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Theory of Atmospheric Radiative Transfer: A Comprehensive Introduction

Figures from the book by

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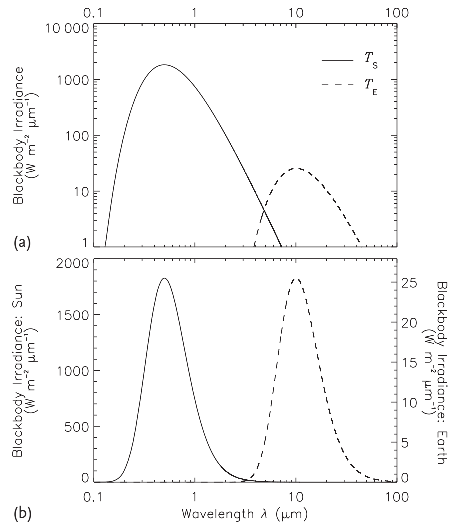
0.1
1
10
100
10 000
1000
100
10
1
Blackbody Irradiance (W m^{-2} µm^{-1})
T_S
T_E
(a)
2000
1500
1000
500
0
Blackbody Irradiance: Sun (W m^{-2} µm^{-1})
25
20
15
10
5
0
Blackbody Irradiance: Earth (W m^{-2} µm^{-1})
(b)
Wavelength λ (µm)

Figure 1.1

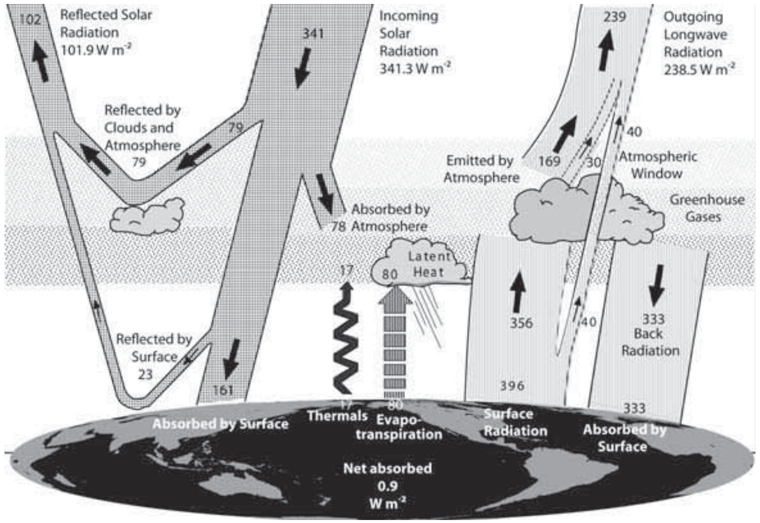
102
Reflected Solar Radiation 101.9 W m-2
341
Incoming Solar Radiation 341.3 W m-2
239
Outgoing Longwave Radiation 238.5 W m-2
Reflected by Clouds and Atmosphere 79
79
40
Atmospheric Window
Emitted by Atmosphere
169
30
Greenhouse Gases
Absorbed by
78 Atmosphere
Latent Heat
17
80
356
40
333 Back Radiation
Reflected by Surface 23
161
396
333
Absorbed by Surface
Thermals
Evapo-transpiration
Surface Radiation
Absorbed by Surface
Net absorbed 0.9 W m-2

Figure 1.2

1. Introduction

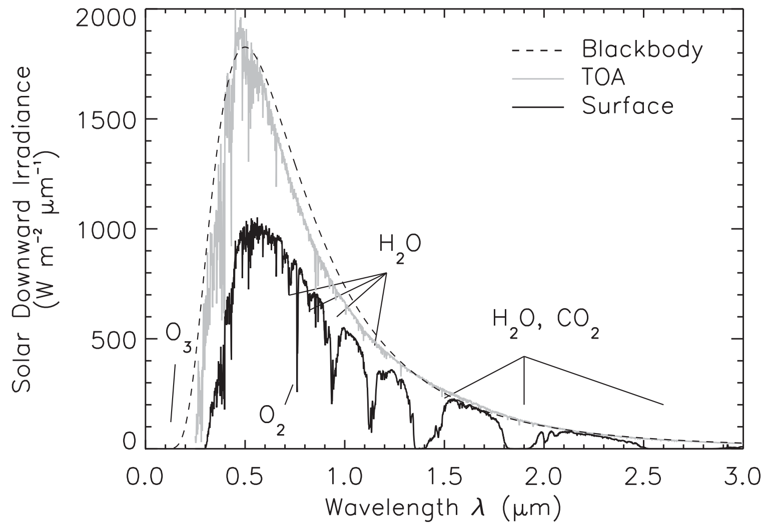

Figure 1.3

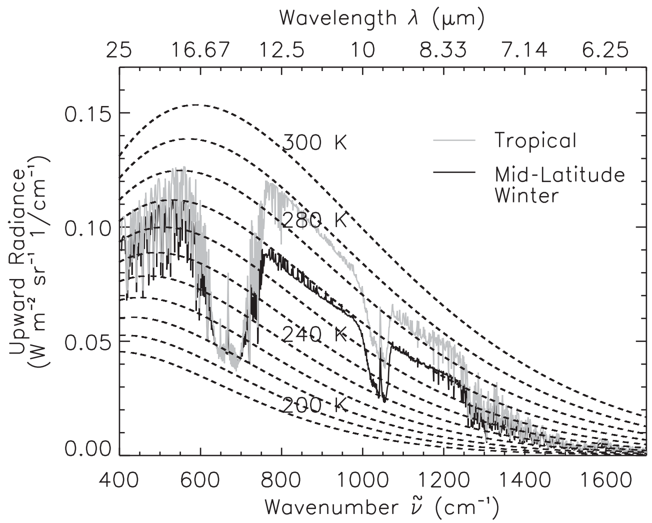
Wavelength λ (μm)
25
16.67
12.5
10
8.33
7.14
6.25
0.15
0.10
0.05
0.00
Upward Radiance
(W m⁻² sr⁻¹ 1/cm⁻¹)
300 K
280 K
240 K
200 K
Tropical
Mid-Latitude Winter
400
600
800
1000
1200
1400
1600
Wavenumber ν̃ (cm⁻¹)

Figure 1.4

1. Introduction

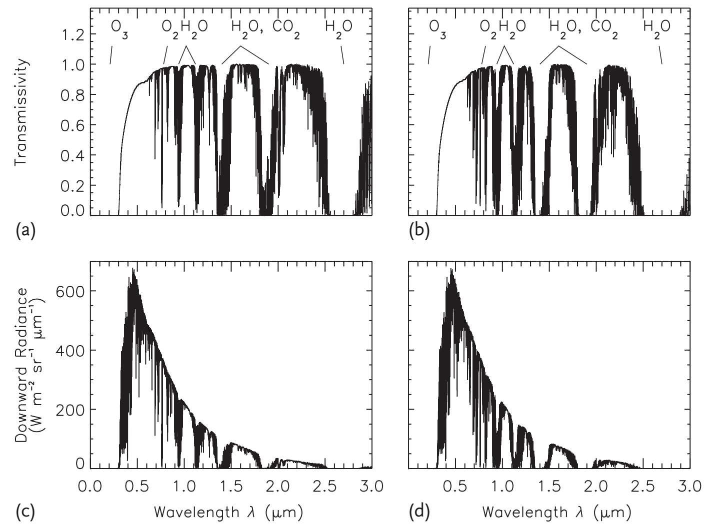

Figure 1.5

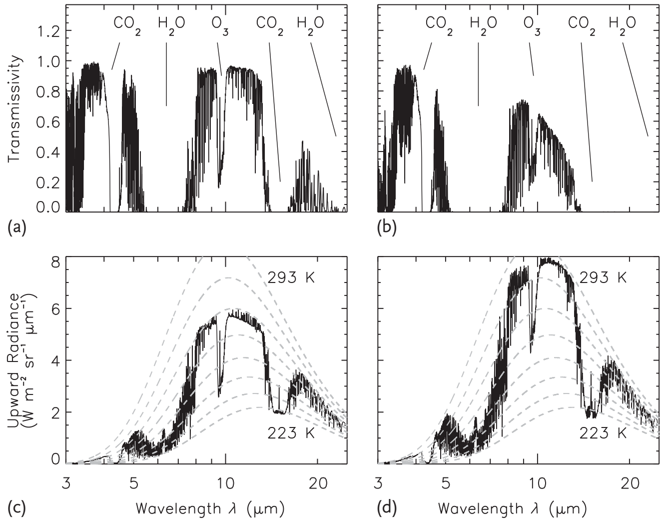
CO2
H2O
O3
CO2
H2O
Transmissivity
1.2
1.0
0.8
0.6
0.4
0.2
0.0
(a)
(b)
8
6
4
2
0
Upward Radiance (W m-2 sr-1 μm-1)
293 K
223 K
3
5
10
20
(c)
(d)
Wavelength λ (μm)

Figure 1.6

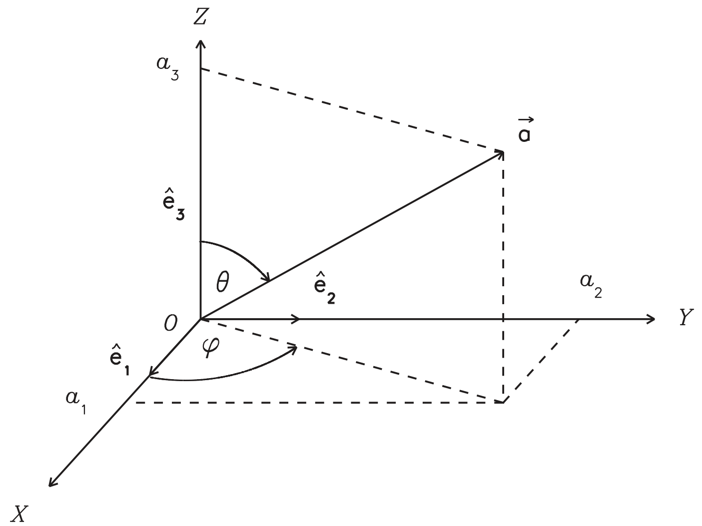

Figure 2.1

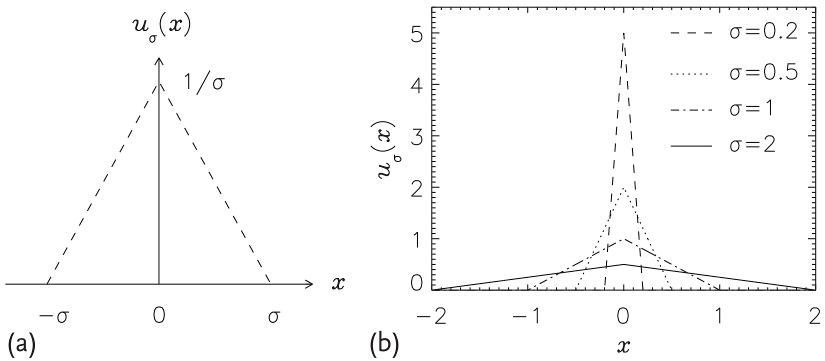
$u_\sigma(x)$
$1/\sigma$
x
$-\sigma$
0
σ
(a)
5
4
3
2
1
0
$u_\sigma(x)$
σ=0.2
σ=0.5
σ=1
σ=2
−2
−1
0
1
2
x
(b)

Figure 2.2

2. Notation and Math Refresher

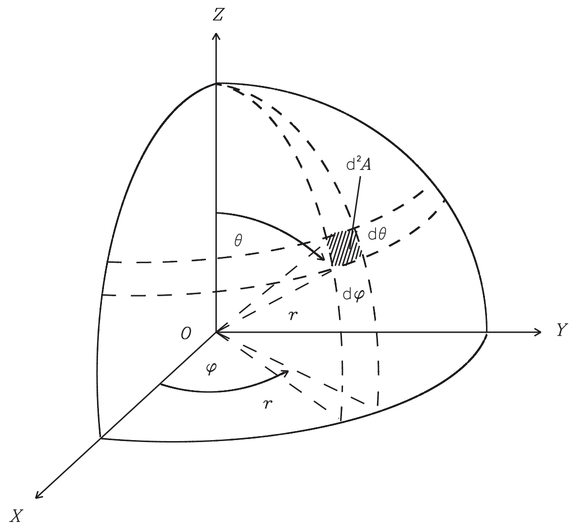

Figure 2.3

2. Notation and Math Refresher

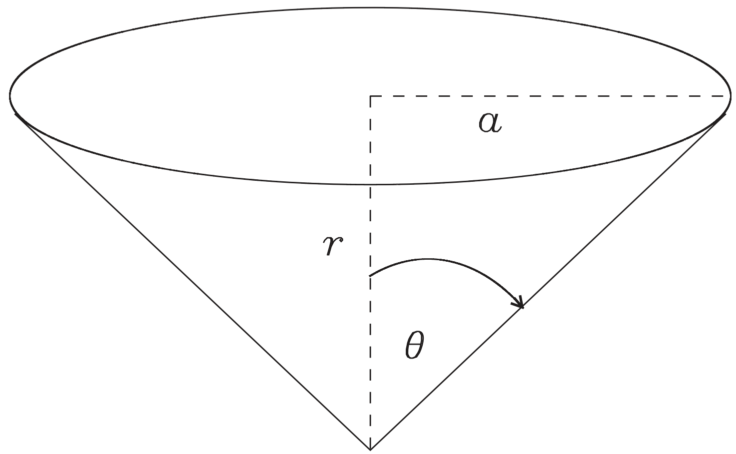

Figure 2.4

3. Fundamentals

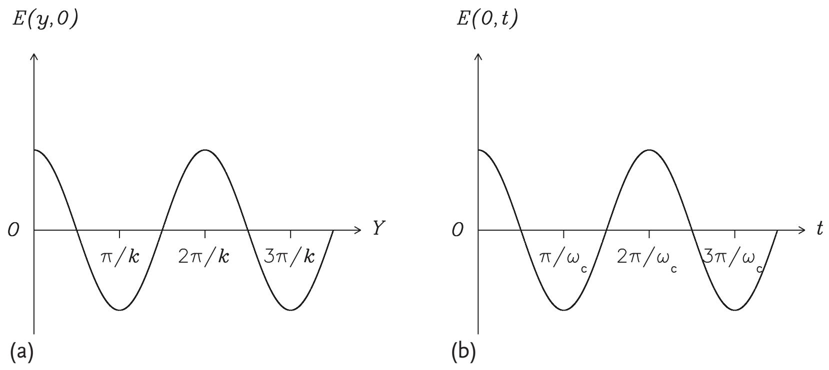

Figure 3.1

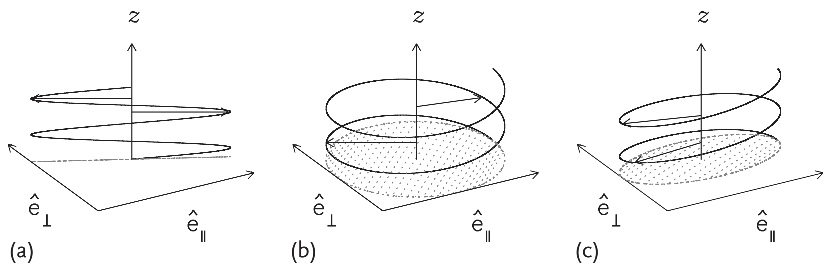
z
z
z
$\hat{e}_\perp$
$\hat{e}_\parallel$
(a)
$\hat{e}_\perp$
$\hat{e}_\parallel$
(b)
$\hat{e}_\perp$
$\hat{e}_\parallel$
(c)

Figure 3.2

3. Fundamentals

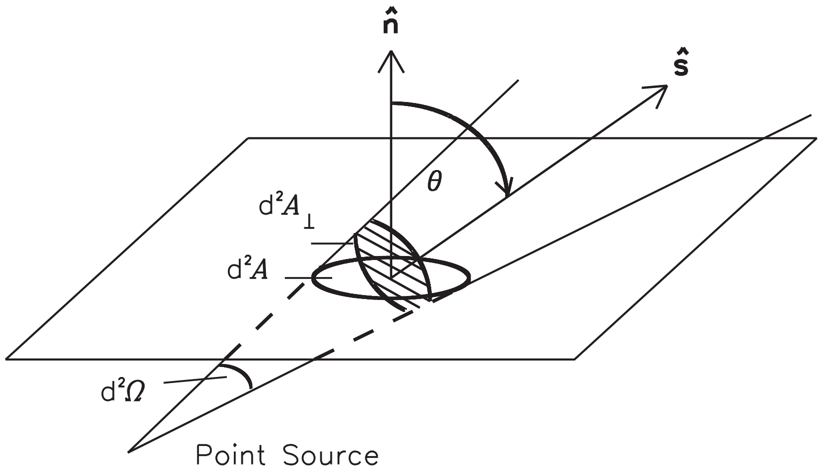

Figure 3.3

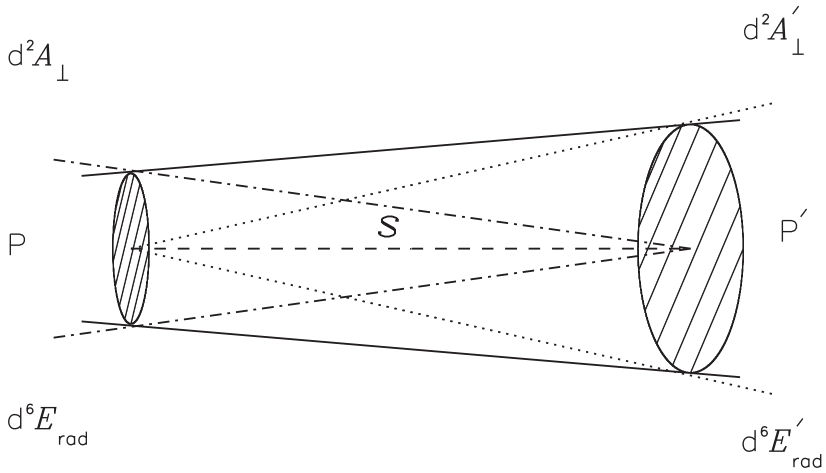

Figure 3.4

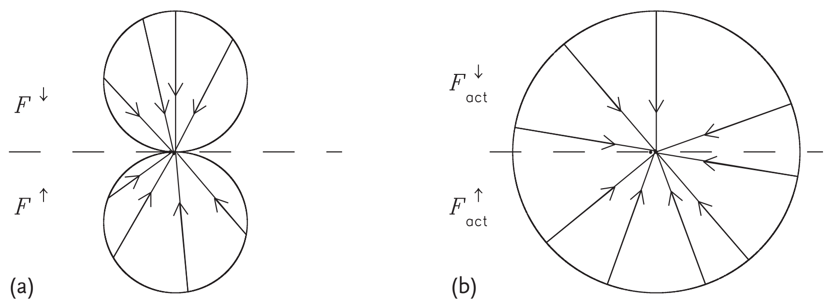

Figure 3.5

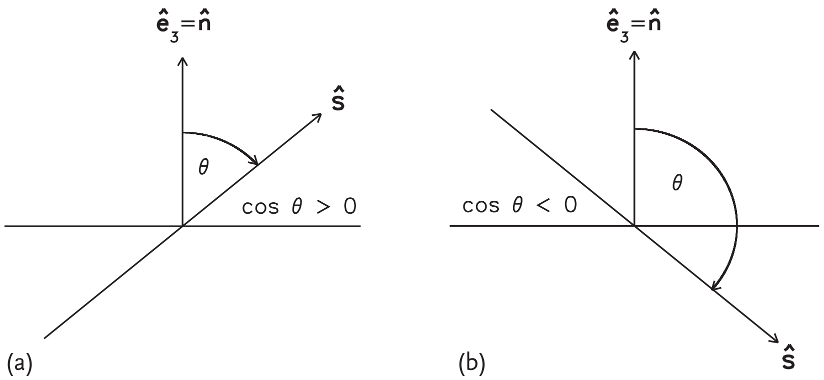

Figure 3.6

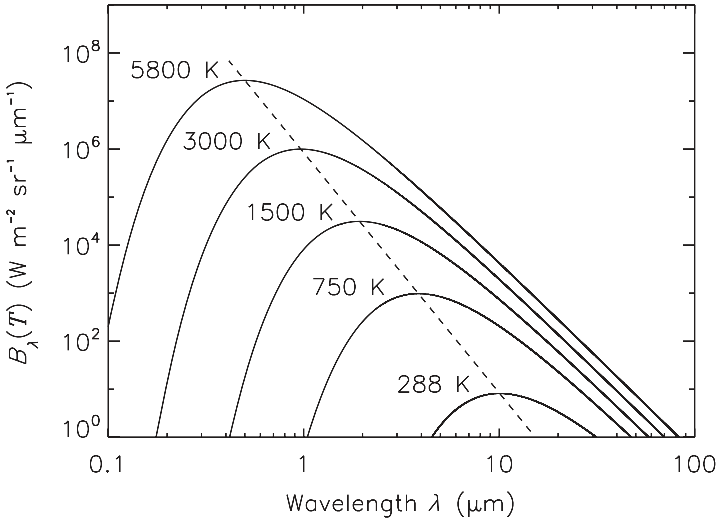
5800 K
3000 K
1500 K
750 K
288 K
10^8
10^6
10^4
10^2
10^0
0.1
1
10
100
$B_\lambda(T)$ (W m^{-2} sr^{-1} μm^{-1})
Wavelength λ (μm)

Figure 3.7

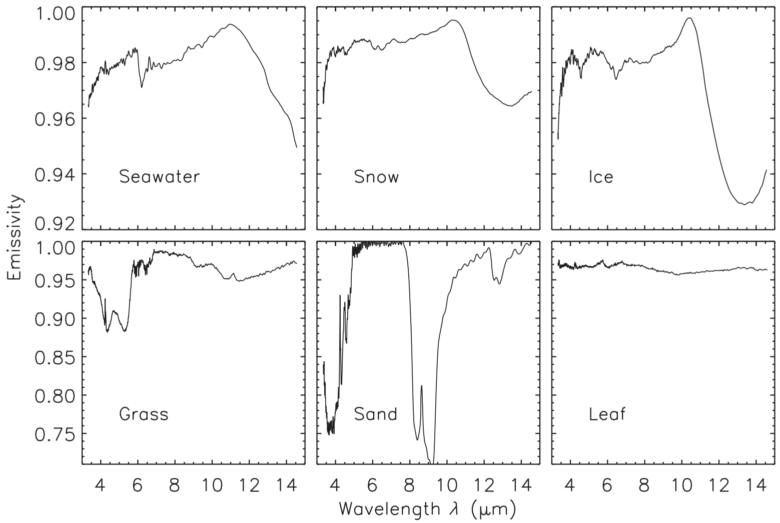
Emissivity
1.00
0.98
0.96
0.94
0.92
Seawater
Snow
Ice
1.00
0.95
0.90
0.85
0.80
0.75
Grass
Sand
Leaf
4 6 8 10 12 14
Wavelength λ (μm)

Figure 3.8

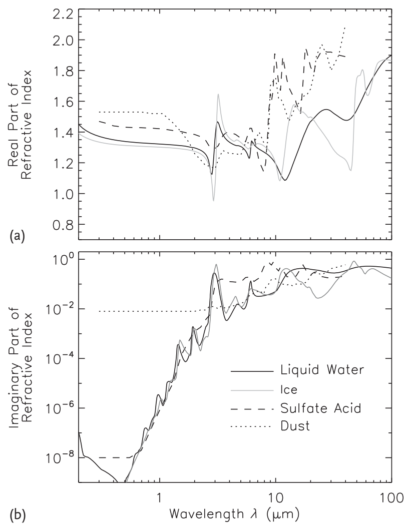
Real Part of Refractive Index
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
(a)
Imaginary Part of Refractive Index
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
Liquid Water
Ice
Sulfate Acid
Dust
1
10
100
Wavelength λ (μm)
(b)

Figure 4.1

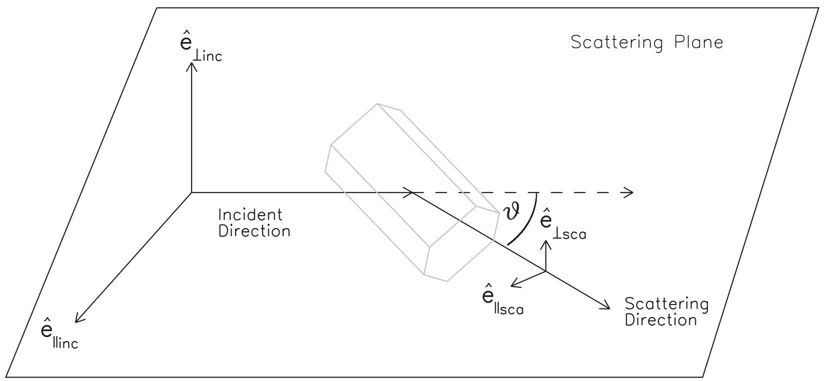
$\hat{e}_{\perp inc}$
Scattering Plane
Incident Direction
ϑ
$\hat{e}_{\perp sca}$
$\hat{e}_{\parallel sca}$
Scattering Direction
$\hat{e}_{\parallel inc}$

Figure 4.2

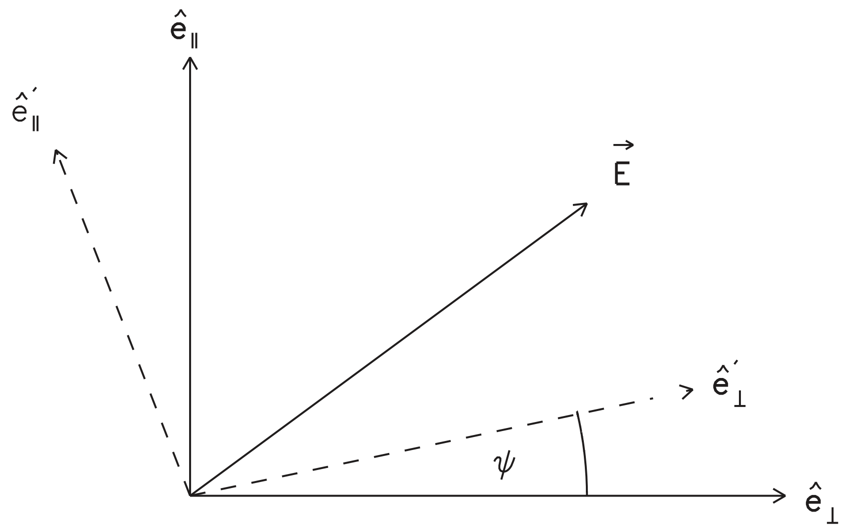

Figure 4.3

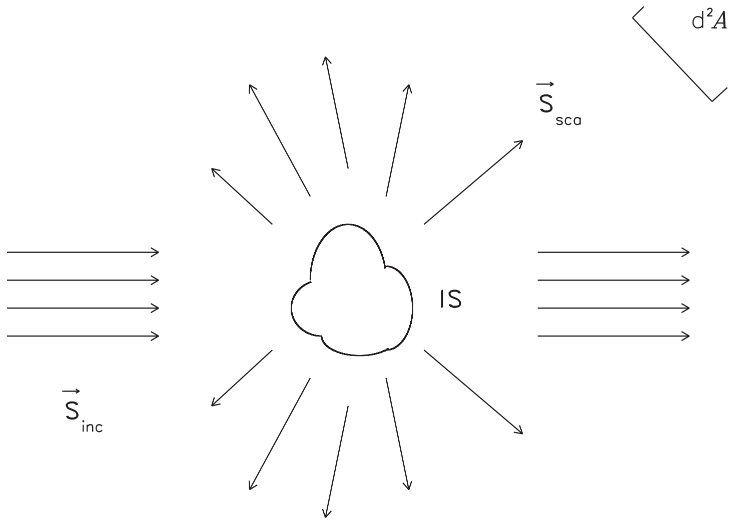

Figure 4.4

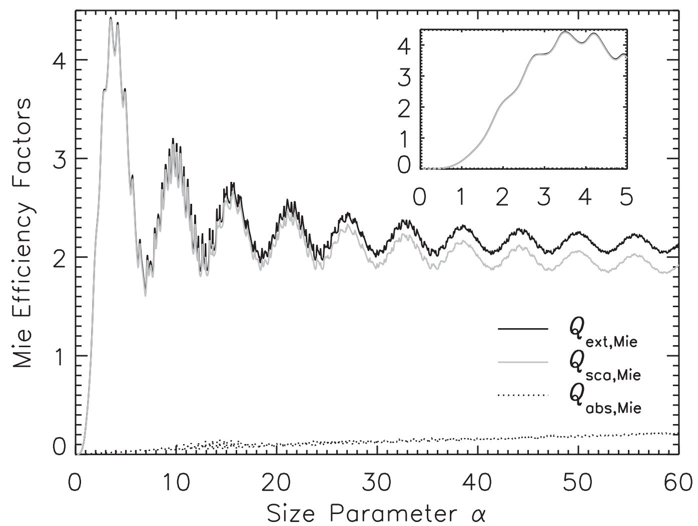
Mie Efficiency Factors
Size Parameter α
0
10
20
30
40
50
60
0
1
2
3
4
0 1 2 3 4 5
$Q_{ext,Mie}$
$Q_{sca,Mie}$
$Q_{abs,Mie}$

Figure 4.5

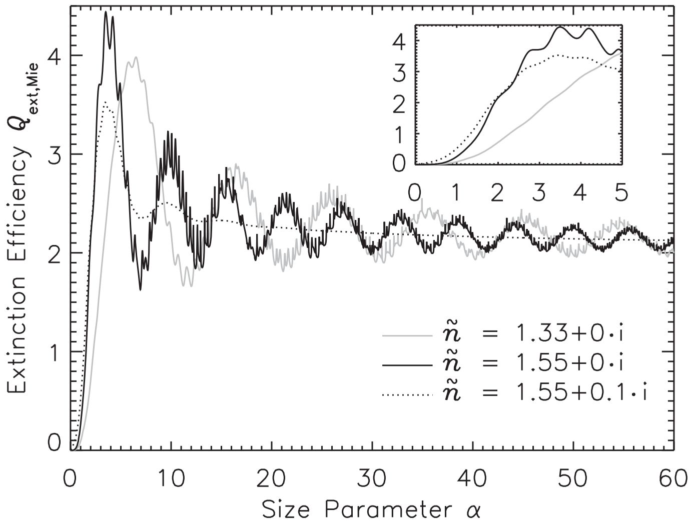
Extinction Efficiency $Q_{ext,Mie}$
Size Parameter α
$\tilde{n}$ = 1.33+0·i
$\tilde{n}$ = 1.55+0·i
$\tilde{n}$ = 1.55+0.1·i
0
1
2
3
4
0
10
20
30
40
50
60
5

Figure 4.6

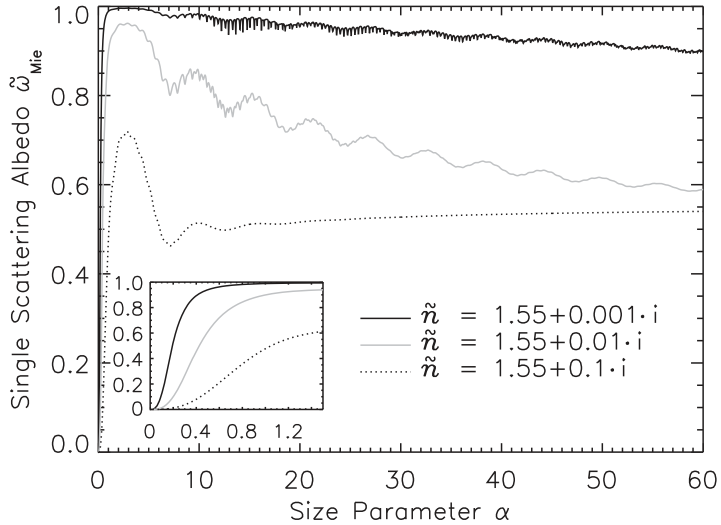
Single Scattering Albedo $\tilde{\omega}_{Mie}$
Size Parameter α
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
0.4
0.8
1.2
$\tilde{n}$ = 1.55+0.001·i
$\tilde{n}$ = 1.55+0.01·i
$\tilde{n}$ = 1.55+0.1·i

Figure 4.7

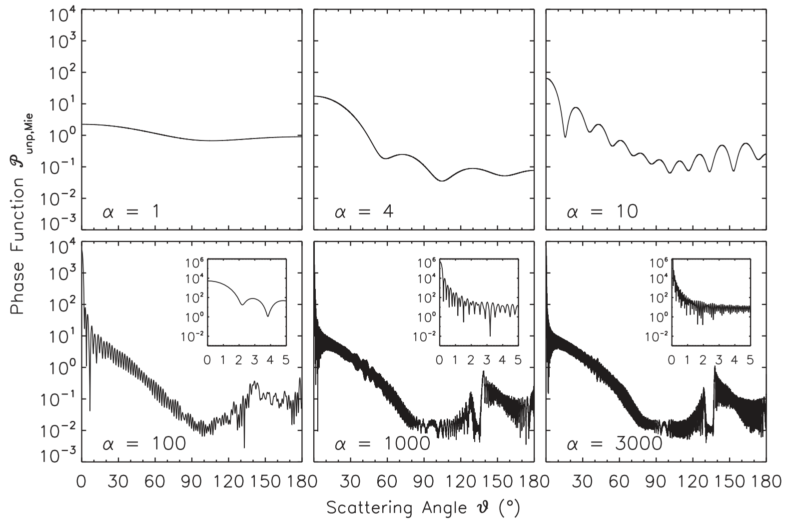
Phase Function $\mathcal{P}_{\mathrm{unp,Mie}}$
α = 1
α = 4
α = 10
α = 100
α = 1000
α = 3000
Scattering Angle ϑ (°)

Figure 4.8

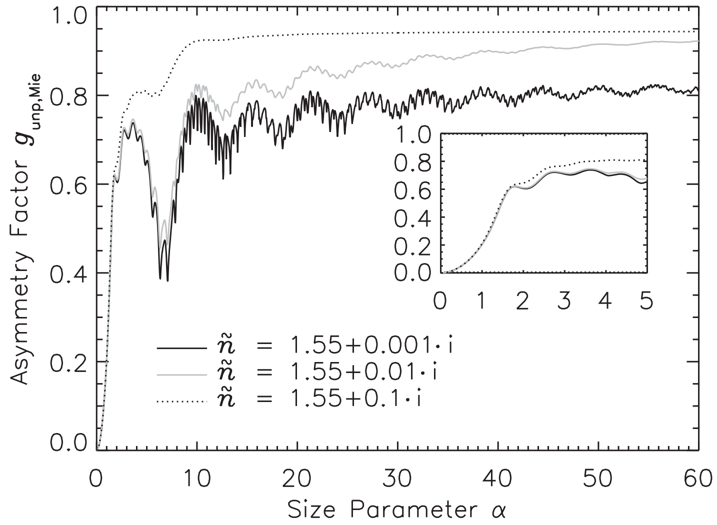
Asymmetry Factor $g_{unp,Mie}$
Size Parameter α
$\tilde{n}$ = 1.55+0.001·i
$\tilde{n}$ = 1.55+0.01·i
$\tilde{n}$ = 1.55+0.1·i
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
0 1 2 3 4 5

Figure 4.9

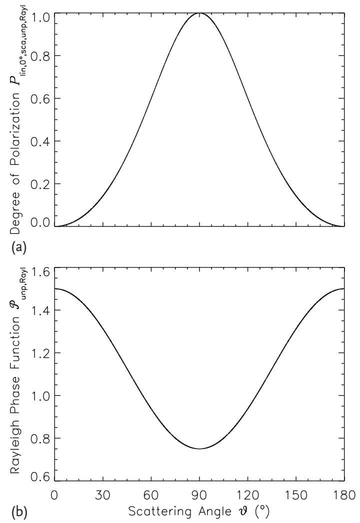
Degree of Polarization $P_{lin,0°,sca,unp,Rayl}$
1.0
0.8
0.6
0.4
0.2
0.0
(a)
Rayleigh Phase Function $\mathcal{P}_{unp,Rayl}$
1.6
1.4
1.2
1.0
0.8
0.6
0
30
60
90
120
150
180
Scattering Angle ϑ (°)
(b)

Figure 4.10

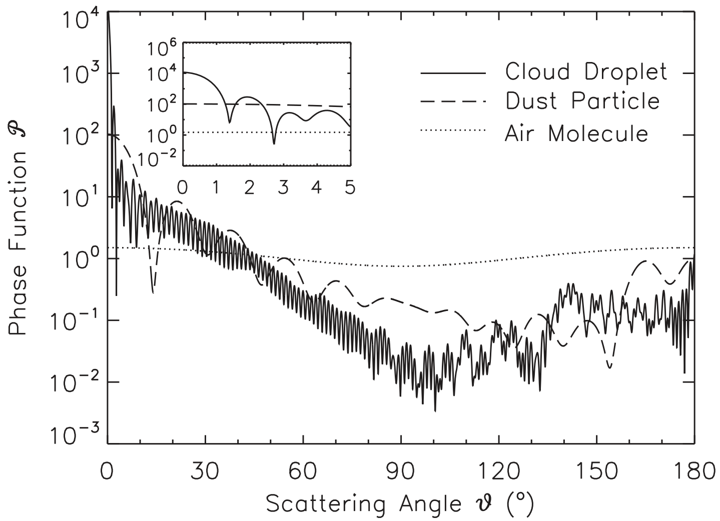
Phase Function $\mathcal{P}$
Scattering Angle ϑ (°)
Cloud Droplet
Dust Particle
Air Molecule
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
30
60
90
120
150
180
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
0 1 2 3 4 5

Figure 4.11

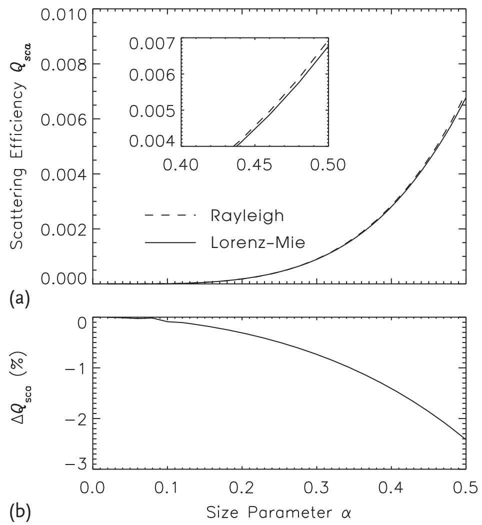
0.010
0.008
0.006
0.004
0.002
0.000
Scattering Efficiency Q_{sca}
0.007
0.006
0.005
0.004
0.40
0.45
0.50
Rayleigh
Lorenz-Mie
(a)
0
−1
−2
−3
ΔQ_{sca} (%)
0.0
0.1
0.2
0.3
0.4
0.5
Size Parameter α
(b)

Figure 4.12

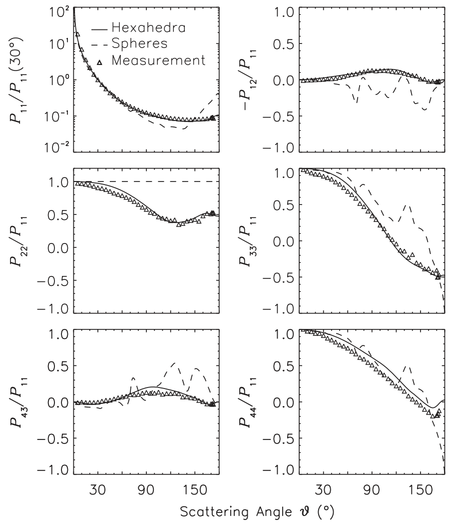
Hexahedra
Spheres
Measurement
$P_{11}/P_{11}(30°)$
$-P_{12}/P_{11}$
P_{22}/P_{11}
P_{33}/P_{11}
P_{43}/P_{11}
P_{44}/P_{11}
Scattering Angle ϑ (°)

Figure 4.13

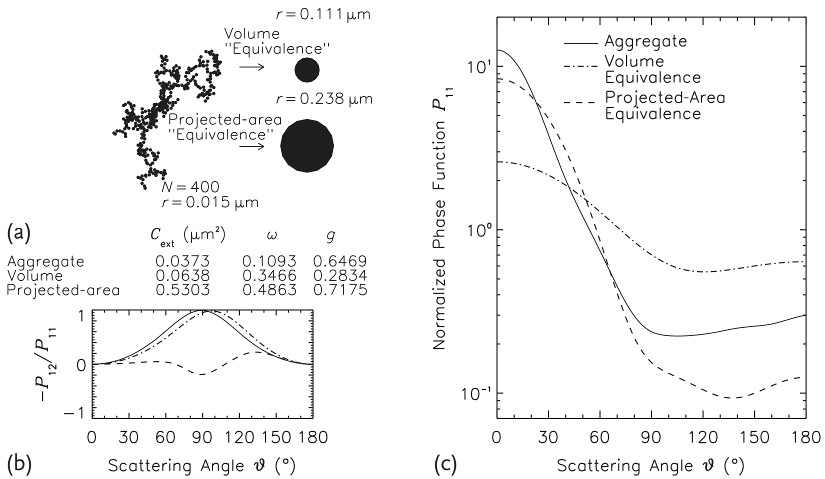
r = 0.111 μm
Volume "Equivalence"
r = 0.238 μm
Projected-area "Equivalence"
N = 400
r = 0.015 μm
(a)
C_ext (μm²) ω g
Aggregate 0.0373 0.1093 0.6469
Volume 0.0638 0.3466 0.2834
Projected-area 0.5303 0.4863 0.7175
−P_12/P_11
(b)
Scattering Angle ϑ (°)
Normalized Phase Function P_11
Aggregate
Volume Equivalence
Projected-Area Equivalence
(c)
Scattering Angle ϑ (°)

Figure 4.14

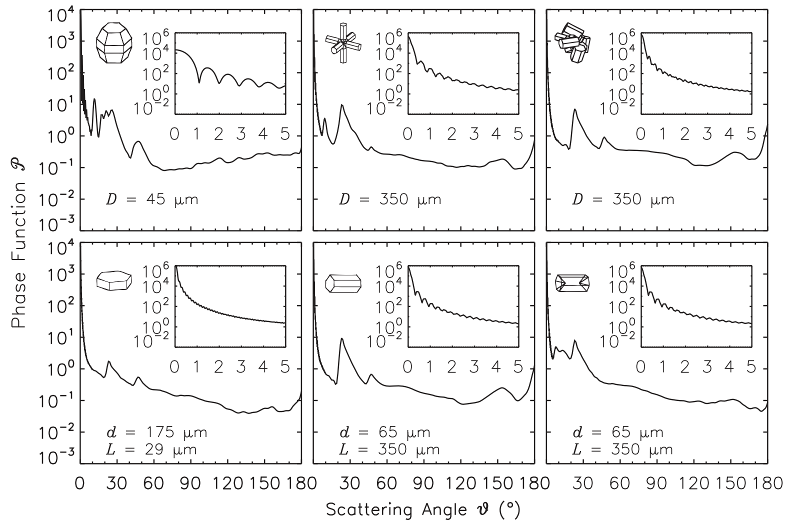
Phase Function $\mathscr{P}$
Scattering Angle ϑ (°)
D = 45 μm
D = 350 μm
D = 350 μm
d = 175 μm
L = 29 μm
d = 65 μm
L = 350 μm
d = 65 μm
L = 350 μm

Figure 4.15

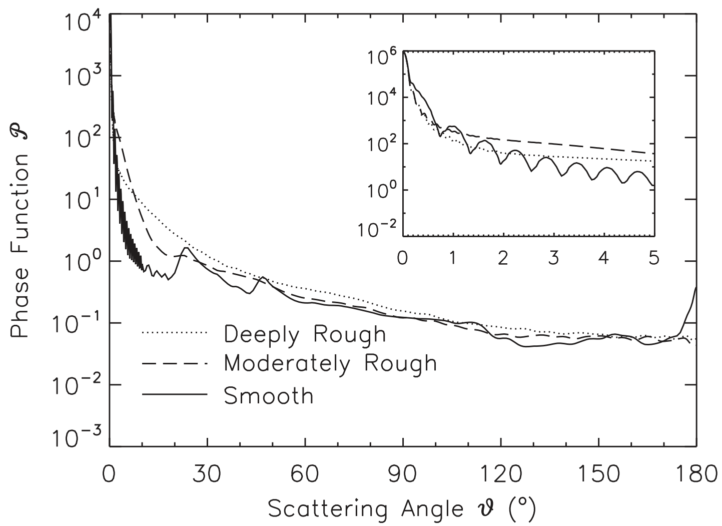
Phase Function 𝒫
Scattering Angle ϑ (°)
Deeply Rough
Moderately Rough
Smooth
0
30
60
90
120
150
180
0
1
2
3
4
5

Figure 4.16

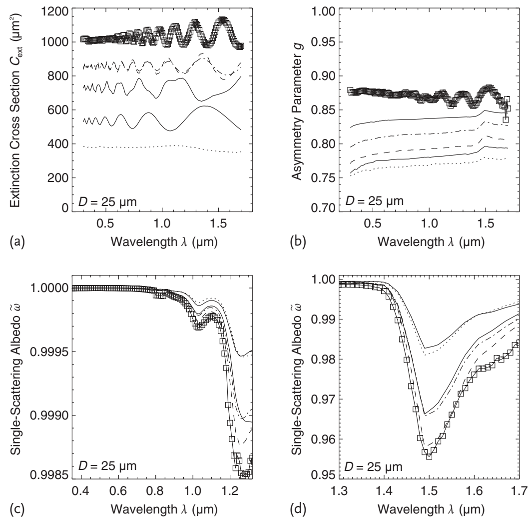
Extinction Cross Section C_{ext} (μm²)
1200
1000
800
600
400
200
0
D = 25 μm
0.5
1.0
1.5
Wavelength λ (μm)
(a)
Asymmetry Parameter g
1.00
0.95
0.90
0.85
0.80
0.75
0.70
D = 25 μm
0.5
1.0
1.5
Wavelength λ (μm)
(b)
Single-Scattering Albedo $\tilde{\omega}$
1.0000
0.9995
0.9990
0.9985
D = 25 μm
0.4
0.6
0.8
1.0
1.2
Wavelength λ (μm)
(c)
Single-Scattering Albedo $\tilde{\omega}$
1.00
0.99
0.98
0.97
0.96
0.95
D = 25 μm
1.3
1.4
1.5
1.6
1.7
Wavelength λ (μm)
(d)

Figure 4.17

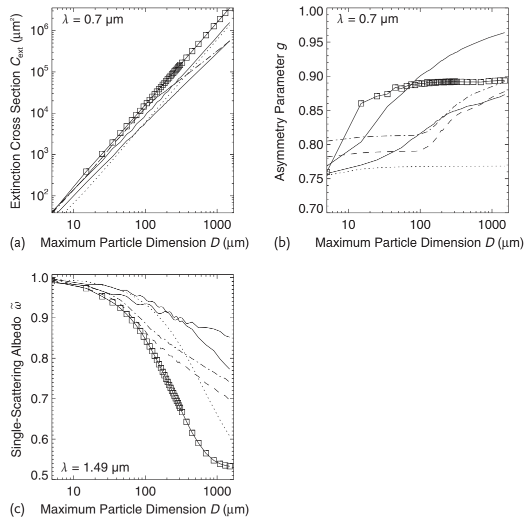
λ = 0.7 μm
Extinction Cross Section C_{ext} (μm²)
(a) Maximum Particle Dimension D (μm)
λ = 0.7 μm
Asymmetry Parameter g
(b) Maximum Particle Dimension D (μm)
λ = 1.49 μm
Single-Scattering Albedo $\tilde{\omega}$
(c) Maximum Particle Dimension D (μm)

Figure 4.18

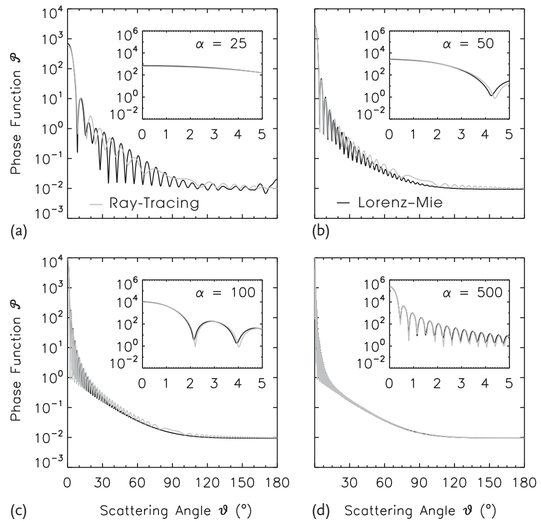
Phase Function $\mathcal{P}$
α = 25
α = 50
α = 100
α = 500
Ray-Tracing
Lorenz-Mie
(a)
(b)
(c)
(d)
Scattering Angle ϑ (°)

Figure 4.19

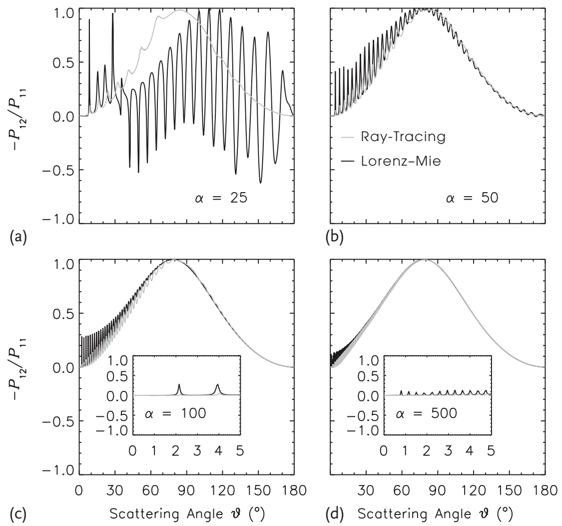
1.0
0.5
0.0
−0.5
−1.0
$-P_{12}/P_{11}$
α = 25
(a)
Ray-Tracing
Lorenz-Mie
α = 50
(b)
α = 100
α = 500
0 30 60 90 120 150 180
Scattering Angle ϑ (°)
(c)
(d)

Figure 4.20

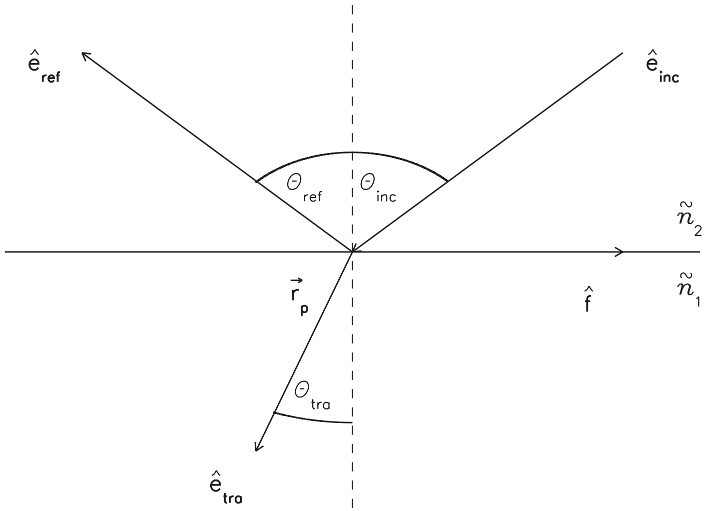

Figure 4.21

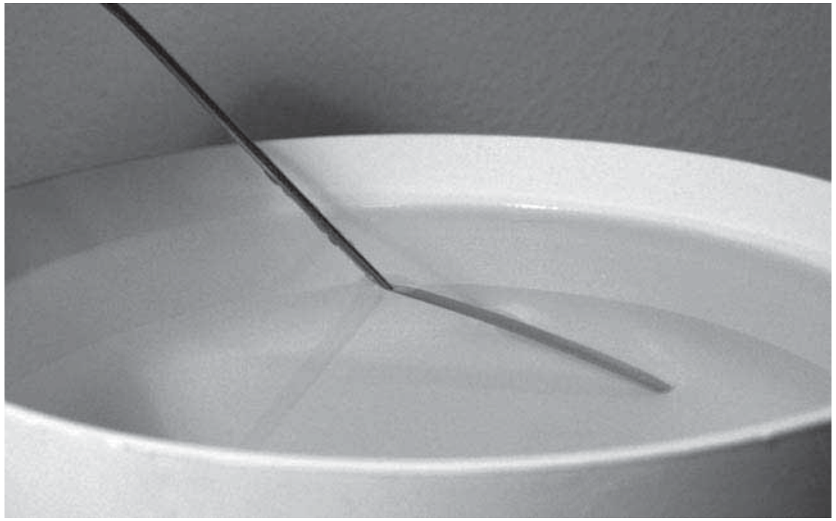

Figure 4.22

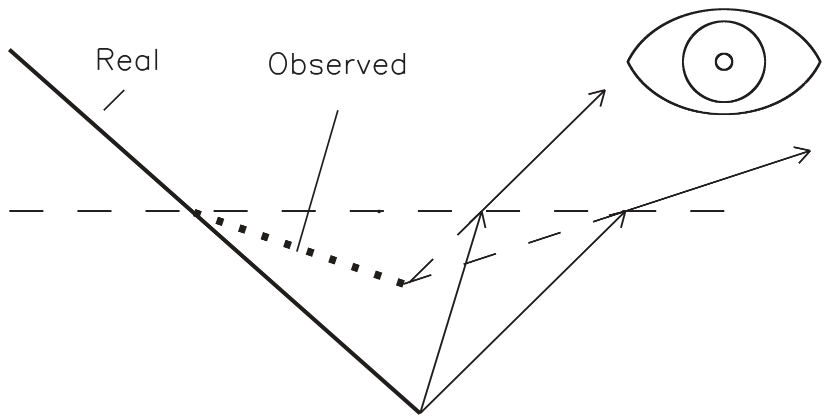

Figure 4.23

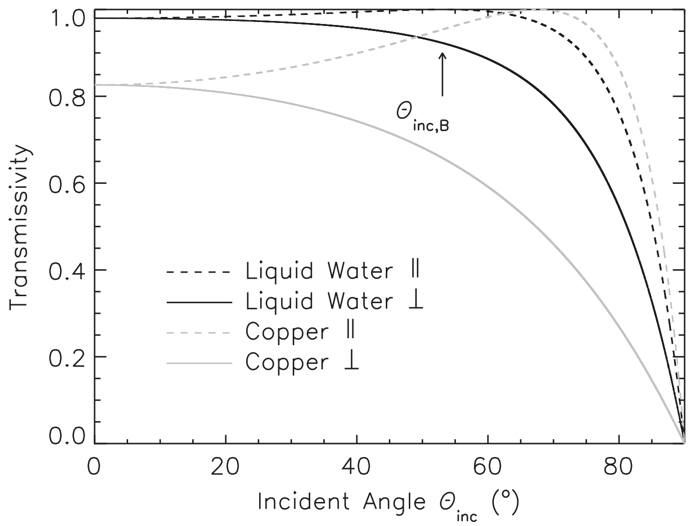
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
Transmissivity
Incident Angle Θ_{inc} (°)
$\Theta_{inc,B}$
Liquid Water ∥
Liquid Water ⊥
Copper ∥
Copper ⊥

Figure 4.24

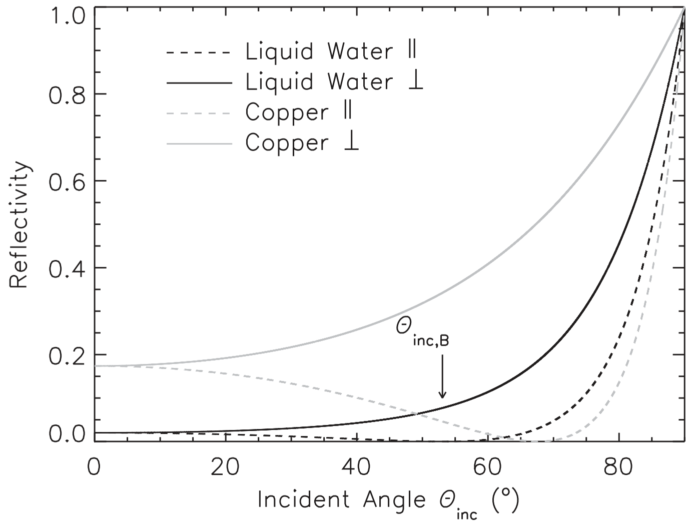
Liquid Water ∥
Liquid Water ⊥
Copper ∥
Copper ⊥
$\Theta_{inc,B}$
Reflectivity
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
Incident Angle Θ_{inc} (°)

Figure 4.25

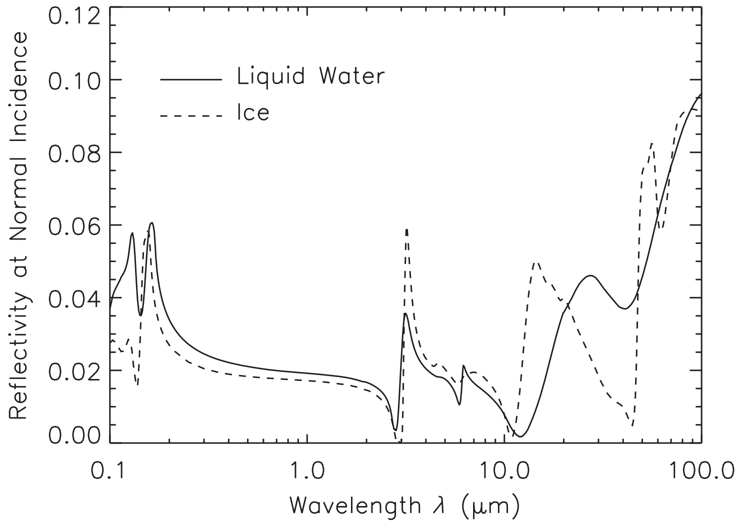
Reflectivity at Normal Incidence
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Liquid Water
Ice
0.1
1.0
10.0
100.0
Wavelength λ (μm)

Figure 4.26

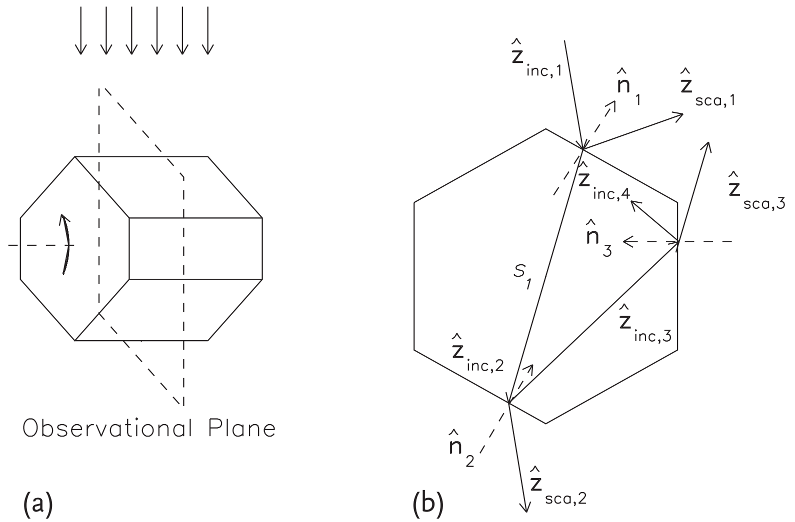
Observational Plane
(a)
ẑ inc,1
n̂ 1
ẑ sca,1
ẑ inc,4
ẑ sca,3
n̂ 3
S 1
ẑ inc,3
ẑ inc,2
n̂ 2
ẑ sca,2
(b)

Figure 4.27

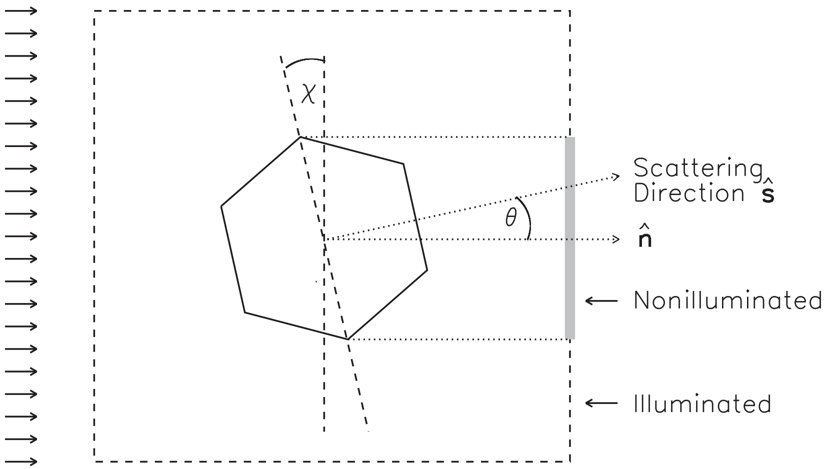

Figure 4.28

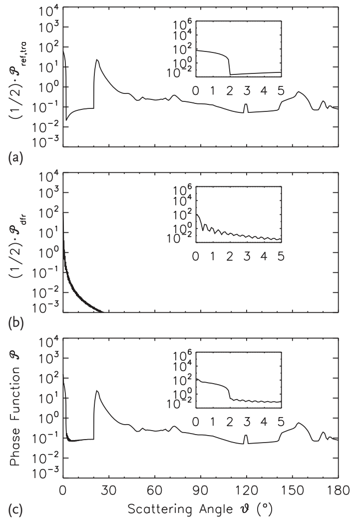

Figure 4.29

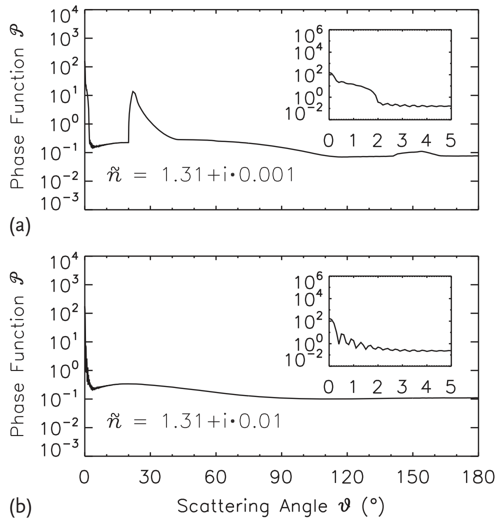
Phase Function $\mathcal{P}$
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$\tilde{n}$ = 1.31+i·0.001
(a)
$\tilde{n}$ = 1.31+i·0.01
0 30 60 90 120 150 180
Scattering Angle ϑ (°)
(b)

Figure 4.30

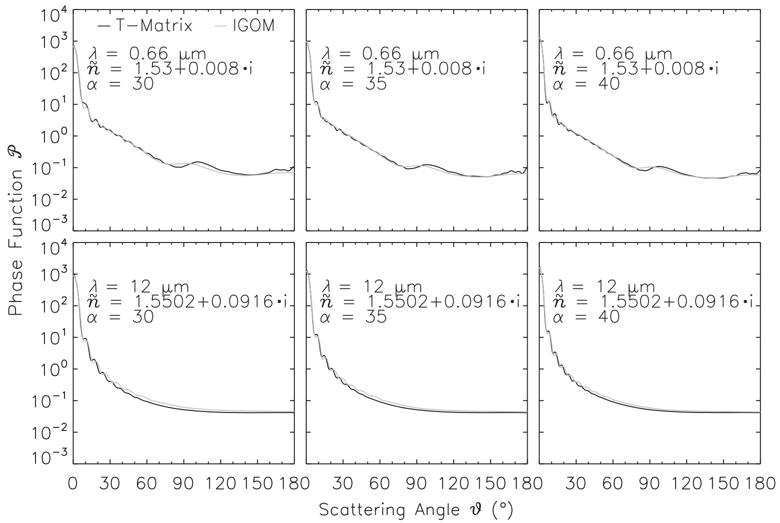
T−Matrix
IGOM
λ = 0.66 μm
ñ = 1.53+0.008·i
α = 30
α = 35
α = 40
λ = 12 μm
ñ = 1.5502+0.0916·i
Phase Function 𝒫
Scattering Angle ϑ (°)
0 30 60 90 120 150 180

Figure 4.31

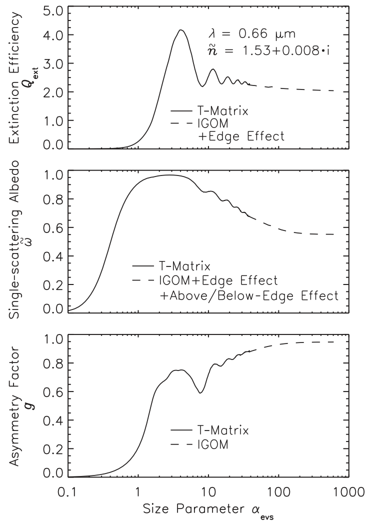
Extinction Efficiency Q_{ext}
5.0
4.0
3.0
2.0
1.0
0.0
λ = 0.66 μm
ñ = 1.53+0.008•i
T-Matrix
IGOM
+Edge Effect
Single-scattering Albedo $\tilde{\omega}$
1.0
0.8
0.6
0.4
0.2
0.0
T-Matrix
IGOM+Edge Effect
+Above/Below-Edge Effect
Asymmetry Factor g
1.0
0.8
0.6
0.4
0.2
0.0
T-Matrix
IGOM
0.1
1
10
100
1000
Size Parameter α_{evs}

Figure 4.32

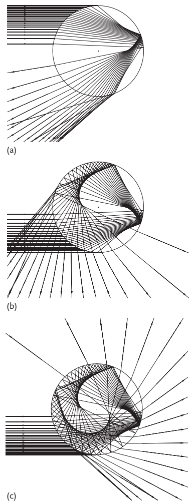

Figure 4.33

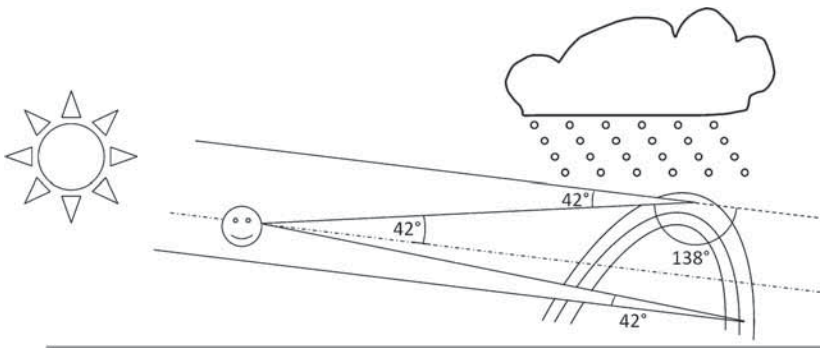

Figure 4.34

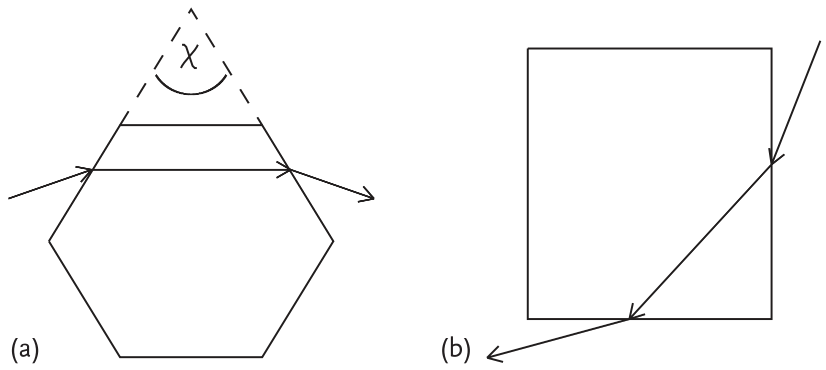

Figure 4.35

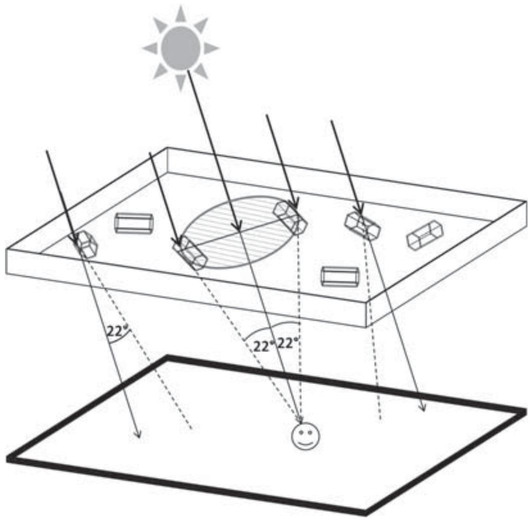

Figure 4.36

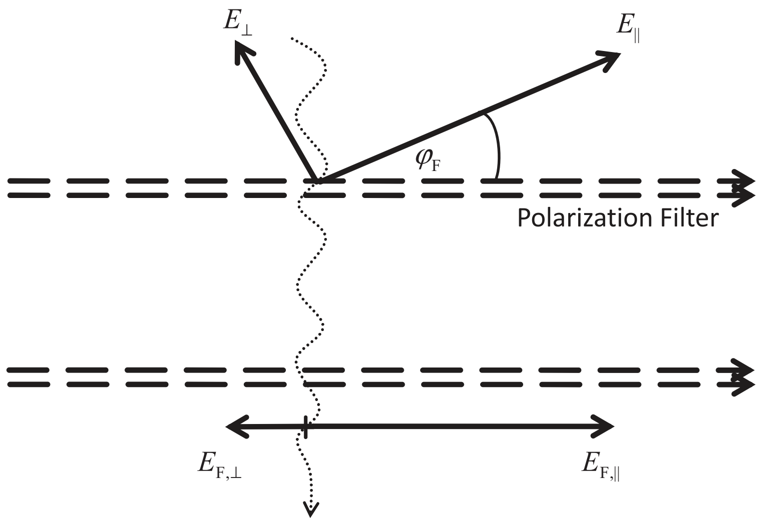

Figure 4.37

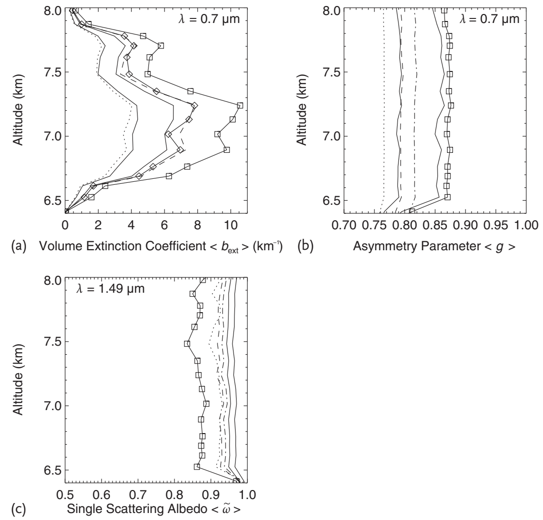
λ = 0.7 μm
8.0
7.5
7.0
6.5
Altitude (km)
0 2 4 6 8 10
(a) Volume Extinction Coefficient < b_{ext} > (km⁻¹)
λ = 0.7 μm
0.70 0.75 0.80 0.85 0.90 0.95 1.00
(b) Asymmetry Parameter < g >
λ = 1.49 μm
0.5 0.6 0.7 0.8 0.9 1.0
(c) Single Scattering Albedo < ω̃ >

Figure 5.1

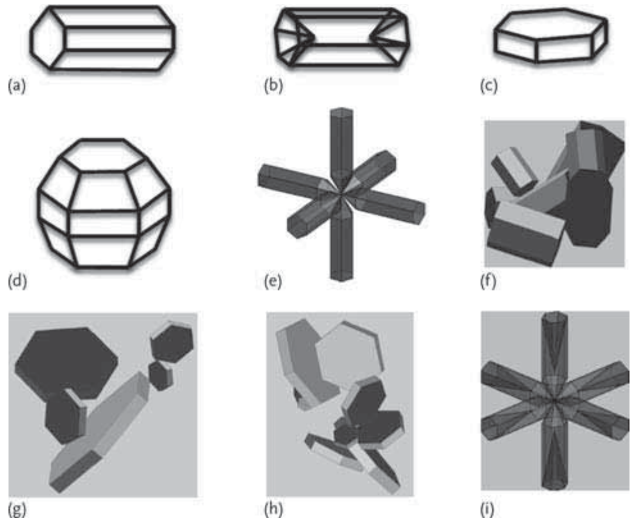

Figure 5.2

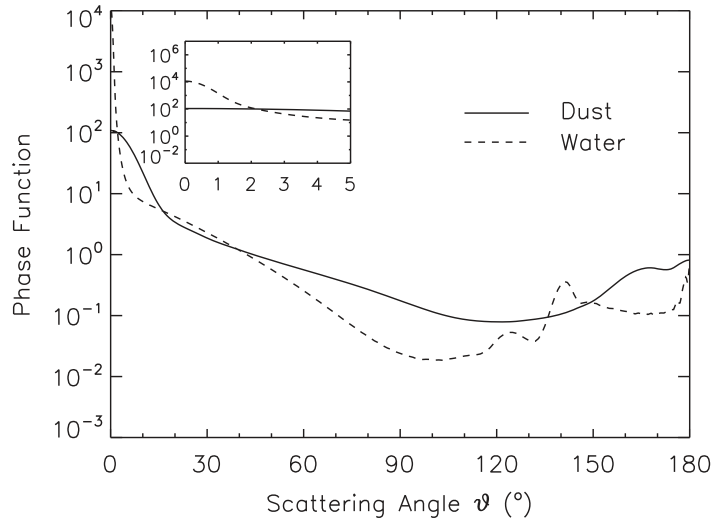
Phase Function
Scattering Angle ϑ (°)
Dust
Water
0
30
60
90
120
150
180
0 1 2 3 4 5

Figure 5.3

5. Volumetric (Bulk) Optical Properties

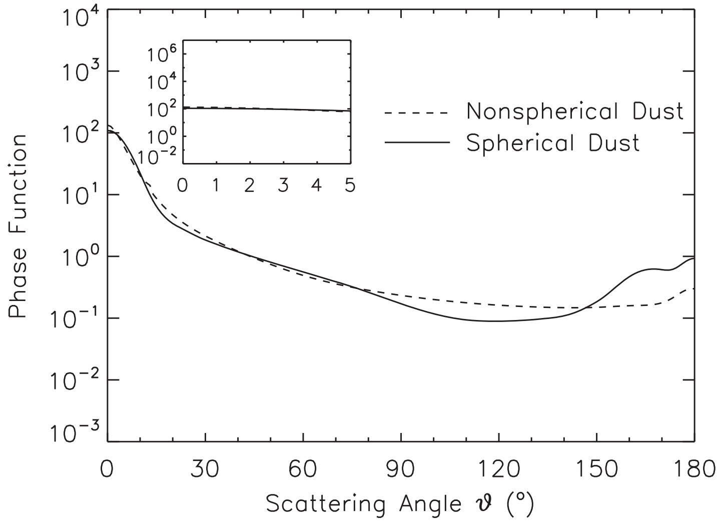

Figure 5.4

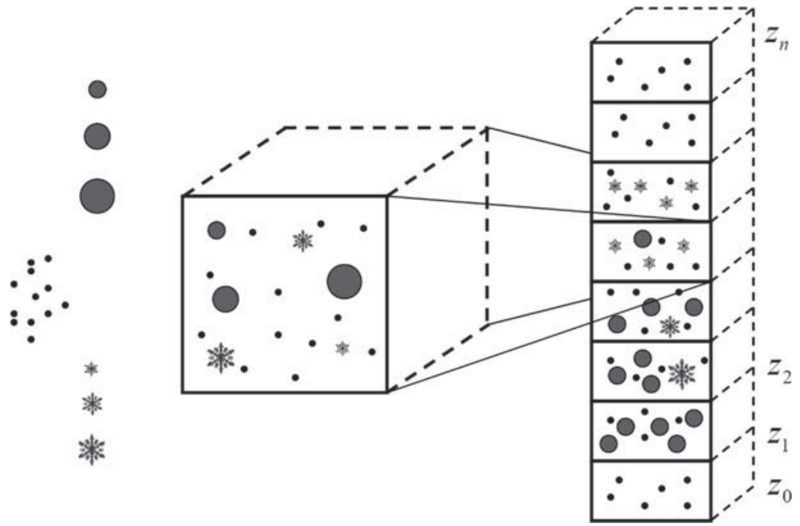

Figure 6.1

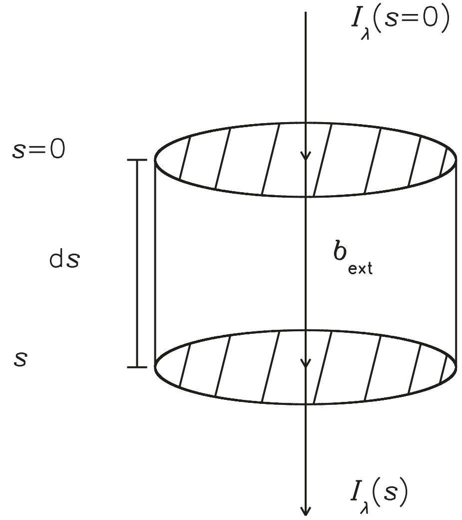

Figure 6.2

6. Radiative Transfer Equation

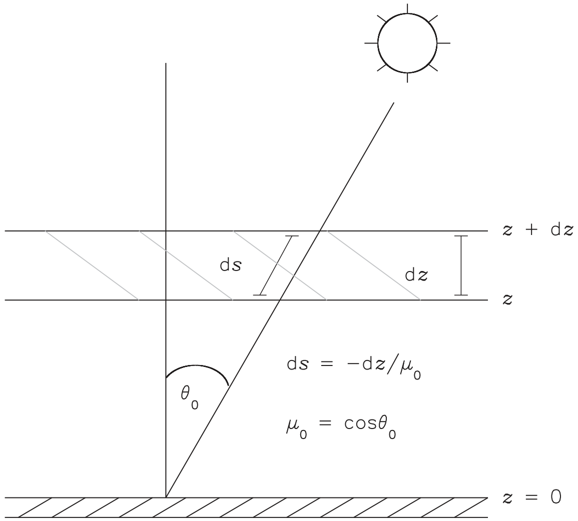

Figure 6.3

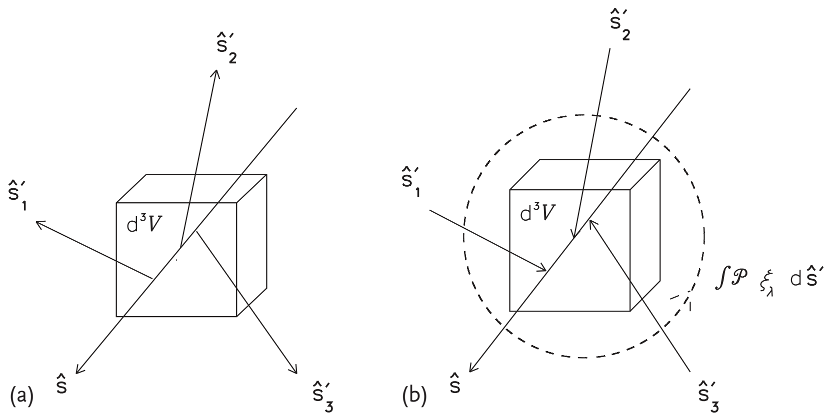

Figure 6.4

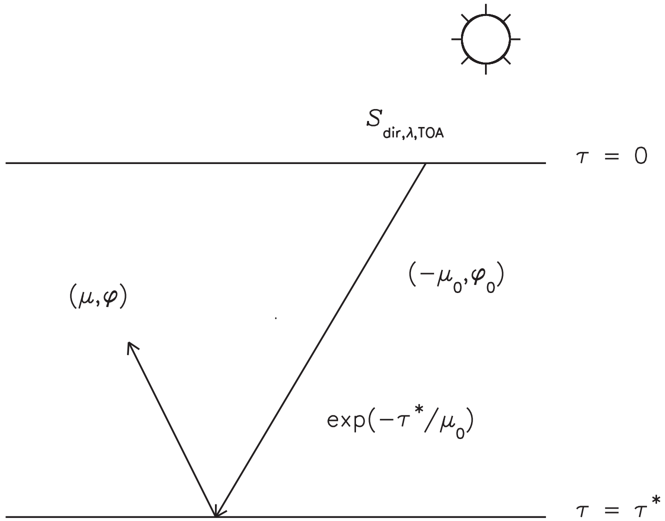
$S_{\mathrm{dir},\lambda,\mathrm{TOA}}$
$\tau = 0$
$(-\mu_0,\varphi_0)$
(μ,φ)
$\exp(-\tau^*/\mu_0)$
$\tau = \tau^*$

Figure 6.5

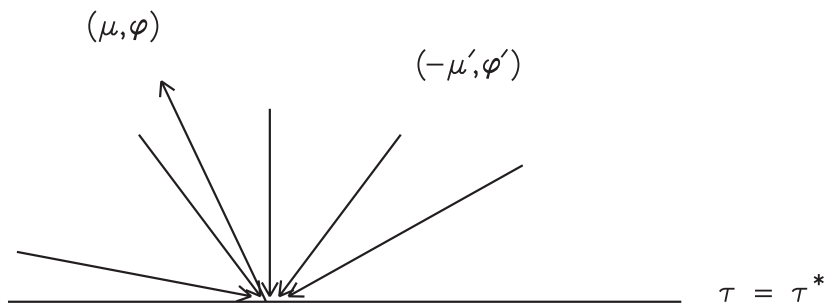

Figure 6.6

6. Radiative Transfer Equation

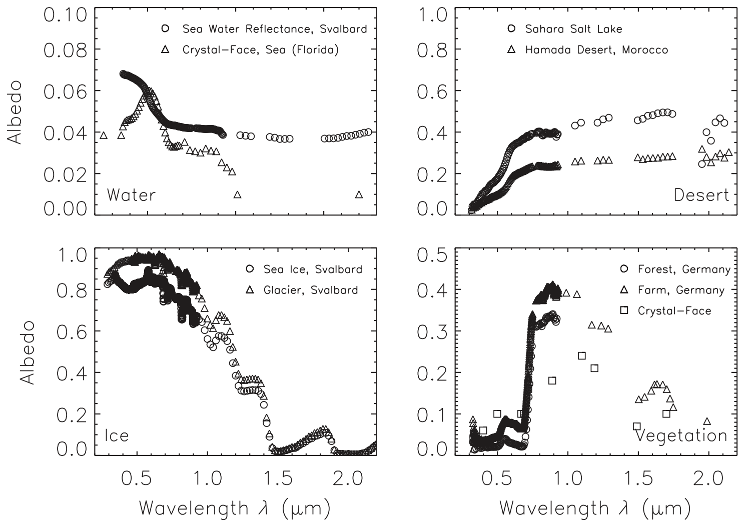

Figure 6.7

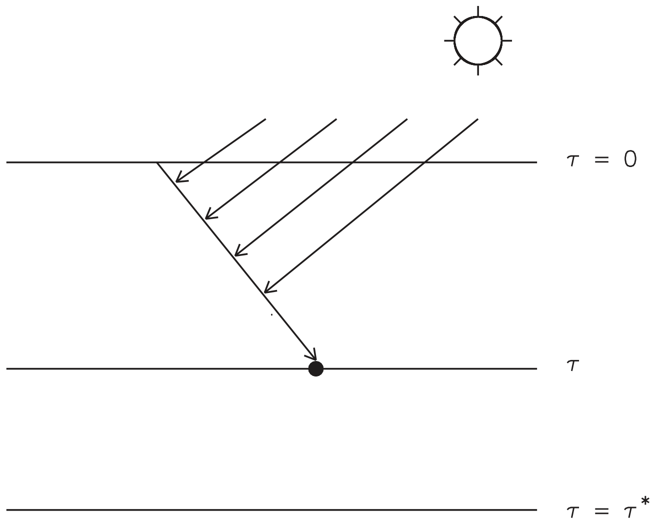

Figure 6.8

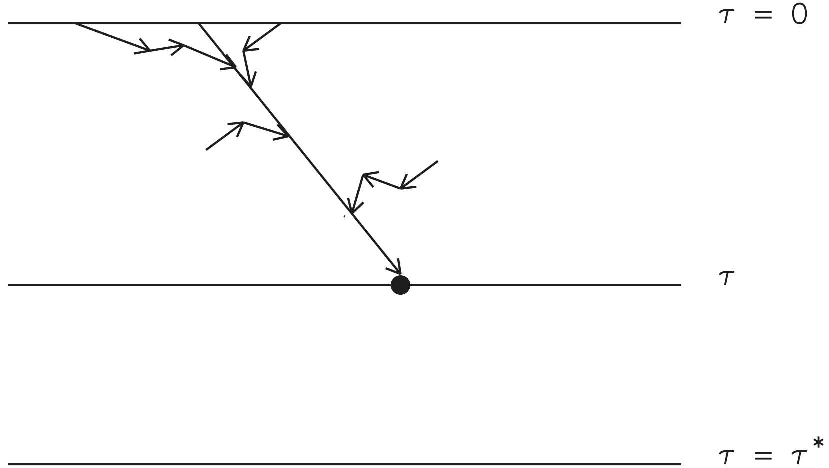

Figure 6.9

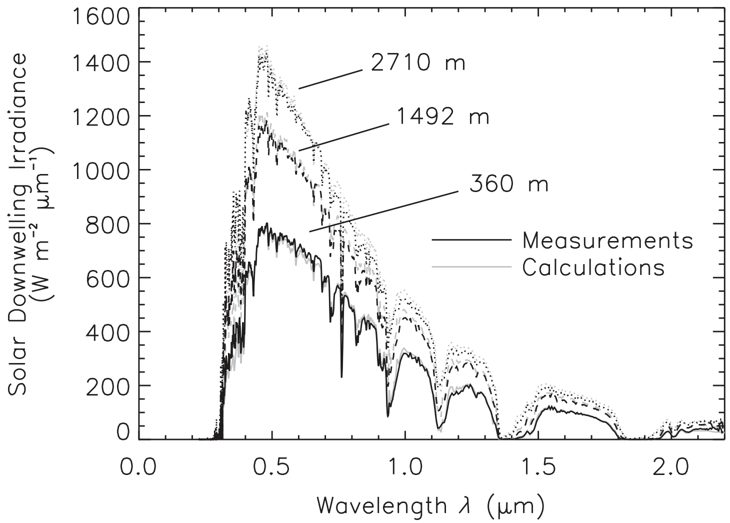
2710 m
1492 m
360 m
Measurements
Calculations
Solar Downwelling Irradiance
(W m^{-2} µm^{-1})
1600
1400
1200
1000
800
600
400
200
0
0.0
0.5
1.0
1.5
2.0
Wavelength λ (µm)

Figure 6.10

6. Radiative Transfer Equation

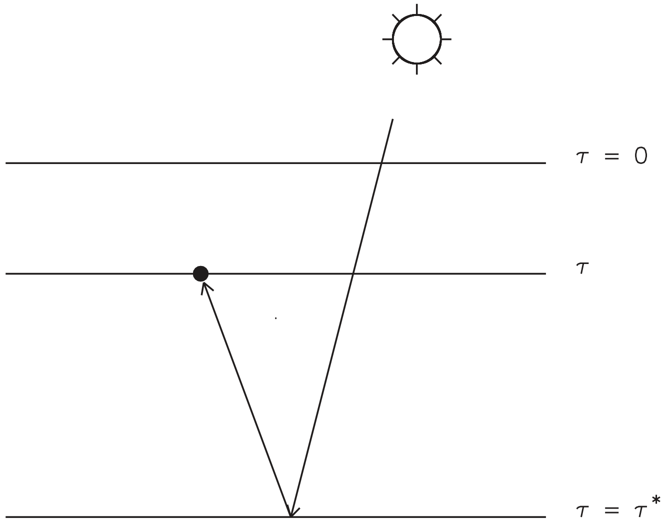

Figure 6.11

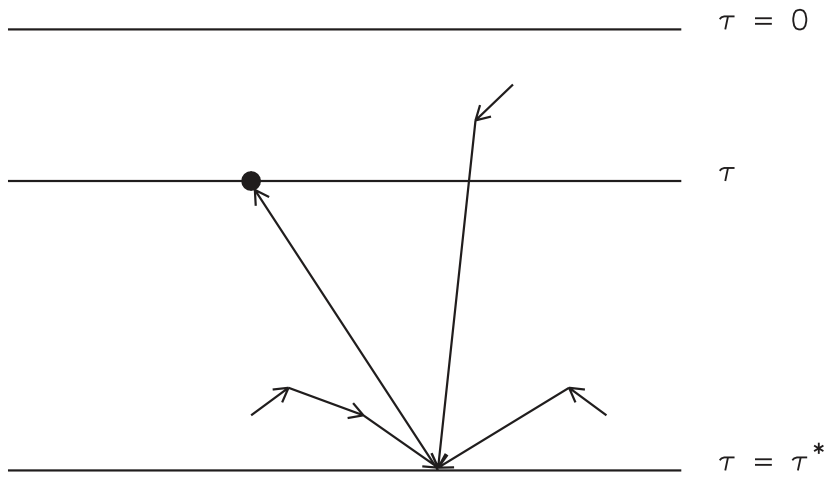

Figure 6.12

6. Radiative Transfer Equation

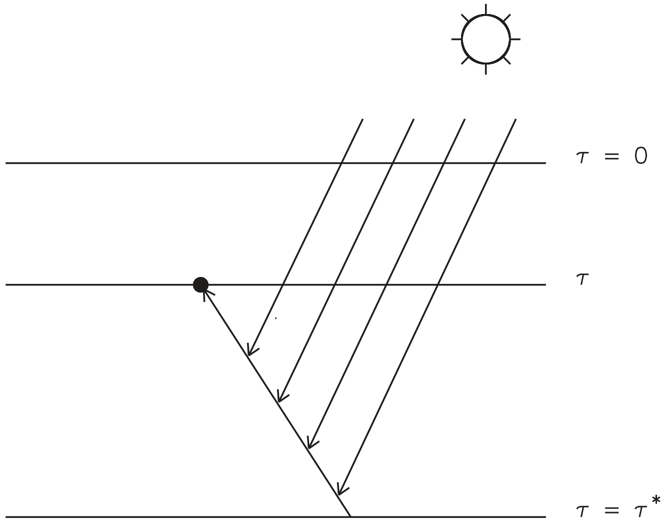

Figure 6.13

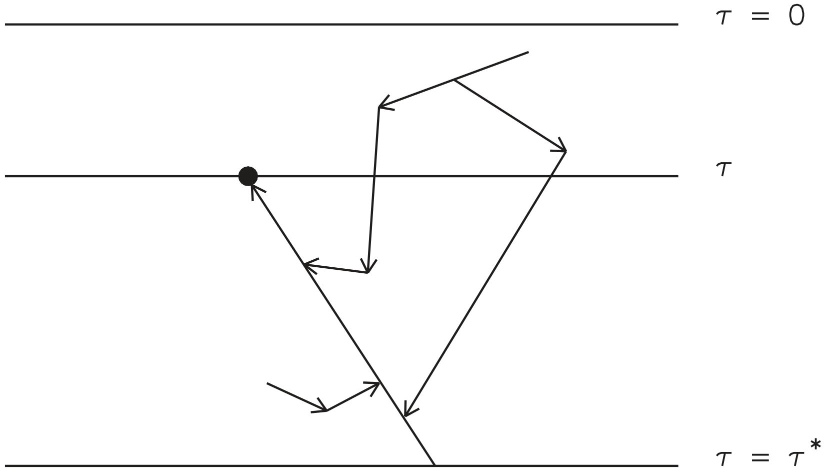

Figure 6.14

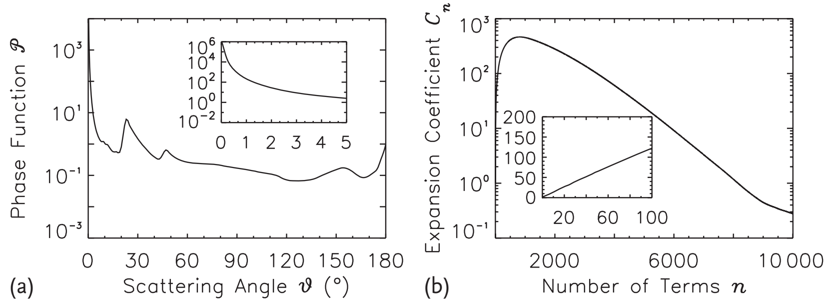
Phase Function $\mathcal{P}$
10^3
10^1
10^{-1}
10^{-3}
10^6
10^4
10^2
10^0
10^{-2}
0 1 2 3 4 5
0 30 60 90 120 150 180
(a)
Scattering Angle ϑ (°)
Expansion Coefficient C_n
10^3
10^2
10^1
10^0
10^{-1}
200
150
100
50
0
20 60 100
2000 6000 10 000
(b)
Number of Terms n

Figure 7.1

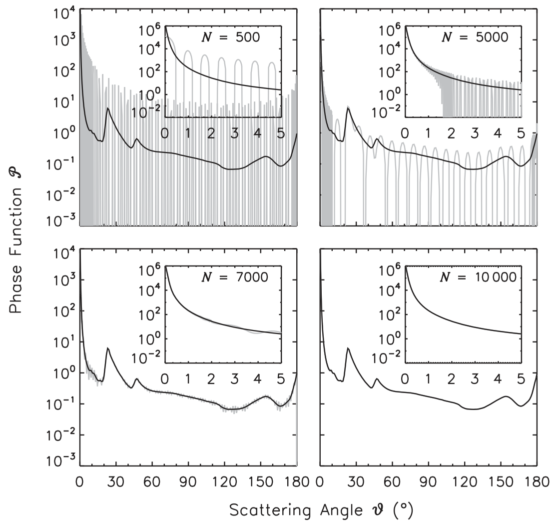
Phase Function $\mathcal{P}$
Scattering Angle ϑ (°)
N = 500
N = 5000
N = 7000
N = 10 000

Figure 7.2

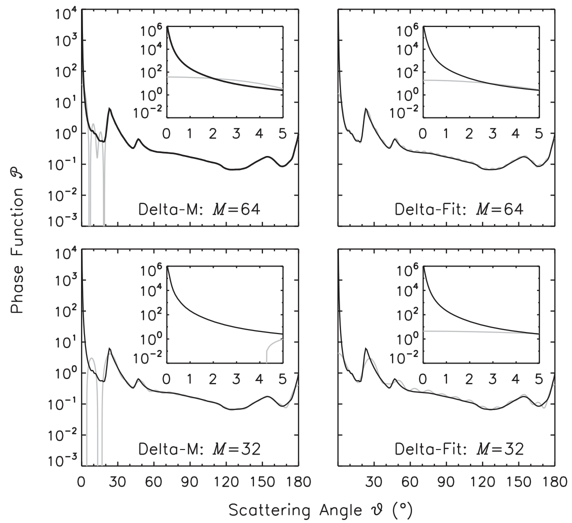
Delta-M: M=64
Delta-Fit: M=64
Delta-M: M=32
Delta-Fit: M=32
Phase Function 𝒫
Scattering Angle ϑ (°)

Figure 7.3

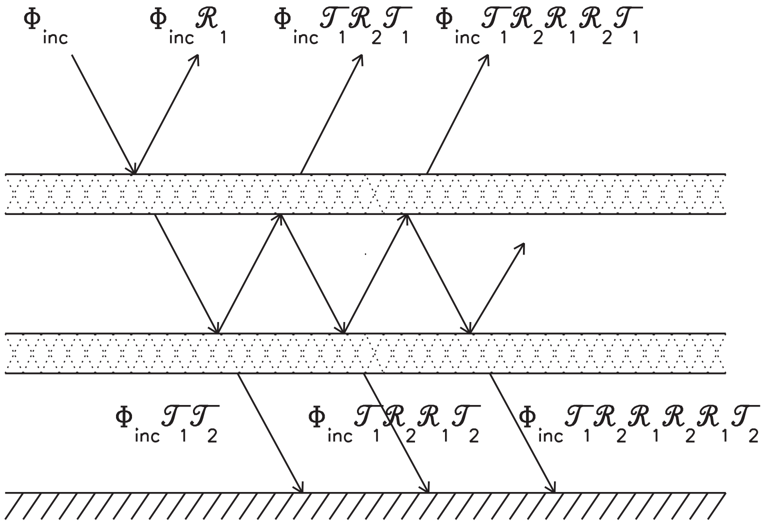
Φ_{inc}
$\Phi_{inc}\mathcal{R}_1$
$\Phi_{inc}\mathcal{T}_1\mathcal{R}_2\mathcal{T}_1$
$\Phi_{inc}\mathcal{T}_1\mathcal{R}_2\mathcal{R}_1\mathcal{R}_2\mathcal{T}_1$
$\Phi_{inc}\mathcal{T}_1\mathcal{T}_2$
$\Phi_{inc}\mathcal{T}_1\mathcal{R}_2\mathcal{R}_1\mathcal{T}_2$
$\Phi_{inc}\mathcal{T}_1\mathcal{R}_2\mathcal{R}_1\mathcal{R}_2\mathcal{R}_1\mathcal{T}_2$

Figure 7.4

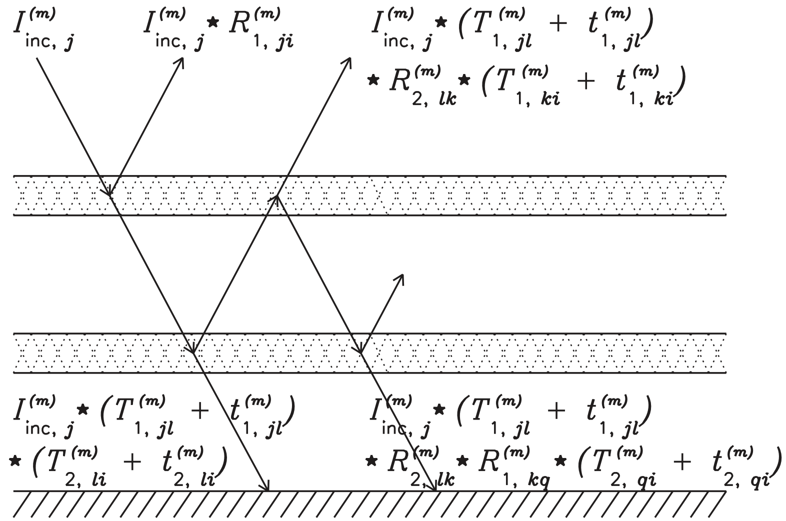
$I_{\mathrm{inc},\,j}^{(m)}$
$I_{\mathrm{inc},\,j}^{(m)} \star R_{1,\,ji}^{(m)}$
$I_{\mathrm{inc},\,j}^{(m)} \star (T_{1,\,jl}^{(m)} + t_{1,\,jl}^{(m)})$
$\star R_{2,\,lk}^{(m)} \star (T_{1,\,ki}^{(m)} + t_{1,\,ki}^{(m)})$
$I_{\mathrm{inc},\,j}^{(m)} \star (T_{1,\,jl}^{(m)} + t_{1,\,jl}^{(m)})$
$\star (T_{2,\,li}^{(m)} + t_{2,\,li}^{(m)})$
$I_{\mathrm{inc},\,j}^{(m)} \star (T_{1,\,jl}^{(m)} + t_{1,\,jl}^{(m)})$
$\star R_{2,\,lk}^{(m)} \star R_{1,\,kq}^{(m)} \star (T_{2,\,qi}^{(m)} + t_{2,\,qi}^{(m)})$

Figure 7.5

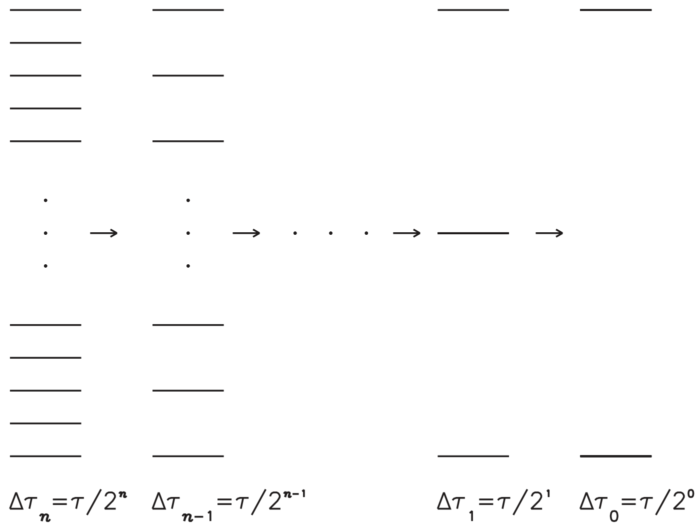
$\Delta\tau_n = \tau/2^n$
$\Delta\tau_{n-1} = \tau/2^{n-1}$
$\Delta\tau_1 = \tau/2^1$
$\Delta\tau_0 = \tau/2^0$

Figure 7.6

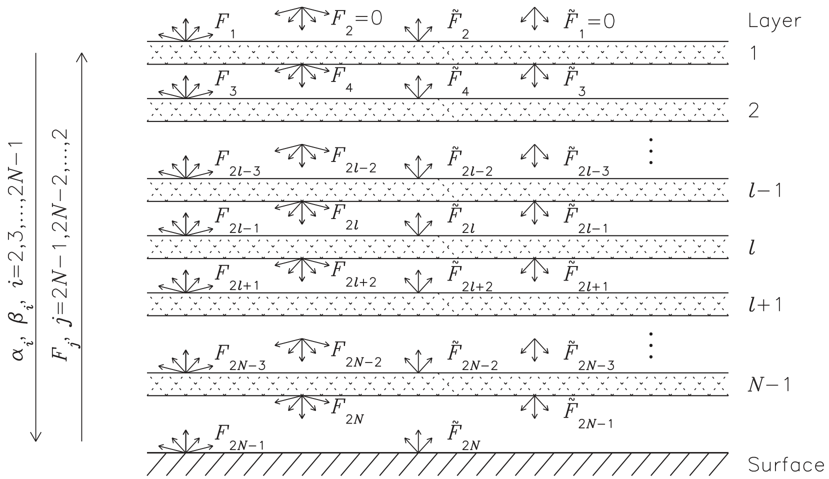
F_1
$F_2=0$
$\tilde{F}_2$
$\tilde{F}_1=0$
Layer
1
2
$l-1$
l
$l+1$
$N-1$
Surface
$\alpha_i,\ \beta_i,\ i=2,3,\ldots,2N-1$
$F_j,\ j=2N-1,2N-2,\ldots,2$

Figure 7.7

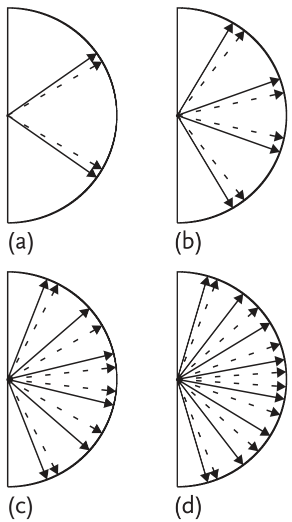

Figure 7.8

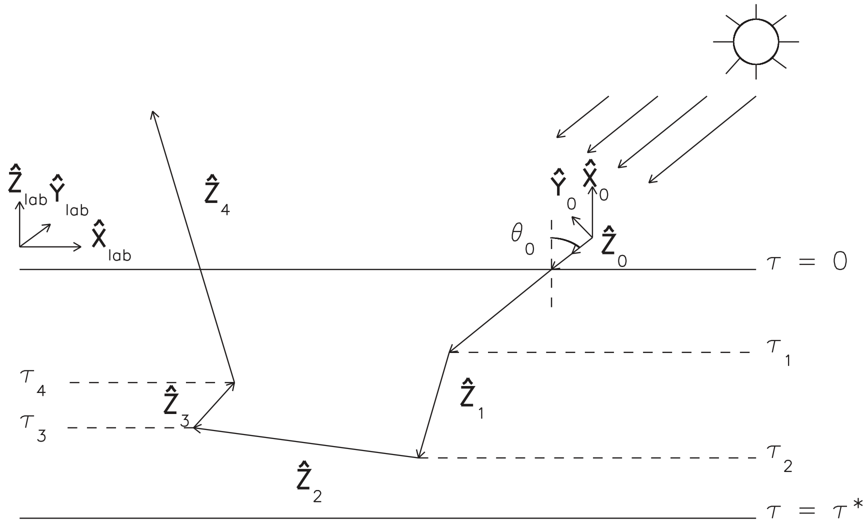

Figure 7.9

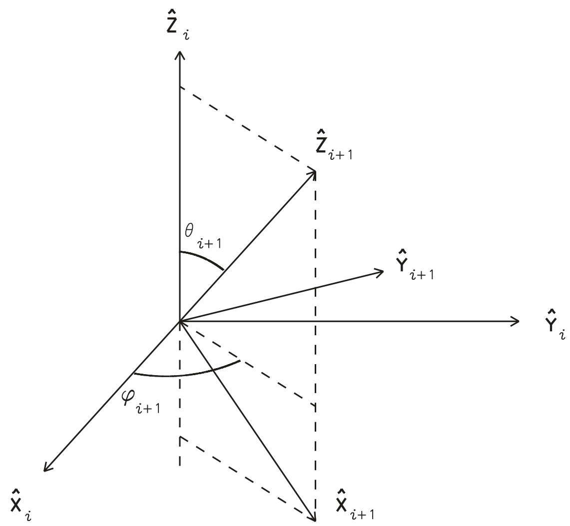

Figure 7.10

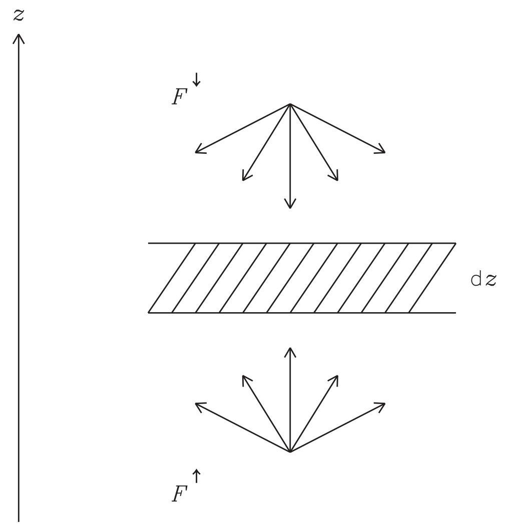

Figure 7.11

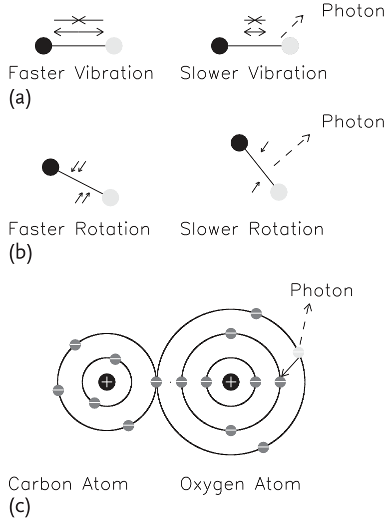
Faster Vibration
Slower Vibration
Photon
(a)
Faster Rotation
Slower Rotation
Photon
(b)
Carbon Atom
Oxygen Atom
Photon
(c)

Figure 8.1

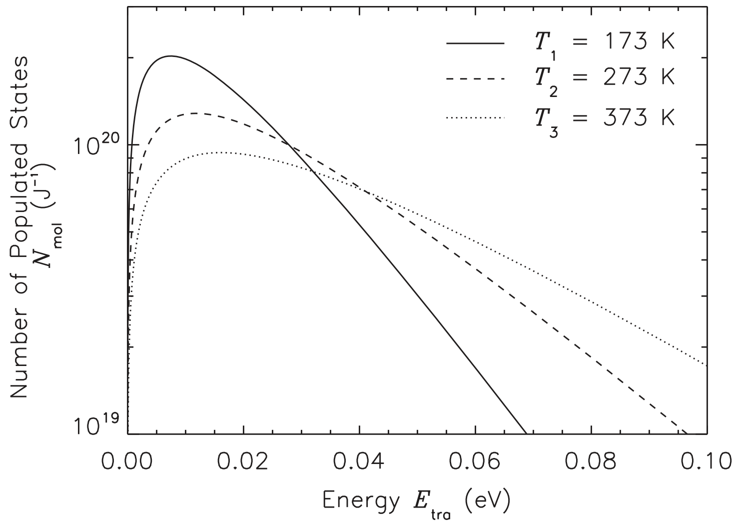
T_1 = 173 K
T_2 = 273 K
T_3 = 373 K
10^{20}
10^{19}
0.00
0.02
0.04
0.06
0.08
0.10
Number of Populated States N_{mol} (J^{-1})
Energy E_{tra} (eV)

Figure 8.2

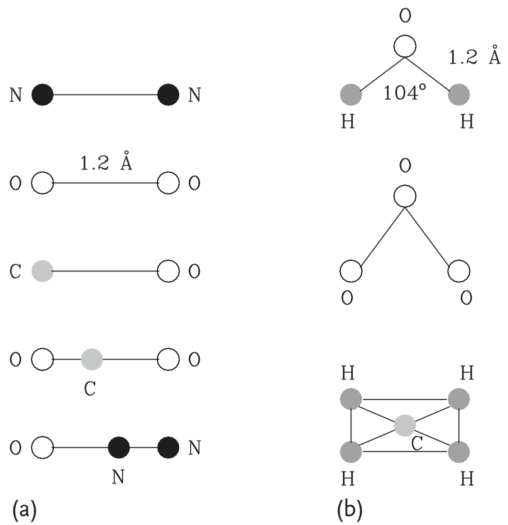

Figure 8.3

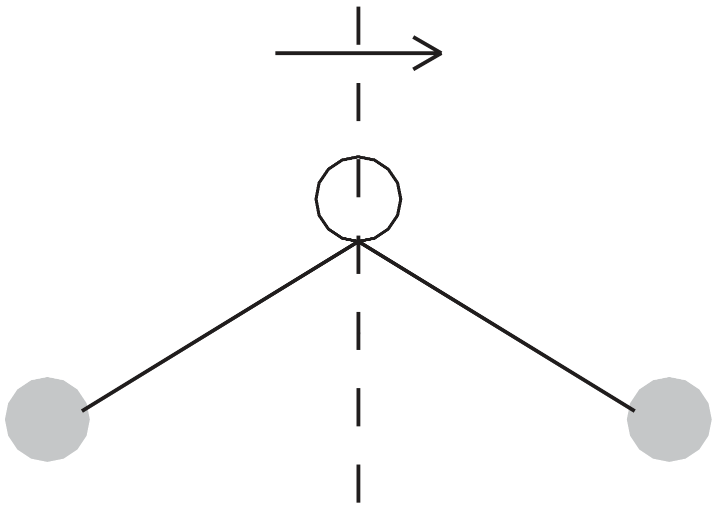

Figure 8.4

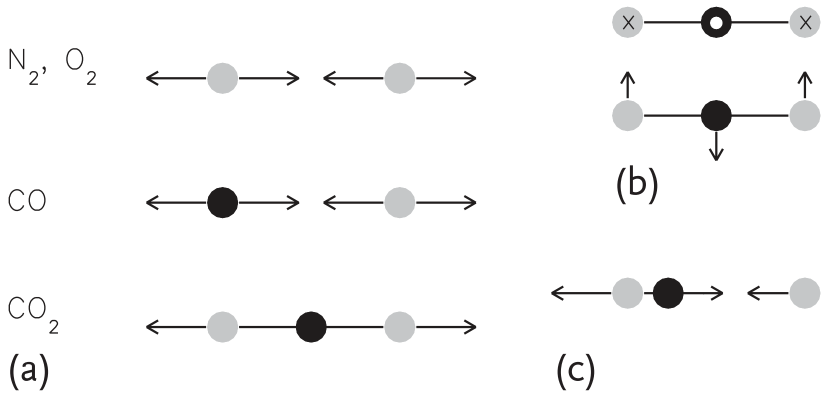

Figure 8.5

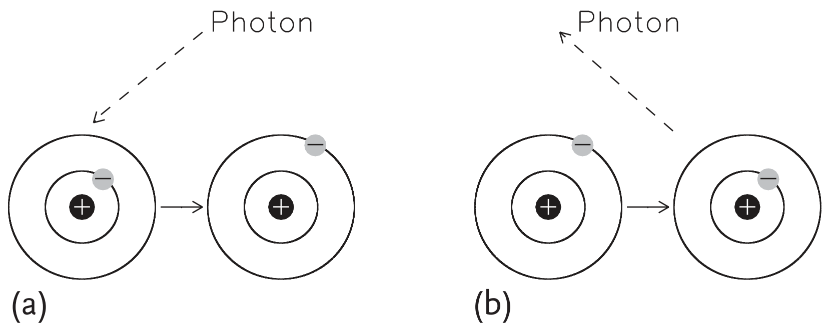

Figure 8.6

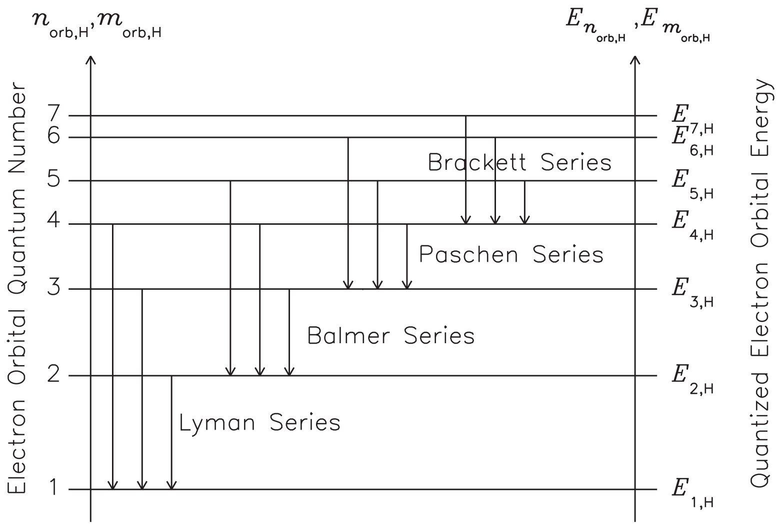
$n_{\text{orb,H}}, m_{\text{orb,H}}$
$E_{n_{\text{orb,H}}}, E_{m_{\text{orb,H}}}$
Electron Orbital Quantum Number
7
6
5
4
3
2
1
Brackett Series
Paschen Series
Balmer Series
Lyman Series
$E_{7,\text{H}}$
$E_{6,\text{H}}$
$E_{5,\text{H}}$
$E_{4,\text{H}}$
$E_{3,\text{H}}$
$E_{2,\text{H}}$
$E_{1,\text{H}}$
Quantized Electron Orbital Energy

Figure 8.7

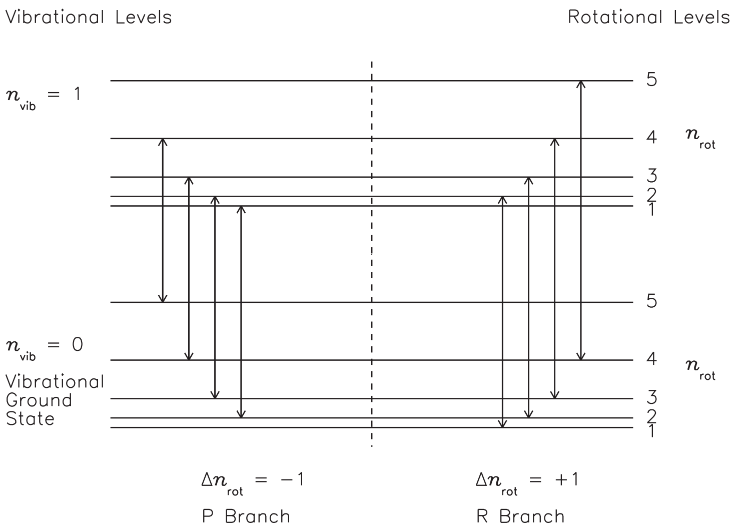
Vibrational Levels
Rotational Levels
n_{vib} = 1
5
4
3
2
1
n_{rot}
5
4
3
2
1
n_{rot}
n_{vib} = 0
Vibrational Ground State
Δn_{rot} = −1
P Branch
Δn_{rot} = +1
R Branch

Figure 8.8

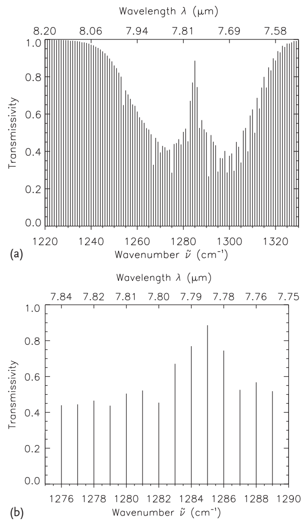
Wavelength λ (μm)
8.20 8.06 7.94 7.81 7.69 7.58
Transmissivity
1.0 0.8 0.6 0.4 0.2 0.0
1220 1240 1260 1280 1300 1320
(a)
Wavenumber ν̃ (cm⁻¹)
Wavelength λ (μm)
7.84 7.82 7.81 7.80 7.79 7.78 7.76 7.75
Transmissivity
1.0 0.8 0.6 0.4 0.2 0.0
1276 1278 1280 1282 1284 1286 1288 1290
(b)
Wavenumber ν̃ (cm⁻¹)

Figure 8.9

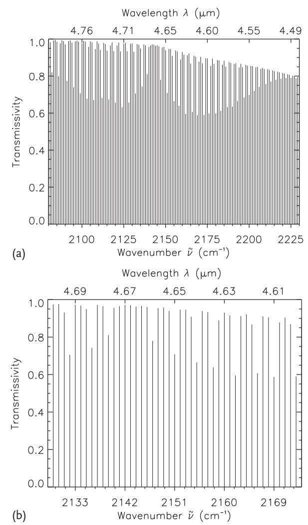
Wavelength λ (μm)
4.76 4.71 4.65 4.60 4.55 4.49
Transmissivity
1.0 0.8 0.6 0.4 0.2 0.0
2100 2125 2150 2175 2200 2225
Wavenumber ν̃ (cm⁻¹)
(a)
Wavelength λ (μm)
4.69 4.67 4.65 4.63 4.61
Transmissivity
1.0 0.8 0.6 0.4 0.2 0.0
2133 2142 2151 2160 2169
Wavenumber ν̃ (cm⁻¹)
(b)

Figure 8.10

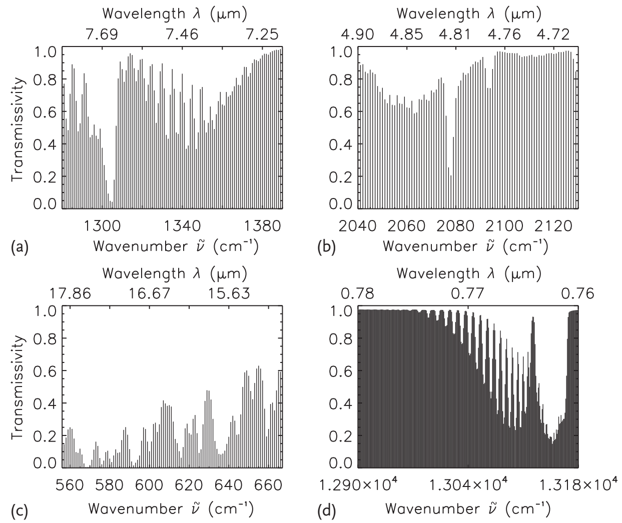
Wavelength λ (μm)
7.69
7.46
7.25
Transmissivity
1.0
0.8
0.6
0.4
0.2
0.0
1300
1340
1380
(a)
Wavenumber ν̃ (cm⁻¹)
Wavelength λ (μm)
4.90
4.85
4.81
4.76
4.72
2040
2060
2080
2100
2120
(b)
Wavenumber ν̃ (cm⁻¹)
Wavelength λ (μm)
17.86
16.67
15.63
560
580
600
620
640
660
(c)
Wavenumber ν̃ (cm⁻¹)
Wavelength λ (μm)
0.78
0.77
0.76
1.290×10⁴
1.304×10⁴
1.318×10⁴
(d)
Wavenumber ν̃ (cm⁻¹)

Figure 8.11

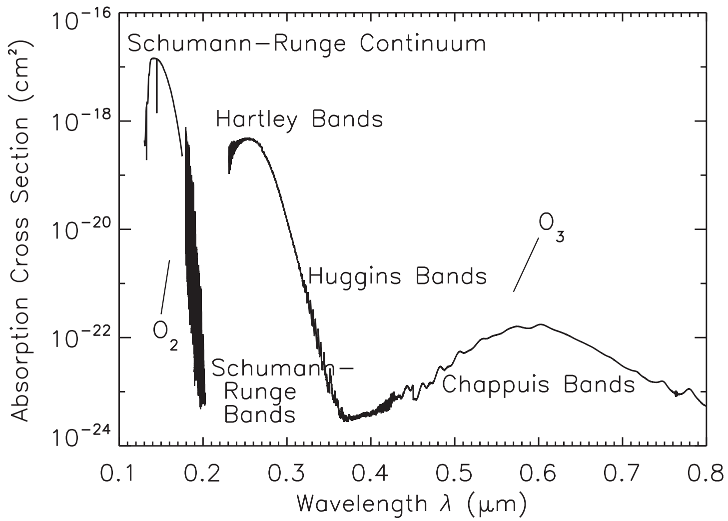
Schumann−Runge Continuum
Hartley Bands
Huggins Bands
O_3
O_2
Schumann− Runge Bands
Chappuis Bands
Absorption Cross Section (cm²)
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
Wavelength λ (μm)

Figure 8.12

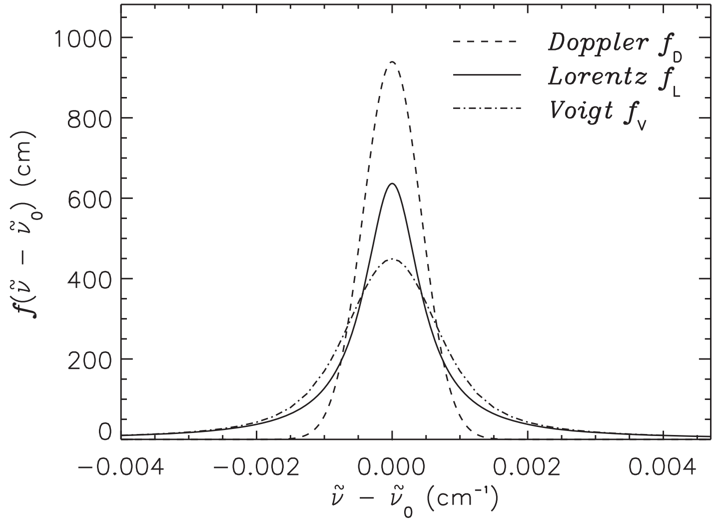
Doppler f_D
Lorentz f_L
Voigt f_V
$f(\tilde{\nu} - \tilde{\nu}_0)$ (cm)
1000
800
600
400
200
0
−0.004
−0.002
0.000
0.002
0.004
$\tilde{\nu} - \tilde{\nu}_0$ (cm^{-1})

Figure 8.13

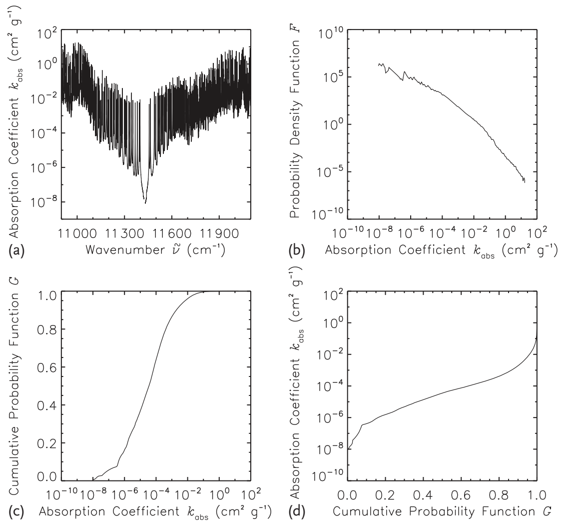
Absorption Coefficient k_{abs} (cm² g⁻¹)
Wavenumber $\tilde{\nu}$ (cm⁻¹)
(a)
Probability Density Function F
Absorption Coefficient k_{abs} (cm² g⁻¹)
(b)
Cumulative Probability Function G
Absorption Coefficient k_{abs} (cm² g⁻¹)
(c)
Absorption Coefficient k_{abs} (cm² g⁻¹)
Cumulative Probability Function G
(d)

Figure 8.14

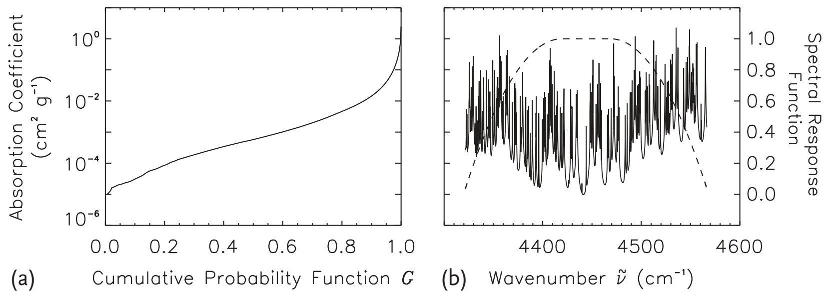
Absorption Coefficient (cm² g⁻¹)
10⁰
10⁻²
10⁻⁴
10⁻⁶
0.0
0.2
0.4
0.6
0.8
1.0
(a)
Cumulative Probability Function G
4400
4500
4600
(b)
Wavenumber ν̃ (cm⁻¹)
1.0
0.8
0.6
0.4
0.2
0.0
Spectral Response Function

Figure 8.15

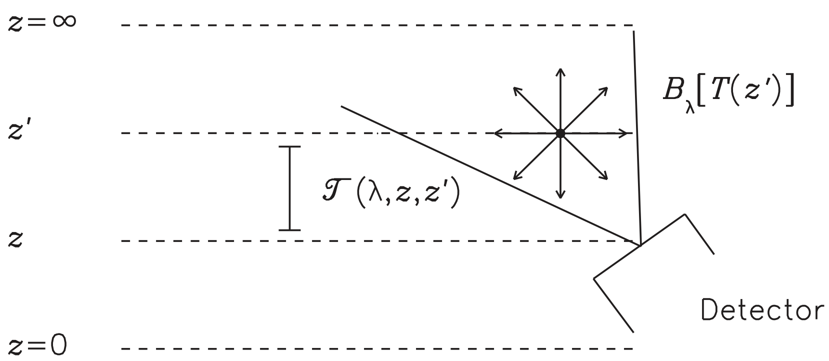

Figure 9.1

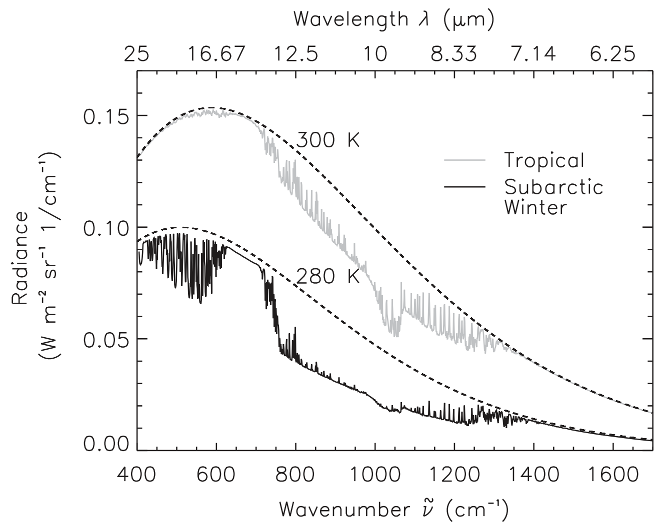
Wavelength λ (μm)
25
16.67
12.5
10
8.33
7.14
6.25
0.15
0.10
0.05
0.00
300 K
280 K
Tropical
Subarctic
Winter
Radiance
(W m⁻² sr⁻¹ 1/cm⁻¹)
400
600
800
1000
1200
1400
1600
Wavenumber ν̃ (cm⁻¹)

Figure 9.2

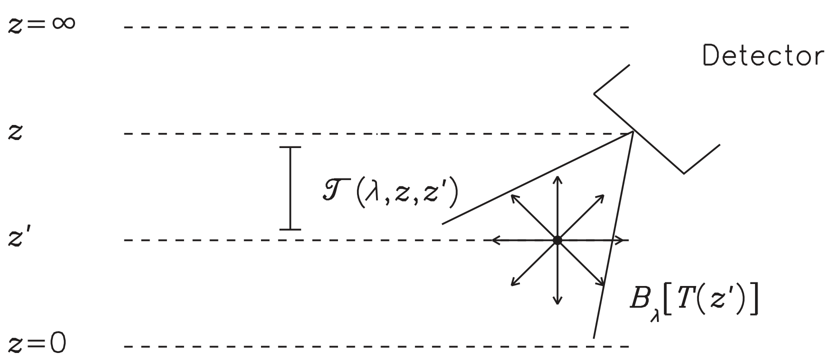

Figure 9.3

9. Terrestrial Radiative Transfer

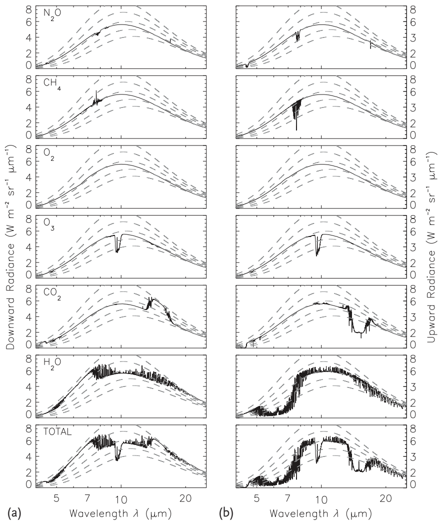

Figure 9.4

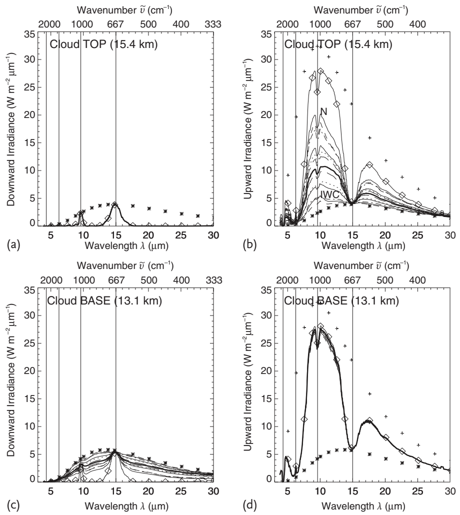
Wavenumber $\tilde{\nu}$ (cm^{-1})
Cloud TOP (15.4 km)
Downward Irradiance (W m^{-2} µm^{-1})
Upward Irradiance (W m^{-2} µm^{-1})
N
IWC
Cloud BASE (13.1 km)
Wavelength λ (µm)
(a)
(b)
(c)
(d)

Figure 9.5

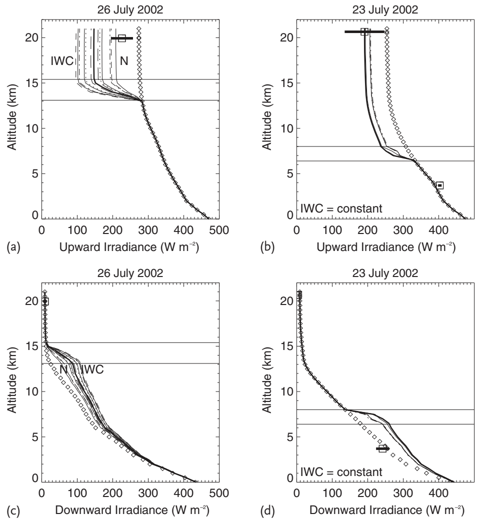
26 July 2002
23 July 2002
IWC
N
IWC = constant
Altitude (km)
Upward Irradiance (W m⁻²)
Downward Irradiance (W m⁻²)
(a)
(b)
(c)
(d)

Figure 9.6

9. Terrestrial Radiative Transfer

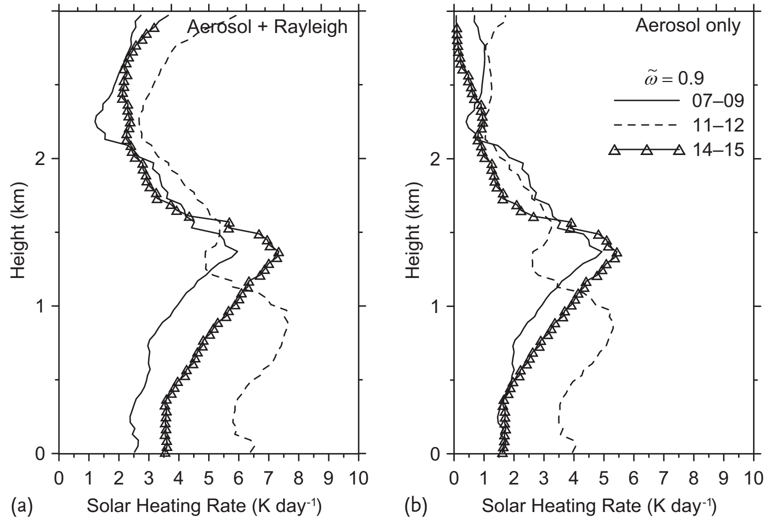

Figure 9.7

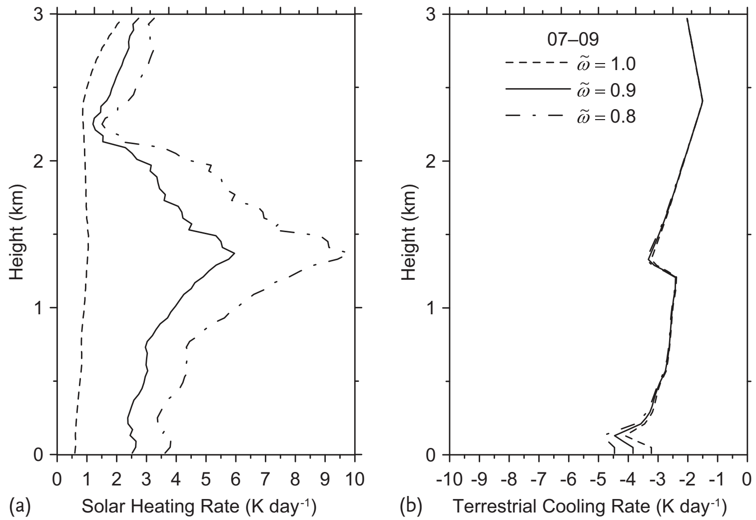
3
2
1
0
Height (km)
0 1 2 3 4 5 6 7 8 9 10
(a)
Solar Heating Rate (K day⁻¹)
07–09
$\tilde{\omega} = 1.0$
$\tilde{\omega} = 0.9$
$\tilde{\omega} = 0.8$
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0
(b)
Terrestrial Cooling Rate (K day⁻¹)

Figure 9.8