

# Contents

<b>1</b>	<b>Historical Background</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.2	Kirchhoff (1859) . . . . .	1
1.2.1	The Birth of Spectroscopy . . . . .	1
1.2.2	The First Law . . . . .	3
1.2.3	The Second Law . . . . .	5
1.3	Stefan (1879) and Boltzmann (1884) . . . . .	7
1.3.1	Experimental Background . . . . .	7
1.3.2	Maxwell's Theory and Thermodynamics . . . . .	8
1.4	Wien (1893) . . . . .	10
1.5	Planck (1900) . . . . .	15
1.5.1	Modeling Matter . . . . .	15
1.5.2	The Quantum Hypothesis . . . . .	16
1.6	Einstein (1905, 1907, and 1916) . . . . .	19
1.6.1	Quantization of Light Absorption (1905) . . . . .	19
1.6.2	Specific Heat of Solids (1907) . . . . .	20
1.6.3	Spontaneous Emission (1916) . . . . .	20
<b>I</b>	<b>Quantized Matter</b>	<b>23</b>
<b>2</b>	<b>Two-Level Medium</b>	<b>25</b>
2.1	Electric Field Equation . . . . .	25
2.2	Material Equations . . . . .	26
2.3	Phenomenology: Incoherent Pumping and Decay . . . . .	30
<b>3</b>	<b>Propagation Regimes</b>	<b>33</b>
3.1	Linear Propagation Regime . . . . .	33
3.2	Nonlinear Susceptibility . . . . .	37
3.3	Nonlinear Steady Propagation . . . . .	39
3.4	Group and Phase Velocity . . . . .	40

<b>4</b>	<b>Coherence and Atomic Interference</b>	<b>43</b>
4.1	Atomic Interference . . . . .	43
4.2	Semiclassical Formulation . . . . .	46
4.3	Electromagnetically Induced Transparency . . . . .	49
4.4	Slow Light . . . . .	52
<b>II</b>	<b>Sine–Gordon Solitons</b>	<b>55</b>
<b>5</b>	<b>Self-Induced Transparency</b>	<b>57</b>
5.1	Derivation of the Area Theorem . . . . .	57
5.2	Properties of the Area . . . . .	61
<b>6</b>	<b>Sine–Gordon Equation</b>	<b>65</b>
6.1	The Bäcklund Transformation . . . . .	65
6.2	$2\pi$ Solitons . . . . .	67
6.3	Short Bibliography on Solitons . . . . .	70
<b>7</b>	<b>Higher-Order Sine–Gordon Solitons</b>	<b>73</b>
7.1	The Bianchi Theorem . . . . .	73
7.2	$4\pi$ Solitons . . . . .	74
7.3	Unstable $0\pi$ Solitons . . . . .	79
7.4	Stable $0\pi$ Solitons . . . . .	79
7.5	Solutions in an Amplifier . . . . .	82
<b>III</b>	<b>Cavity Nonlinear Optics</b>	<b>85</b>
<b>8</b>	<b>Laser Theory</b>	<b>87</b>
8.1	Introduction . . . . .	87
8.2	Single-Mode Ring Laser . . . . .	91
8.2.1	Steady States . . . . .	94
8.2.2	Rate Equations . . . . .	95
8.2.3	Good Cavity Limit . . . . .	97
8.3	Single-Mode Fabry–Perot Laser . . . . .	98
8.3.1	Semiclassical Equations . . . . .	98
8.3.2	Population Gratings in Steady State . . . . .	101
8.3.3	Rate Equations . . . . .	102
8.4	Warning . . . . .	104
8.5	Short Bibliography on Cavity Optics . . . . .	105

<b>9</b>	<b>Optical Bistability I</b>	<b>113</b>
9.1	Introduction . . . . .	113
9.2	Steady-State Solutions . . . . .	114
9.3	Optical Devices . . . . .	115
9.4	Generic Description . . . . .	118
9.5	Nonlinear Stability . . . . .	119
9.6	Address Pulses . . . . .	121
9.7	Pulse Area Law . . . . .	124
9.8	Appendix: the Schmitt Trigger . . . . .	126
<b>10</b>	<b>Optical Bistability II</b>	<b>129</b>
10.1	Delay–Differential Equations . . . . .	129
10.2	Discrete Maps . . . . .	130
10.3	Deterministic Chaos . . . . .	133
10.4	Bibliography . . . . .	136
10.4.1	Ikeda Delay–Differential Equations . . . . .	136
10.4.2	Deterministic Chaos . . . . .	137
<b>IV</b>	<b>Weakly Nonlinear Systems: <math>\chi^{(2)}</math> Media</b>	<b>141</b>
<b>11</b>	<b>Frequency Mixing</b>	<b>143</b>
11.1	Tensor $\rightarrow$ Vector $\rightarrow$ Scalar Description . . . . .	143
11.2	Multiple Time-Scales . . . . .	144
11.3	$\chi^{(2)}$ Media . . . . .	146
11.4	Bibliography . . . . .	148
11.4.1	Founding Papers of Nonlinear Optics . . . . .	148
11.4.2	Nonlinear Optics ( $\chi^{(2)}$ and $\chi^{(3)}$ Media) . . . . .	148
<b>12</b>	<b>Second Harmonic Generation</b>	<b>153</b>
12.1	Formulation . . . . .	153
12.2	Free-Running Second Harmonic Generation . . . . .	155
12.2.1	Perfect Matching and $G = 0$ . . . . .	158
12.2.2	Perfect Matching but $G \neq 0$ . . . . .	160
12.2.3	Imperfect Phase Matching . . . . .	161
12.2.4	Phase Matching . . . . .	162
12.3	Intra-Cavity Second Harmonic Generation . . . . .	164
<b>13</b>	<b>Sum and Difference Frequency Generation</b>	<b>171</b>
13.1	Sum Frequency Generation . . . . .	171
13.1.1	Formulation . . . . .	171
13.1.2	Free-Running Sum Frequency Generation . . . . .	172

13.2	Difference Frequency Generation . . . . .	176
13.2.1	Two Intense Input Fields . . . . .	177
13.2.2	One Intense Input Field . . . . .	178
<b>14</b>	<b>Optical Parametric Oscillator</b>	<b>181</b>
14.1	Formulation . . . . .	181
14.2	Threshold Condition . . . . .	182
14.3	Degenerate Optical Parametric Oscillator . . . . .	184
14.4	Ring and Fabry–Perot Cavities . . . . .	186
<b>V</b>	<b>Weakly Nonlinear Systems: <math>\chi^{(3)}</math> Media</b>	<b>189</b>
<b>15</b>	<b>Multiwave Frequency Mixing</b>	<b>191</b>
15.1	Introduction . . . . .	191
15.2	Monochromatic Input Field . . . . .	192
15.3	Three-Wave Mixing . . . . .	193
15.4	Optical Phase Conjugation . . . . .	194
15.4.1	Fresnel’s Laws . . . . .	194
15.4.2	Optical Phase Conjugation . . . . .	195
15.5	Degenerate Four-Wave Mixing . . . . .	200
<b>16</b>	<b>Nonlinear Schrödinger Solitons</b>	<b>203</b>
16.1	Introduction . . . . .	203
16.2	Formulation . . . . .	203
16.3	Mathematical Digression . . . . .	205
16.3.1	The $\Delta$ Operator . . . . .	206
16.3.2	The $\nabla(\nabla\cdot)$ Operator . . . . .	206
16.3.3	The Operator $\partial^2/\partial t^2$ . . . . .	206
16.3.4	The Integral Operator . . . . .	207
16.3.5	Expansion of the Operator $\varphi(\omega)$ . . . . .	208
16.3.6	Third-Order Polarization . . . . .	209
16.3.7	Summing Up . . . . .	209
16.4	Perturbation Expansion . . . . .	210
16.4.1	The $\mathcal{O}(\mu^0)$ Equation . . . . .	210
16.4.2	The $\mathcal{O}(\mu)$ Equation . . . . .	211
16.4.3	The $\mathcal{O}(\mu^2)$ Equation . . . . .	211
16.5	The Nonlinear Schrödinger Equation . . . . .	213
16.6	Basic Solutions of the Nonlinear Schrödinger Equation . . . . .	214
16.7	Comments . . . . .	216

<b>17 Higher-Order Nonlinear Schrödinger Solitons</b>	<b>219</b>
17.1 Inverse Scattering Method . . . . .	219
17.2 First-Order Solutions . . . . .	221
17.2.1 Turing/Modulational Instability (Time-Periodic Solutions) . . . . .	221
17.2.2 Aperiodic Solution . . . . .	221
17.2.3 Space-Periodic Solutions . . . . .	224
17.2.4 Space- and Time-Periodic Solutions . . . . .	225
17.3 Second-Order Solutions . . . . .	225
17.3.1 Colliding Bright Solitons . . . . .	225
17.3.2 Breathers (Space-Periodic) . . . . .	230
17.3.3 Aperiodic Solution . . . . .	231
17.4 Colliding Dark Solitons . . . . .	231
17.5 Existence and Stability . . . . .	233
17.6 Short Bibliography on Patterns . . . . .	235
<b>References</b>	<b>237</b>
<b>Index</b>	<b>241</b>

