Theory of Atmospheric Radiative Transfer: 
A Comprehensive Introduction

Figures from the book by

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1. Introduction

Wendisch & Yang Theory of Atmospheric Radiative Transfer: A Comprehensive Introduction
February 20, 2012 2/108
1. Introduction
1. Introduction

Figure 1.4

Wendisch & Yang Theory of Atmospheric Radiative Transfer: A Comprehensive Introduction
February 20, 2012 5/108
1. Introduction

Figure 1.5

1. Introduction

Figure 1.6

Figure 2.2
2. Notation and Math Refresher

Figure 2.3
Figure 2.4
3. Fundamentals

Figure 3.1

(a) $E(y, 0)$

(b) $E(0, t)$
3. Fundamentals

Figure 3.2

(a) $$\hat{e}_1$$

(b) $$\hat{e}_1$$

(c) $$\hat{e}_1$$
Point Source

Figure 3.3
Figure 3.4
Figure 3.5
3. Fundamentals

Figure 3.6

(a) $\hat{e}_3 = \hat{n}$

(b) $\hat{e}_3 = \hat{n}$

$\cos \theta > 0$

$\cos \theta < 0$
3. Fundamentals

Figure 3.7

Figure 3.8

3. Fundamentals
Figure 4.1

Figure 4.5

Figure 4.6

Extinction Efficiency $Q_{ext,max}$ vs. Size Parameter $\alpha$

- $\tilde{n} = 1.33 + 0.1i$
- $\tilde{n} = 1.55 + 0.1i$
- $\tilde{n} = 1.55 + 0.1i$

Figure 4.6
Figure 4.7


Single Scattering Albedo $\omega_a$

Size Parameter $\alpha$

$\tilde{n} = 1.55 + 0.001 \cdot i$

$\tilde{n} = 1.55 + 0.01 \cdot i$

$\tilde{n} = 1.55 + 0.1 \cdot i$
Figure 4.8
Figure 4.10
Figure 4.11

- Cloud Droplet
- Dust Particle
- Air Molecule

Scattering Angle θ (°)
Phase Function ϕ
Figure 4.12

(a) Scattering Efficiency $Q_s$

(b) $\Delta Q_{ext}$ (%)

Rayleigh
Lorenz-Mie
Figure 4.13
4. Interactions of EM Radiation and Individual Particles

Figure 4.14

- (a) Volume "Equivalence"
- (b) Projected-Area "Equivalence"
- (c) Normalized Phase Function $F_\pi$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Aggregate</th>
<th>Volume</th>
<th>Projected-area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\infty}$ ($\mu m^3$)</td>
<td>0.2373</td>
<td>0.2058</td>
<td>0.5303</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.1123</td>
<td>0.4663</td>
<td>0.4663</td>
</tr>
<tr>
<td>$g$</td>
<td>0.6486</td>
<td>0.2654</td>
<td>0.7175</td>
</tr>
</tbody>
</table>

Scattering Angle $\Theta$ (°)

Scattering Angle $\Theta$ (°)
Figure 4.15

Figure 4.16

4. Interactions of EM Radiation and Individual Particles

Figure 4.17
4. Interactions of EM Radiation and Individual Particles

- **Extinction Cross Section** $C_{\text{ext}} (\mu m^2)$
  - $\lambda = 0.7 \mu m$

- **Asymmetry Parameter** $g$
  - $\lambda = 0.7 \mu m$

- **Single-Scattering Albedo** $\omega$
  - $\lambda = 1.49 \mu m$

Figure 4.18

---

[Graphs showing data for different particle dimensions and wave lengths.]
4. Interactions of EM Radiation and Individual Particles

Ray-Tracing Lorenz–Mie

(a) (b)

Figure 4.19

4. Interactions of EM Radiation and Individual Particles

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Ray-Tracing
Lorenz–Mie

Figure 4.20

(a) (b)

(c) (d)

Figure 4.21
4. Interactions of EM Radiation and Individual Particles
Figure 4.24

Transmissivity vs. Incident Angle $\theta_{inc}$

- Liquid Water $\parallel$
- Liquid Water $\perp$
- Copper $\parallel$
- Copper $\perp$

$\theta_{inc,0}$
4. Interactions of EM Radiation and Individual Particles

Figure 4.25

Reflectivity vs. Incident Angle ($\theta_{\text{inc}}$) for different materials:
- Liquid Water
- Liquid Metal
- Copper

Legend:
- Liquid Water II
- Liquid Water ⊥
- Copper II
- Copper ⊥
4. Interactions of EM Radiation and Individual Particles

Wendisch & Yang Theory of Atmospheric Radiative Transfer: A Comprehensive Introduction
February 20, 2012 46/108
Figure 4.28

Scattering Direction $\mathbf{s}$

$\hat{n}$

Non-Illuminated

Illuminated
Figure 4.29
Figure 4.31: T-Matrix Calculations for Different Particles
4. Interactions of EM Radiation and Individual Particles

Figure 4.32
4. Interactions of EM Radiation and Individual Particles

Figure 4.33
Figure 4.34
Figure 4.35

(a) (b)
Figure 4.37

Wendisch & Yang Theory of Atmospheric Radiative Transfer: A Comprehensive Introduction
February 20, 2012 56/108
5. Volumetric (Bulk) Optical Properties

- Volume Extinction Coefficient \( b_{\text{ext}} \) (km\(^{-1}\))
- Asymmetry Parameter \( g \)
- Single Scattering Albedo \( \omega_\sim \)

Figure 5.1

Figure 5.2
Figure 5.3

Scattering Angle $\theta$ (°)

Phase Function

- Dust
- Water

$10^2$, $10^4$, $10^6$, $10^{10}$
Figure 5.4

Phase Function vs. Scattering Angle $\Theta$ ($^\circ$)

- Solid line: Spherical Dust
- Dashed line: Nonspherical Dust

Logarithmic scale on the y-axis:

- $10^{-3}$
- $10^{-2}$
- $10^{-1}$
- $10^{0}$
- $10^{1}$
- $10^{2}$
- $10^{3}$
- $10^{4}$
6. Radiative Transfer Equation
Figure 6.2

\[ I_a(s = 0) \]

\[ s = 0 \]

\[ ds \]

\[ b_{ext} \]

\[ I_a(s) \]

Figure 6.2
Figure 6.3

\[ ds = -dz/\mu_0 \]
\[ \mu_0 = \cos \theta_0 \]
Figure 6.4

Figure 6.5

\begin{equation}
\frac{S_{d\nu,\lambda,\tau\alpha}}{\tau = 0} \quad \exp(-\tau^* / \mu_0)
\end{equation}

\begin{align}
S_{d\nu,\lambda,\tau\alpha} & = \frac{S_{d\nu,\lambda,\tau\alpha}}{\tau = 0} \\
& = \exp(-\tau^* / \mu_0)
\end{align}
Figure 6.7
6. Radiative Transfer Equation

Figure 6.8

\[ T = 0 \]

\[ T = T^* \]

\[ T \]
Figure 6.9
6. Radiative Transfer Equation

Figure 6.10

Solar Downwelling Irradiance (W/m²)

Wavelength λ (µm)

2710 m
1492 m
360 m

Measurements
Calculations

Figure 6.10
Figure 6.11

\[ \begin{align*}
\tau &= 0 \\
\tau &= \tau^* \\
\end{align*} \]
6. Radiative Transfer Equation

\[ \tau = 0 \]

\[ \tau \]

\[ \tau = \tau^* \]

Figure 6.12
Figure 6.13

\[ \tau = 0 \]

\[ \tau = \tau^* \]

The figure illustrates the radiative transfer equation, showing the variation of optical depth (\(\tau\)) with atmospheric layers, indicating how radiation propagates through the atmosphere.
6. Radiative Transfer Equation

\[ \tau = 0 \]

\[ \tau \]

\[ \tau = \tau^* \]

Figure 6.14
Figure 7.1

(a) Phase Function $\phi$ for different scattering angles $\theta$.
(b) Expansion Coefficient $C$ versus the number of terms $n$.

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7. Numerical and Approximate Solution Techniques
Wendisch & Yang
February 20, 2012 75/108
Figure 7.2

Phase Function $\phi$

Scattering Angle $\theta$ (°)

Numerical and Approximate Solution Techniques

Volume 2: A Comprehensive Introduction to Atmospheric Radiative Transfer

March 20, 2012
Figure 7.4
\[
\Delta T_n = \frac{\tau}{2^n} \quad \Delta T_{n-1} = \frac{\tau}{2^{n-1}} \quad \Delta T_1 = \frac{\tau}{2} \quad \Delta T_0 = \frac{\tau}{2^0}
\]
Figure 7.8
Figure 7.9
Figure 7.10
Figure 7.11
Figure 8.1

(a) Faster Vibration  (b) Slower Vibration

(c) Faster Rotation  (d) Slower Rotation

Carbon Atom  Oxygen Atom
Figure 8.2

Figure 8.3
8. Absorption and Emission by Atmospheric Gases

Figure 8.5

N₂, O₂

CO

CO₂

(a)

(b)

(c)
8. Absorption and Emission by Atmospheric Gases

Figure 8.6

(a) and (b) show the interaction of photons with molecules in the atmosphere, illustrating the processes of absorption and emission.
Figure 8.7

- Lyman Series
- Balmer Series
- Paschen Series
- Brackett Series
Figure 8.9
Figure 8.10
Figure 8.11
Figure 8.13
Figure 8.14

(a) Absorption Coefficient $k_{\text{abs}}$ (cm$^{-1}$)
(b) Probability Density Function $f$
(c) Correlation Probability Function $G$
(d) Cumulative Probability Function $F$

Figure 8.15

(a) Cumulative Probability Function $G$

(b) Wavenumber $\tilde{v}$ (cm$^{-1}$)

Absorption Coefficient ($\text{cm}^{-1} \text{g}^{-1}$)
9. Terrestrial Radiative Transfer

Figure 9.1

\[ z = \infty \]

\[ z' \]

\[ z \]

\[ z = 0 \]

\[ B_{\lambda}[T'(z')] \]

\[ \mathcal{T}(\lambda, z, z') \]

Detector

Figure 9.1
Figure 9.2

[Wavelength $\lambda$ (\textmu m)]

- Radiance [W m$^{-2}$ sr$^{-1}$ cm$^{-1}$]

- 300 K
- 280 K
- Tropical
- Subarctic Winter

Wavenumber $\tilde{\nu}$ (cm$^{-1}$)

Figure 9.2
Figure 9.3
Figure 9.5
Figure 9.6
Figure 9.7

(a) Aerosol + Rayleigh

(b) Aerosol only

Figure 9.8

(a) Solar Heating Rate (K day⁻¹)

(b) Terrestrial Cooling Rate (K day⁻¹)

07–09

--- θ = 1.0

--- θ = 0.9

--- θ = 0.8

Height (km)