. Since U and S must be functions of n_s, for F to be constant and remain at its minimum, the density of the condensate n_s must remain constant in time. Small fluctuations near the mean value might be present, but they average to zero and do not cause any change of the mean superfluid density which defines the mean supercurrent. The key fact is that in a superconductor at a temperature below its critical temperature the superfluid density is larger than zero provided that the thermodynamic equilibrium is established.

The phase gradient of the condensate wavefunction also does not change with time if the electric and chemical potentials are constant within the sample. Gor'kov has shown theoretically [10], using his microscopic theory [7, 10], that the phase of the superconductor wavefunction changes in time as

$$\phi(\boldsymbol{r}, t) = \frac{2e\mu(\boldsymbol{r})t}{\hbar} + \phi(\boldsymbol{r}, 0) \tag{1.4}$$

where $\phi(\boldsymbol{r}, 0)$ is the phase at time zero and $\mu(\boldsymbol{r})$ is the local value of the electrochemical potential, which is defined by the equation $E_{N+2} - E_N = 2e\mu(\boldsymbol{r})$. Here, E_N is the energy of the condensate containing N electrons, and E_{N+2} is the energy of the same condensate, after introducing an additional superpair at position $\boldsymbol{r}$.

3) Such normalization is traditionally used to stress the superconducting parity effect, that is, the fact that the number of electrons in a BCS-condensed state is an even number. It is curious to note that the BCS quantum state is such that the number of the pairs is not exactly defined, but it is subject to quantum fluctuations. For a large, macroscopic sample, the *uncertainty* of the number of pairs is by many orders of magnitude smaller than the number itself.

4) Compare this with the time-evolution of wavefunctions of a single electron. Such wavefunctions can change in time because of scattering over impurities or phonons, or other perturbations. Thus, the fact that the condensate is able to maintain a constant nonzero velocity or momentum is not trivial and requires some discussion. For example, it would be interesting to understand why a flowing condensate cannot dissipate its momentum to phonons.

The absolute value of the phase does not have any physical significance since it cannot be measured. On the other hand, the phase difference can be measured. Let us define the phase difference between two points, $\boldsymbol{r}_2$ and $\boldsymbol{r}_1$, as $\phi = \phi_2 - \phi_1 = \phi \equiv \phi(\boldsymbol{r}_2, t) - \phi(\boldsymbol{r}_1, t)$. The time-evolution equation (1.4) can be transformed for the phase difference as

$$\phi = \frac{2e\Delta\mu t}{\hbar} + \phi(0)$$

where $\Delta\mu = (\mu(\boldsymbol{r}_2) - \mu(\boldsymbol{r}_1))$ is the difference of electrochemical potentials and $\phi(0) = \phi(\boldsymbol{r}_2, 0) - \phi(\boldsymbol{r}_1, 0)$ is the phase difference at time zero.

The electrochemical potential is the sum of the chemical potential and the local electric potential. Assume that the chemical potential is constant everywhere in the superconducting sample. Then, the difference of electrochemical potentials becomes the difference of electric potentials, which is the voltage V between two points. Therefore, $\Delta\mu = V$, and the time-evolution equation becomes

$$\phi = \frac{2eVt}{\hbar} + \phi(0)$$

Finally, one can differentiate it with respect to time and obtain

$$\hbar\frac{d\phi}{dt} = 2eV \tag{1.5}$$

where V is the voltage between two points specified by the arbitrary chosen radius vectors $\boldsymbol{r}_2$ and $\boldsymbol{r}_1$. It was Anderson and Dayem [11] who first introduced this popular presentation of the phase evolution equation, in which the phase difference, ϕ, rather than the local value of the phase itself, $\phi(\boldsymbol{r}, t)$, is used. Since, fundamentally, the time-evolution of the phase of the macroscopic superconducting wavefunction was first derived by Gor'kov (1958) (P.W. Anderson, private communication, 2007), we elect to call (1.5) as the Gor'kov phase-evolution equation. It was also named by various authors as the Gor'kov–Josephson equation [12, 14], or the AC Josephson equation [1, 15], or simply the phase-evolution equation. Fundamentally, it is analogous to the time-dependent Schrödinger equation (see more on this analogy below).

Incidentally, note that (1.5) is the only equation in the field of superconductivity which is exact; all others are only approximate. This is why the Gor'kov equation is used in metrology, in which case the phase rotation is synchronized with the external electromagnetic field of a known frequency f, so $d\phi/dt = 2\pi f$. The factor 2π occurs because as the phase completes one cycle, it changes exactly by 2π. Then, according to (1.5), $2\pi f\hbar = 2eV$. Thus, by measuring voltage, the fundamental constants ratio $\hbar/e$ can be determined as $\hbar/e = V/\pi f$.

If the voltage is zero, the phase difference between any two points on the wire, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, does not depend on time. Furthermore, if the electric field is zero, $\boldsymbol{E} = 0$, then the phase gradient is also time-independent. Remember that the phase difference and the phase gradient are proportional to each other as $\phi = (\boldsymbol{r}_2 - \boldsymbol{r}_1)\nabla\phi$, assuming that the two points are close to each other. In this notation, the

voltage is also zero, $V = (\boldsymbol{r}_2 - \boldsymbol{r}_1)\boldsymbol{E} = 0$. So, if $d\phi/dt = 0$ and $\boldsymbol{r}_2 \neq \boldsymbol{r}_1$, then $d(\nabla\phi)/dt = 0$. (Note that the phase gradient $\nabla\phi$ is a vector.)

Thus, we have argued that the supercurrent is time-independent under zero electric field because the supercurrent is a product of the phase gradient $\nabla\phi$ and the density of the condensate n_s, both of which are time-independent, as was discussed above.

To develop a physical intuition and qualitatively understand the physical origin of the time-evolution equations of the phase of the wavefunction, we note that the phase evolution given by (1.4) and (1.5) is analogous to the evolution in time of the phase of a single quantum particle in the ground state. Below, we develop this analogy. Consider a particle in a ground state with energy E_0. Its complete wavefunction satisfies the time-dependent Schrödinger equation $i\hbar\partial_t\Psi = \hat{H}\Psi$, where, for convenience, we use the notation for the partial time derivative as $\partial_t\Psi \equiv \partial\Psi/\partial t$. However, in the ground state, we can also write the time-independent Schrödinger equation as $\hat{H}\Psi = E_0\Psi$. Combining these two equations, $i\hbar\partial_t\Psi = E_0\Psi$. The solution, that is, the wavefunction of the considered quantum particle in the ground state, is well known, namely, $\Psi(\boldsymbol{r}, t) = \psi(\boldsymbol{r})\exp(-iE_0t/\hbar)$, where $\psi(\boldsymbol{r})$ is the time-independent complex function that defines the spatial distribution of the particle probability amplitude, t is the time, $\boldsymbol{r}$ is the radius-vector of the particle, and the imaginary unit satisfies the equality $(-i)i \equiv 1$. Let us find the phase of this wavefunction. First, remember that any complex number X_c can be presented in the form $X_c = X_a\exp(i\phi_x)$. The real number ϕ_x is called the phase of X_c. The absolute value or the magnitude of X_c is $X_a = \sqrt{X_c^* X_c}$. Accordingly, for the wavefunction in the ground state, the first, time-independent factor can be presented as $\psi(\boldsymbol{r}) = |\psi(\boldsymbol{r})|\exp(i\phi_0)$. Here, ϕ_0 is the phase at time zero. Thus, the entire wavefunction is $\Psi(\boldsymbol{r}, t) = |\psi(\boldsymbol{r})|\exp[i(\phi_0 - E_0t/\hbar)]$. So the phase of the single-particle wavefunction is $\phi = \phi_0 - E_0t/\hbar$. This expression is analogous to the equation describing the phase of the superconducting condensate, that is, (1.4).

To develop the analogy further, assume that the quantum particle under investigation is a single electron exposed to a spatially constant electric potential μ. Then, the Hamiltonian is $\hat{H} = (-\hbar^2/2m)\nabla^2 + e\mu$. Thus, the ground state energy is $E_0 = e\mu$ and, therefore, the phase of the wavefunction is $\phi = \phi_0 - e\mu t/\hbar$ which is already very similar to (1.4).

To understand the origin of the factor 2 in front of μ in (1.4), remember the parity effect. The BCS condensate always contains an even number of electrons. Each pair has the charge $2e$ and the mass $2m$. Thus, the Hamiltonian for a single pair is $\hat{H} = (-\hbar^2/4m)\nabla^2 + 2e\mu$, the energy of the ground state is $E_0 = 2e\mu$, and, therefore, the phase of the wavefunction depends on time as $\phi = \phi_0 - 2e\mu t/\hbar$. The result is in agreement with (1.4), which follows from the BCS and the Gor'kov theory. Since in a superconductor all pairs behave coherently, one expects that the phase evolution of one pair is the same as the phase evolution of the phase-coherent ensemble of pairs.

An important property of a BCS condensate, either stationary or moving with respect to the crystal lattice, is that its spectrum of excitations is usually "gapped," that is, a finite amount of energy, Δ, is required to create an excited state. Such ex-

cited states are called quasiparticles, or Bogoliubov quasiparticles, or bogoliubons (see [1], p. 61). According to the BCS theory, the gap is $\Delta = 1.76 k_B T_c$, where $k_B = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant and T_c is the critical temperature of the superconductor. The T_c is the temperature below which superconductivity develops. For completeness, we should mention that gapless superconductivity is in general also possible [16], so the presence of a gap in the spectrum of excitations is not a necessary condition for zero resistance (for more details, see [1], p. 390).

It is interesting to compare superconductors to semiconductors, in which the spectrum of excitations is also gapped. The difference is that in semiconductors, the gapped state, that is, the state in which the valence band is completely filled and the conduction band is completely empty, is characterized by zero total current. To create a nonzero electrical current in a semiconductor, some number of electrons must be excited from the valence band to the conduction band. Such excited states are not gapped since the electron(s) present in the conduction band can change energy by an infinitesimal amount, for example, under the action of external electric field or impurities. With time, the excited electrons give up their energy to phonons and relax back to the lower-energy valence band. As soon as all excited electrons relax, the electrical current decays to zero. In a superconductor, however, the supercurrent is associated not with excitations, but with the condensate itself. Even in the ground state, the supercurrent can be large. For example, if a superconducting wire loop is exposed to a perpendicular magnetic field, the velocity of the condensate is proportional to the magnetic vector-potential, which, in turn, is proportional to the magnetic flux piercing the loop. Such a magnetically induced supercurrent is called Meissner current. It is possible because all electrons in superconductors behave coherently, as a single quantum particle (single electron). For a single electron, the velocity is proportional to vector-potential, assuming that the phase gradient is zero. The Meissner current does not decay since it is associated with the ground state, that is, the BCS condensate. The ground state cannot relax because there are no states having lower energies. The electrons in a normal metal also participate in persistent currents if a magnetic field is applied. However, these currents are all different, and their signs are different since the electrons are not coherent in a normal metal. Thus, they all add to an extremely small value, of the order of a current of one electron. In a superconductor, a macroscopic number of electrons participate in a collective persistent current. In such cases, the currents of all condensed electrons add up. That is why Meissner currents can be much stronger than persistent currents in normal metals.

The ability of a superconductor to carry a dissipationless current, that is, a current under zero applied voltage, disappears if the superconductor is shaped into a *thin* cylinder or a thin wire, or, in other words, if the superconductor is quasi-one-dimensional (see Figure 1.1). This is because if the diameter of the superconductor is small, the rate of strong thermal fluctuations, which bring short segments of the wire into the normal state, is essentially greater than zero at finite temperatures.

Such fluctuations, first predicted by William Little in 1967 [17] and called Little's phase slips (LPS), occur stochastically at random spots on a superconducting wire and interrupt the dissipationless flow of the condensate. Each such local fluc-

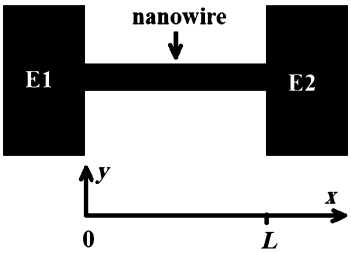

Figure 1.1 Illustration of a typical experimental realization of a transport experiment on a nanowire. The wire is connected at its ends to two macroscopic superconducting electrodes E1 and E2. The wire itself is a thin cylinder having a diameter much smaller than the magnetic field penetration depth, and also smaller than the superconductor's coherence length. It will always be assumed that the x-axis is directed along the wire and the wire starts at $x = 0$ and ends at $x = L$. Since the wire is assumed to be thinner than the coherence length, ξ, it follows that the order parameter is approximately constant within any cross-section of the wire, taken perpendicular to the wire axis. Thus, the assumption that the wire has an exact cylindrical geometry is not essential, that is, the cross-section can be of any shape, without having any qualitative effect on the wire behavior. To qualify as quasi-one-dimensional, the dimensions of the cross-section of the wire must be smaller than $\pi\sqrt{2}\xi$ since, in this case, vortices are not energetically favorable on the wire [18]. The term "nanowire" is usually applied to wires which are much thinner than 1 μm in diameter.

tuation allows the phase difference between the ends of the wire to "slip" by 2π (in other words, to decrease by 2π), causing the supercurrent to diminish. To undergo a phase slip, the free energy of the condensate must increase somewhat to overcome a certain energy barrier (usually denoted ΔF). This barrier equals the condensation energy density multiplied by the volume of the normal region associated with the LPS. As with any barrier crossing process, the LPS are driven by thermal fluctuations at sufficient temperatures. Such phase slips are referred to as thermally activated phase slips (TAPS).

As the temperature is lowered, the rate of TAPS exhibits a rapid decline described by the Arrhenius activation law. The resistance of the wire is linearly proportional to the rate of TAPS. Thus, as the temperature is reduced, the wire resistance drops exponentially, or, to be more precise, according to the Arrhenius law [21], namely, as $R \sim \exp(-\Delta F(T)/k_B T)$, where $\Delta F(T)$ is some effective barrier, which will be discussed in detail below. Such activation dependence of resistance on temperature was confirmed in experiments by Lukens, Warburton, and Webb [22] and Newbower, Beasley, and Tinkham [23]. Although the resistance of a superconducting wire is exponentially low at low temperatures, nevertheless, strictly speaking, it does not become zero at any finite temperature because TAPS has a nonzero probability at any finite temperature. In other words, there is no thermodynamic phase transition in a thin wire. As the temperature is reduced below the thermodynamic critical temperature T_c, the resistance decreases continuously, never reaching zero.

There is no qualitative difference in the state of the wire above T_c and below T_c. Above T_c, superconducting fluctuations occur. Thus, the wire is not completely normal. Below T_c, there are normal-state fluctuations (i.e., the LPS), so the wire is

not fully superconducting. Thus, the wire undergoes a crossover from a predominantly normal state above T_c to a predominantly superconducting state below T_c, but not a phase transition. In fact, the crossover does not happen at $T = T_c$, but at a temperature T_1 such that $\Delta F(T_1) \sim k_B T$. It should be emphasized that nothing experimentally noticeable happens with the wire either at $T = T_c$. Thus, when fitting data, the T_c should be treated as a fitting parameter. The parameter T_c controls the behavior of the resistance through the effective barrier ΔF since $\Delta F = 0$ at T_c and increases with cooling. The T_c is not a parameter that is directly measurable in thin wires. This is in contrast with bulk superconductors, in which the T_c is simply the temperature at which the resistance drops to zero.

Generally, one expects that at low temperatures, the thermal activation rate decreases exponentially with cooling while the quantum tunneling rate should remain roughly constant, thus becoming dominant below a certain crossover temperature, typically denoted T^*. Therefore, TAPS, occurring in superconducting wires below T_c, should be succeeded by tunneling of phase slips at sufficiently low temperatures, namely, at $T < T^*$. Such tunneling phase slips are usually called quantum phase slips (QPS) since, qualitatively speaking, they derive their existence from quantum fluctuations and the Heisenberg uncertainty of the energy. If the system undergoing quantum tunneling possesses many internal degrees of freedom which get involved into the tunneling event, then the tunneling is called "macroscopic." For example, tunneling of a condensate involving many electrons or tunneling of a large molecule composed of many atoms would be considered as a macroscopic quantum tunneling (MQT). Thus, tunneling of Little's phase slips is an example of MQT since a large number of electrons occur in the virtual normal core of QPS. According to this classification, QPS is a particular case of MQT.

Macroscopic quantum tunneling is one of the advanced research topics of modern physics, as it belongs to the transitional region between classical and quantum mechanics. Note that at the fundamental level, the relationship between classical and the quantum theories is still not fully understood because of the quantum mechanics' reliance on classical mechanics for its justification. This fact is exemplified by the problem of quantum measurement, which requires the wavefunction to collapse when a quantum system is measured with a classical measuring apparatus. If the system is strictly isolated, such a collapse is difficult to justify.

In older textbooks, such collapse was explained by making an explicit assumption that the measurement apparatus is classical, not quantum, by definition. The statement that the apparatus is classical infers that it cannot exist in a quantum superposition of macroscopically distinct states. For example, a voltmeter cannot, in principle, exist in a superposition of states having different readings, for example, $V = 0$ and $V = 1\,\mathrm{V}$ simultaneously. It must "choose" one particular reading.

Although such an assumption seems reasonable, it remains desirable to formulate quantum mechanics in a self-sufficient manner. Within quantum theory, the system can be in any quantum superposition of allowed states. For example, electrons can be described by extended wavefunctions, meaning that they are not located in any particular point of space, but rather they can occupy many remote points of space simultaneously. And, although counterintuitive, a voltmeter isolated from

any interaction with the external world should be able to accept a state of quantum superposition of states having different readings. Such would be a Schrödinger cat state.

Therefore, a search for fundamental physical phenomena causing wavefunctions of large isolated objects to collapse continues [24]. Of course, the puzzle of collapse is a puzzle only insofar as the measurement apparatus is allowed to only interact with the quantum system, but not with the environment. On the other hand, if the measurement apparatus interacts with its environment, say with the rack supporting it or with a physicist looking at it, then quantum theory alone predicts that the wavefunction of the apparatus collapses because of decoherence [25]. Yet, when an isolated system is considered, such as a hypothetical Schrödinger cat [26] or, as a different example, the whole Universe, which is presumed to include everything with which anything can interact, then the expected collapse of the wavefunction remains unjustified theoretically. These fundamental difficulties led to such impressive ideas as the many-worlds interpretation of quantum mechanics by Hugh Everett, which is currently a mainstream interpretation [27].

Initiated by Leggett, the field of macroscopic quantum physics has seen widespread development [28–39]. Definitive experimental evidence that a macroscopic system can behave according to the laws of quantum mechanics has been obtained by Clarke and collaborators [33]. Evidence of MQT was also found in experiments using magnetic nanoparticles, in which case the entire particle reverses its magnetization within a single quantum tunneling event [35]. These experiments quite convincingly demonstrate that rather large systems (large when compared with single atoms) can exist in quantum superpositions of macroscopically distinct states. Recent fundamental recognition [40, 41] of the potential advantages of computational methods based on quantum bits (qubits) has initiated the search for practical implementations of systems which can maintain for a sufficiently long time a quantum superposition of macroscopically distinct states. Such systems can be built and can indeed operate as qubits [42–44]. It was also proposed that superconducting nanowires could be used as active elements in flux qubits, provided that quantum tunneling of the phase difference can occur coherently in nanowires [45, 50, 51]. According to Khlebnikov [45], "the process [of such tunneling in thin superconducting wires] may be suitable for forming quantum superpositions of flux states." Mooij and Nazarov also proposed that QPS could be used to build current standards, and thus could advance the field of exact metrology [51]. Consequently, understanding QPS is an important topic of modern quantum physics.

The search for QPS in superconductor nanowires was first undertaken in experiments by the Mooij group [52], although QPS was not observed. Later, Giordano's experiments [53] gave evidence that QPS might exist. The difficulty of observing QPS is related, in general, to the fact that the tunneling rate is exponentially suppressed, not only by the width and the height of the tunnel barrier and the large effective mass of macroscopic systems, but also by their strong interaction with the environment [30].

Qualitatively speaking, the suppression of the quantum tunneling by the environment can be classified as the quantum Zeno effect. This effect refers to a situation in which an unstable system, if somehow "observed" continuously, can never decay by tunneling. Thus, it is possible to strongly slow down the evolution of the system by continually measuring its state. The quantum Zeno effect is quite general. It refers to a situation in which the Schrödinger-type time-evolution of a quantum system is strongly slowed not only by measurements, but also by quantum decoherence caused by various interactions with the environment. The name originates from Zeno's arrow paradox, which states that an arrow in flight is not observed to move at any single instance, and therefore cannot possibly be moving at all. Of course, arrows can move in space very well. Thus, some sort of paradox is present since the qualitative reasoning leads to a different conclusion. The paradox was resolved by Newton and Leibniz with the invention of calculus, which is a mathematical apparatus allowing exact logical analysis of infinitesimal displacements.

A quantitative description of the environmental effects on a macroscopic quantum system was introduced by Caldeira and Leggett [28, 30, 32]. According to their theoretical approach, the interaction with the environment can be modeled as an interaction with a gapless ensemble of harmonic oscillators. The strength of such an interaction can be characterized by the classical coefficient of viscosity η. The prediction of the theory is that if a system interacts with an environment (or, as is sometimes said, is subjected to "quantum dissipation," or it couples to a "bath of harmonic oscillators"), then its tunneling rate is suppressed by a factor $\exp[-A_{\mathrm{CL}}\eta(\Delta q)^2/\hbar]$ relative to the case in which the tunneling system is perfectly isolated from any environment, but tunnels through the same energy barrier. Here, A_{CL} is a numerical factor of order unity, η is the viscosity coefficient defined in the classically accessible region, and Δq is the size of the classically inaccessible region, that is, the tunnel barrier width. The theory is valid only if the distribution of the oscillators representing the environment is gapless; that is, the distribution of the oscillator frequencies reaches zero frequency. So the reservoir of oscillators must be infinite in size. This is the reason why quantum systems coupled to such dissipative reservoirs are able to undergo quantum phase transitions, such as the dissipative Schmid–Bulgadaev transition.

As will be discussed in detail later, for superconducting devices, the effective viscosity that sets the environmental suppression of the rate of QPS depends on the normal conductance of the system. The normal conductance is well defined only if the device is shunted with a macroscopic normal resistor, which, ideally, should not depend on temperature. A normal resistor contains gapless normal electrons which act as an ensemble of harmonic oscillators damping the QPS. In superconducting wires, the damping effect might occur because QPS, like Abrikosov vortices, have normal cores in which the superconducting gap goes to zero.

One of the biggest remaining puzzles is the origin of superconductor-insulator transitions (SIT) in which a nanowire loses its ability to carry any measurable constant supercurrent. A qualitative difference between the superconducting state and a nonsuperconducting state exists only at $T = 0$. At higher temperatures,

a nanowire is always resistive because of TAPS. Proving that an SIT does occur as some parameter of the wire is changed is difficult because of the obvious fact that zero temperature is inaccessible experimentally. Thus, conclusions about the occurrence of a quantum transition are usually achieved indirectly. For example, resistance versus temperature, $R(T)$, curves could be extrapolated to zero temperature. To argue that an SIT does exist, it is necessary to show that the sample exhibits at least two qualitatively distinct types of behavior – a superconducting regime and an insulating regime. The transition between the two distinct regimes is usually induced by some control parameter, for example, the wire normal-state resistance R_n or its diameter d.

The SIT in thin wires has been analyzed theoretically by many groups. Andrei Zaikin and collaborators were the first to suggest a model of the SIT in 1D by making a quantum analogue of the well-known Kosterlitz–Thouless transition [168].

If the ensemble of samples studied is such that all samples are qualitatively similar and differ only quantitatively, then the system is said to undergo a crossover, but not a quantum phase transition. For example, suppose a series of experiments on a group of nanowires shows that for all samples as $T \to 0$, then $R(T) \to R_0$, and R_0 is some sample-specific constant, such that $0 < R_0 < \infty$. All samples would then saturate at a constant resistance with cooling. Such results would indicate that there is no SIT in the studied type of samples. On the other hand, a crossover might still be present, if, for example, the experiments show that R_0 gradually changes from $R_0 \ll R_n$ to $R_0 \gg R_n$, as the wire diameter is gradually reduced. The Giordano models of QPS predicts such crossover behaviors [108]. It predicts that any wire has a QPS rate above zero and therefore its resistance is greater than zero at zero temperature, although, within this model, R_0 depends exponentially on the wire diameter. Other quantum models predict that a superconductor-insulator phase transition should occur in thin superconducting wires [168, 194]. Experimental evidence in favor of SIT was published by Bollinger et al. [130].

In many theories of SIT, a quantum tunneling of Little's phase slips is the key factor in determining whether the wire is superconducting or insulating. The basic idea is as follows. If the QPS is suppressed completely (at zero temperature), then the wire stays phase coherent indefinitely and the supercurrent does not decay; thus, the wire is classified as superconducting. On the other hand, if the dissipation and other factors are not sufficiently strong to suppress QPS, the QPS occur and cause the supercurrent to decay, thus making the nanowire resistive (either normal or insulating). Although, in many cases, the experimentally observed transition in thin wires is called SIT, a better name might be SRT, that is, superconductor-resistor transition. The reason that a short wire can be superconducting is related to the net rate of QPS being zero at $T = 0$. The wire can also act as a resistor if the QPS rate is greater than zero. But, it is difficult to prove and/or expect that for a short wire the resistance is infinite. So, the insulating state is usually defined merely by the fact that the resistance increases with cooling. Such behavior, although it resembles insulators in some sense, might better be called a resistive state, not an insulating state. Therefore, in each concrete case of an SIT observation, it is important to explain the meaning of the I-state and the S-state. On the other hand, as was

stated above, the meaning of the S-state is always the same within this book – it is a state of zero resistance at $T = 0$.

A qualitatively different approach to SIT is the idea that certain factors, such as enhanced electron–electron repulsion, or unpaired spins, or dangling bonds on the surface of the wire can become more and more influential as the diameter is reduced. As a result, these factors can suppress T_c of the wire to zero, thus leading to an SIT for long wires (in which the normal state is localized and thus insulating) or an SRT for short wires. In latter chapters, we will consider the existing evidence for quantum transitions in thin wires and some of the theoretical models.