1
The Dark Energy Problem

The discovery that the expansion of the universe is accelerating was first made by Riess *et al*. (1998) and Perlmutter *et al*. (1999), with supporting evidence for this observation strengthening over time.

The cause for the observed acceleration is unknown, and is usually referred to as "the dark energy problem". It could be due to an unknown energy component in the universe (i.e., "dark energy"), or the modification of gravity as described by Einstein's general relativity (i.e., "modified gravity"). Solving the mystery of the observed cosmic acceleration is one of the most exciting challenges in cosmology today.

1.1
Evidence for Cosmic Acceleration

To understand the evidence for cosmic acceleration, we need to first introduce the basis of standard cosmology. We will then discuss the first and current evidence for cosmic acceleration.

1.1.1
The Basic Cosmological Picture

We live in an expanding universe, a fact first discovered by Hubble in 1929. Our physical universe can be described by the Robertson–Walker metric, the simplest metric that describes a homogeneous, isotropic, and expanding universe:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\tilde{k}r^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right] \tag{1.1}$$

where c is the speed of light, t is cosmic time, $a(t)$ is the cosmic scale factor, and $\tilde{k}$ is the curvature constant. The universe is flat for $\tilde{k} = 0$, open for $\tilde{k} < 0$ and closed for $\tilde{k} > 0$. The spatial location of an object is given by (r, θ, ϕ) in spherical coordinates.

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The physical wavelength of light emitted at time t is given by

$$\lambda_{\text{phys}} = a(t)\lambda \,, \tag{1.2}$$

where λ is the comoving wavelength. As the universe expands, $a(t)$ increases with time. Comoving quantities do not change with the expansion of the universe. The expansion of the universe leads to an increase in the observed wavelength (i.e., a redshift) of light from a distant source. The cosmological redshift is defined as

$$z \equiv \frac{1}{a(t)} - 1 \,. \tag{1.3}$$

The redshift z is usually used as the indicator for cosmic time, because it can be measured for a given astrophysical object. If the light emitted by a distant object is stretched by a factor of $(1+z)$ in wavelength upon arrival at the observer, the object is said to be at a distance corresponding to redshift z.

The coordinate distance $r(z)$ from Eq. (1.1) gives the observer's *comoving distance* to an object located at redshift z. Our physical distance to the object is the *angular diameter distance* given by

$$d_{\text{A}}(z) \equiv a(t)r = \frac{r(z)}{1+z} \,. \tag{1.4}$$

If we know the intrinsic luminosity of an object, then measuring its apparent brightness allows us to infer our *luminosity distance* to the object

$$\left[\frac{d_{\text{L}}(z)}{10\,\text{pc}}\right]^2 = \frac{F_{\text{int}}}{F} \,, \tag{1.5}$$

where F is the observed flux from the object, and F_{int} is its "intrinsic flux", defined to be the flux from the object received by an observer located at a distance of 10 pc away from the object. In astronomical observations, magnitude is used as the unit for the observed flux. The magnitude difference between two objects with observed fluxes F_1 and F_2 is defined as

$$m_1 - m_2 \equiv 2.5 \log\left(\frac{F_2}{F_1}\right) . \tag{1.6}$$

Thus, Eq. (1.5) becomes

$$m - M = 2.5 \log\left(\frac{F_{\text{int}}}{F}\right) = 5 \log\left[\frac{d_{\text{L}}(z)}{\text{Mpc}}\right] + 25 \,, \tag{1.7}$$

where m and M are the apparent and absolute magnitudes of the object respectively, and $m-M$ is known as the *distance modulus*. Due to the redshifting of the light from the object, and the time dilation effect, the luminosity distance and the comoving distance to the object are related by

$$d_{\text{L}}(z) = (1+z)r(z) \,. \tag{1.8}$$

The expansion rate of the universe at time t is known as the *Hubble parameter* $H(t)$, defined as

$$H(t) \equiv \frac{\dot{a}}{a} . \tag{1.9}$$

The Hubble parameter $H(t)$ and the cosmic scale factor $a(t)$ are functions of time (i.e., redshift) that depend on the composition of the universe, as well as the global spatial curvature of the universe. Setting $\mathrm{d}^2 s = 0$ and considering radial dependence only (i.e., considering the radial propagation of photons), Eq. (1.1) gives a relation between distance and redshift that depends on the Hubble parameter, which in turn depends on the composition and spatial curvature of the universe. For a flat universe, we have

$$d_{\mathrm{L}}(z) = c(1+z) \int_0^z \mathrm{d}z' \frac{1}{H(z')} , \quad \text{(flat universe)} . \tag{1.10}$$

In the standard cosmological model, the universe began in a very hot and very dense state, known as the *Big Bang*. It is likely that the universe went through a period of extremely rapid expansion (known as *inflation*) in the first tiny fraction of a second in the history of the universe. The universe was radiation dominated after the end of inflation, then became matter dominated at $z \sim 3000$.

For a universe consisting of matter, radiation, and a cosmological constant, the Hubble parameter $H(z)$ is

$$H^2(z) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_{\mathrm{m}}(1+z)^3 + \Omega_{\mathrm{r}}(1+z)^4 + \Omega_{\mathrm{k}}(1+z)^2 + \Omega_\Lambda\right], \tag{1.11}$$

where the *Hubble constant* H_0 is defined as the value of the Hubble parameter today. The density fractions Ω_{m}, Ω_{r}, Ω_Λ, and Ω_{k} are defined by

$$\Omega_{\mathrm{m}} \equiv \frac{\rho_{\mathrm{m}}(t_0)}{\rho_{\mathrm{c}}^0} , \quad \Omega_{\mathrm{r}} \equiv \frac{\rho_{\mathrm{rad}}(t_0)}{\rho_{\mathrm{c}}^0} , \quad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_{\mathrm{c}}^0} , \quad \Omega_{\mathrm{k}} \equiv -\frac{\tilde{k}}{H_0^2} , \tag{1.12}$$

where $\rho_{\mathrm{m}}(t_0)$ and $\rho_{\mathrm{rad}}(t_0)$ are the matter and radiation densities today, and ρ_Λ is the energy density due to a *cosmological constant* (also known as *vacuum energy*). The *critical density* ρ_{c}^0 is defined as

$$\rho_{\mathrm{c}}^0 \equiv \frac{3H_0^2}{8\pi G} . \tag{1.13}$$

Requiring the consistency of Eq. (1.11) at $z = 0$, that is, $H(z = 0) = H_0$, gives

$$\Omega_{\mathrm{m}} + \Omega_{\mathrm{r}} + \Omega_{\mathrm{k}} + \Omega_\Lambda = 1 . \tag{1.14}$$

Note that $\Omega_{\mathrm{r}} \ll \Omega_{\mathrm{m}}$, thus the Ω_{r} term is usually omitted at $z \ll 3000$.

The matter-energy content of the universe is parametrized as an ideal fluid with density ρ and pressure p. Each component of the universe can be described by its equation of state w, defined as

$$w \equiv \frac{p}{\rho} \,. \tag{1.15}$$

Matter has $w = 0$. Radiation has $w = 1/3$. A cosmological constant corresponds to $w = -1$.

1.1.2
First Direct Observational Evidence for Cosmic Acceleration

Until about a decade ago, observational data favored a universe that is dominated by matter today. If we allow the universe to have an arbitrary spatial curvature (see Eq. (1.1)), and a possibly nonzero cosmological constant Λ (see Eq. (1.11)), then we can define a "deceleration parameter" of the cosmic expansion:

$$q_0 \equiv -\frac{\ddot{a}(t_0)/a(t_0)}{H_0^2} = \frac{\Omega_{\mathrm{m}}}{2} - \Omega_\Lambda \,, \tag{1.16}$$

where $a(t_0)$ is the cosmic scale factor today, and we have used Eq. (1.11). Equation (1.16) shows clearly that a matter-dominated universe (with $\Omega_{\mathrm{m}} > 2\Omega_\Lambda$) should be decelerating today.

For decades, astronomers tried to measure the cosmic "deceleration parameter" using Type Ia supernovae (SNe Ia) as cosmological distance indicators. SNe Ia can be calibrated to be good *standard candles*, with very small scatter in their intrinsic peak luminosity. Thus the measured apparent peak brightness of SNe Ia can be used to infer the distances to the SNe Ia. The observed spectra of the SNe Ia can be used to measure their redshifts. This yields an observed distance-redshift relation of SNe Ia that can be shown in a Hubble diagram. In a Hubble diagram, the distance modulus $m - M$ of a SN Ia, the difference between the observed apparent peak brightness of a SN Ia and its absolute peak brightness, is plotted as a function of redshift of the SN Ia. The measured distance modulus of a SN Ia gives a measurement of the luminosity distance $d_{\mathrm{L}}(z)$ to the SN Ia at redshift z, see Eq. (1.7). This measured $d_{\mathrm{L}}(z)$ can then be compared with the theoretical prediction (e.g., Eqs. (1.10) and (1.11)) to infer the values of cosmological parameters, and constrain q_0.

Unexpectedly, the quest to measure the cosmic "deceleration parameter" led to the discovery that the universe is accelerating today. This means that the universe is dominated by something that is not matter-like today, or general relativity does not give a complete description of the present universe. This discovery was made using the observed peak brightness of SNe Ia as cosmological distance indicators, independently by two teams of astronomers, Riess *et al.* (1998) and Perlmutter *et al.* (1999). Figure 1.1 shows the joint confidence contour for Ω_{m} and Ω_Λ from Riess *et al.* (1998) and Perlmutter *et al.* (1999). The discovery of cosmic acceleration was made at high statistical significance.

Figure 1.2 shows Hubble diagrams of SNe Ia from both teams. These show the observed distance-redshift relations, that is, the distance modulus $m - M$ versus

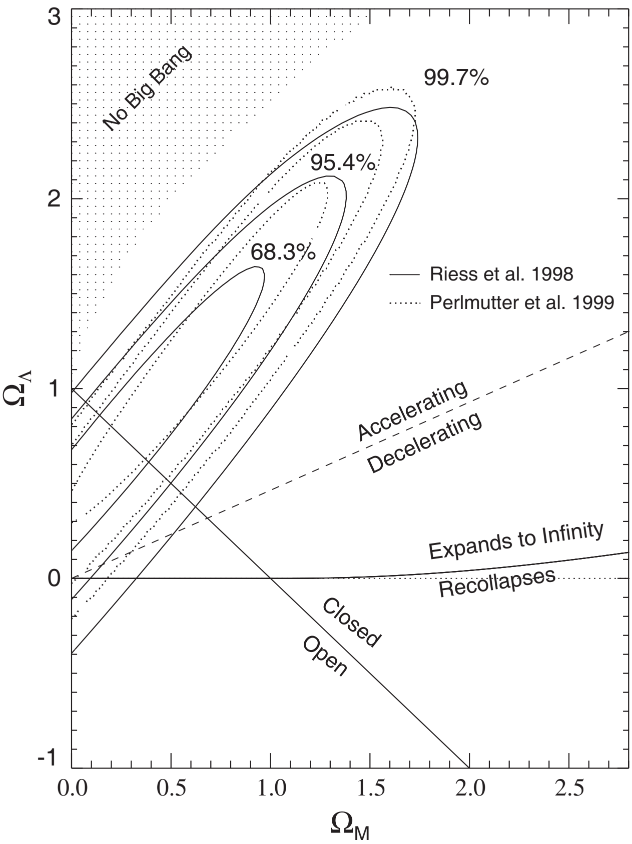

Figure 1.1 The discovery of cosmic acceleration by Riess *et al.* (1998) and Perlmutter *et al.* (1999) (Riess, 2000).

redshift z, compared to several theoretical models. It is difficult to see which cosmological model is favored. Figure 1.3 shows Hubble diagrams of flux-averaged data from Wang (2000a) using SNe Ia from both teams (Riess *et al.*, 1998; Perlmutter *et al.*, 1999). Flux averaging removes the weak gravitational lensing systematic effect of demagnification or magnification of SNe Ia due to the distribution of matter in the universe, since the total number of photons is unchanged by gravitational lensing (Wang, 2000a; Wang and Mukherjee, 2004). Flux averaging also makes data more transparent. It is interesting to note that flux averaging leads to slightly larger error ellipses that are shifted toward smaller Ω_Λ, making the first evidence for cosmic acceleration less strong than that suggested by Figure 1.1 (Wang, 2000a).

Another thing to note is that there were systematic differences in the data from the two teams, although both teams found cosmic acceleration using their own data and that of low-redshift measurements by Hamuy *et al.* (1996). Wang (2000a) compared the data of 18 SNe Ia published by both teams, see Figure 1.4. The error bars are the combined errors in the apparent B magnitude m_B^{eff} measured by Perlmutter

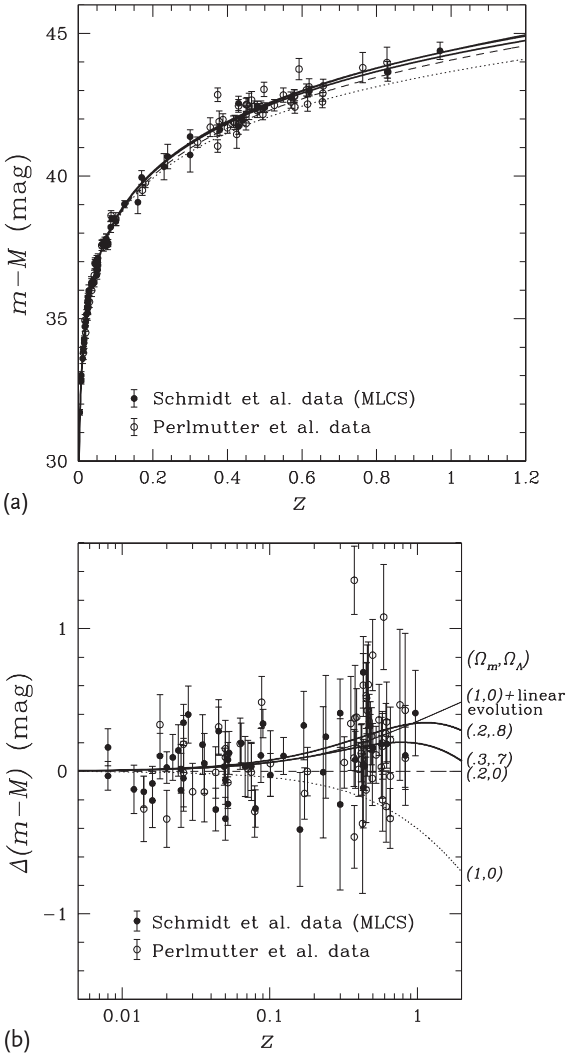

Figure 1.2 Supernova data from two independent teams, Riess *et al.* (1998), and Perlmutter *et al.* (1999) (from Wang, 2000a). Panel (b) is the same as panel (a), but with an open universe model ($\Omega_\mathrm{m} = 0.2$, $\Omega_\Lambda = 0$) subtracted.

et al. (1999), and the distance modulus μ_0^{MLCS} measured by Riess *et al.* (1998). The difference of m_B^{eff} and μ_0^{MLCS} should be a constant (the SN Ia peak absolute magnitude) with zero scatter, if there were no differences in analysis techniques, and no internal dispersion in the SN Ia peak brightness. Wang (2000a) found a mean SN Ia peak absolute magnitude of $M_B = -19.33 \pm 0.25$. This scatter of 0.25 mag can be accounted for by the internal dispersion of each data set of about 0.20 mag in the

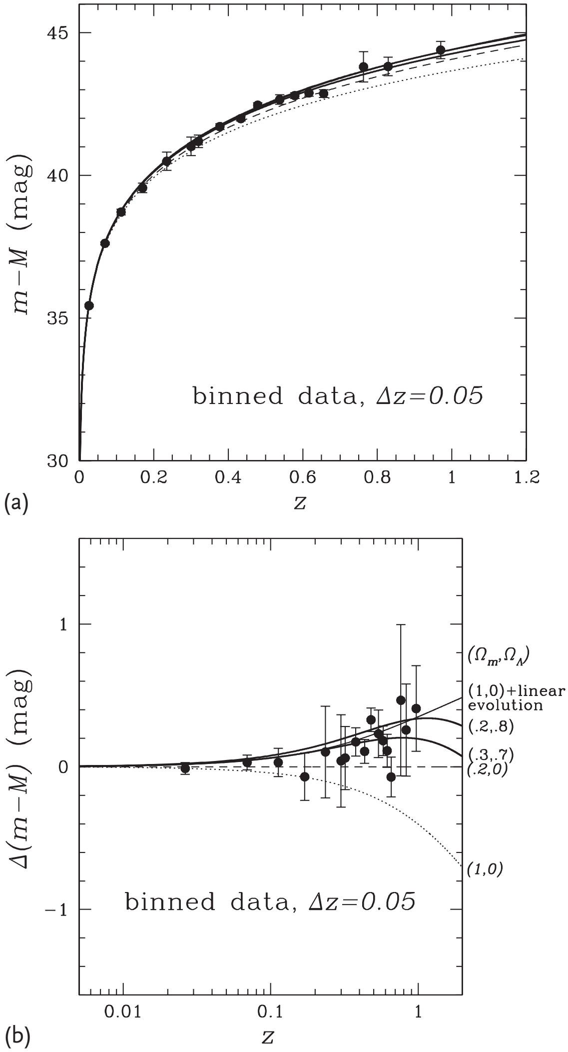

Figure 1.3 The data from Figure 1.2 flux-averaged (Wang, 2000a). Panel (b) is the same as panel (a), but with an open universe model ($\Omega_\mathrm{m} = 0.2$, $\Omega_\Lambda = 0$) subtracted.

calibrated SN Ia peak absolute magnitudes, and an additional uncertainty of about 0.15 mag that is introduced by the difference in analysis techniques (Wang, 2000a).

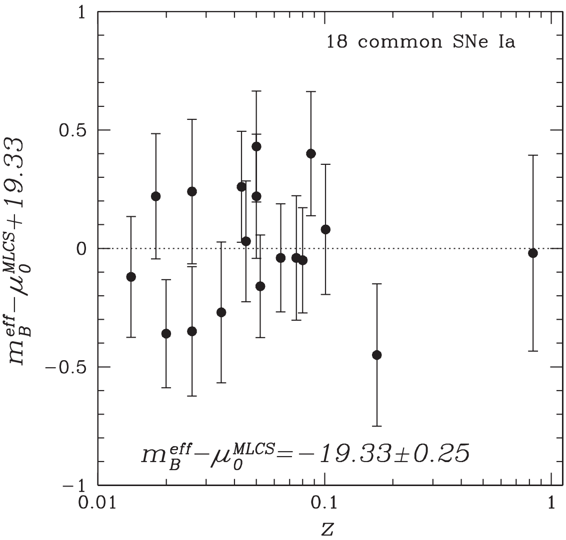

Figure 1.4 The difference between the apparent B magnitude $m_B^{\rm eff}$ measured by Perlmutter *et al.* (1999), and the distance modulus $\mu_0^{\rm MLCS}$ measured by Riess *et al.* (1998) for the same 18 SNe Ia (from Wang, 2000a). The error bars are the combined errors in $m_B^{\rm eff}$ and $\mu_0^{\rm MLCS}$.

1.1.3
Current Observational Evidence for Cosmic Acceleration

The direct evidence for cosmic acceleration has strengthened over time, as a result of the observations of more SNe Ia, and an improvement in the analysis technique. The analysis by Wang (2000a) demonstrated the importance of analyzing all the SNe Ia using the same analysis technique. This was first done by Riess *et al.* (2004), who compiled a "gold" set of 157 SNe Ia. Most recently, Kowalski *et al.* (2008) compiled a "union" set of 307 SNe Ia in 2008, using data from Hamuy *et al.* (1996); Riess *et al.* (1998, 1999); Perlmutter *et al.* (1999); Tonry *et al.* (2003); Knop (2003); Krisciunas *et al.* (2004a,b); Barris *et al.* (2004); Jha *et al.* (2006); Astier *et al.* (2006); Riess *et al.* (2007), and Miknaitis *et al.* (2007).

Other observational data have provided strong indirect evidence for the existence of dark energy. Cosmic microwave background anisotropy data (CMB) have indicated that the global geometry of the universe is close to being flat, that is, $\Omega_{\rm tot} \sim 1$ (de Bernardis *et al.*, 2000). The observed abundance of galaxy clusters first revealed that we live in a low matter density universe ($\Omega_{\rm m} \sim 0.2$–0.3) (Bahcall, Fan, and Cen, 1997). The CMB and galaxy cluster data together require the existence of dark energy that dominates the universe today.

To see the observational evidence for cosmic acceleration without assuming a cosmological constant, it is useful to measure the expansion history of the universe, the Hubble parameter $H(z)$, from data. Figure 1.5 shows the Hubble parameter $H(z)$, as well as $\dot{a}$, measured from a combination of current observation-

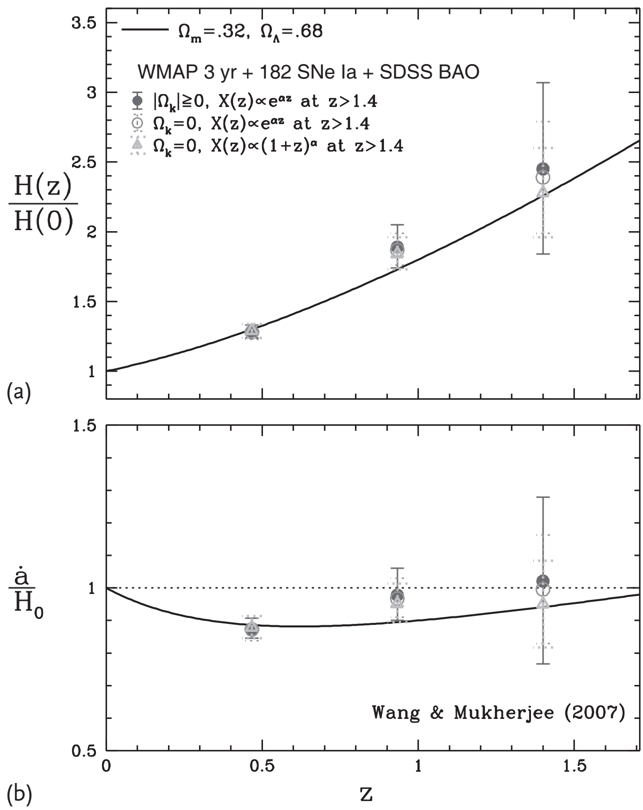

Figure 1.5 Expansion history of the universe measured from current data by Wang and Mukherjee (2007). Data used: CMB data from *WMAP* three-year observations (Spergel *et al.*, 2007); 182 SNe Ia (compiled by Riess *et al.* (2007), including data from the Hubble Space Telescope (HST) obtained by Riess *et al.* (2007), the Supernova Legacy Survey (SNLS) data obtained by Astier *et al.* (2006), as well as nearby SNe Ia); *SDSS* baryon acoustic oscillation measurement (Eisenstein *et al.*, 2005). Note that $X(z) \equiv \rho_X(z)/\rho_X(0)$ in the figure legends, with $\rho_X(z)$ denoting the dark energy density. Panels (a) and (b) use the same data but differ in y axis: $\dot{a} = H(z)a$.

al data by Wang and Mukherjee (2007): CMB data from *WMAP* 3 year observations (Spergel *et al.*, 2007); 182 SNe Ia (compiled by Riess *et al.* (2007), including data from the Hubble Space Telescope (HST) obtained by Riess *et al.* (2007), the Supernova Legacy Survey (SNLS) data obtained by Astier *et al.* (2006), as well as nearby SNe Ia); Sloan Digital Sky Survey (*SDSS*) baryon acoustic oscillation scale measurement (Eisenstein *et al.*, 2005). Clearly, the universe transitioned from cosmic deceleration (matter domination) to cosmic acceleration around $z \sim 0.5$.

The observed cosmic acceleration could be due to an unknown energy component (dark energy, e.g., Quintessence Models references; Linde (1987)), or a modification to general relativity (modified gravity, e.g., Modified Gravity Models references; Dvali, Gabadadze, and Porrati (2000); Freese and Lewis (2002)). The fol-

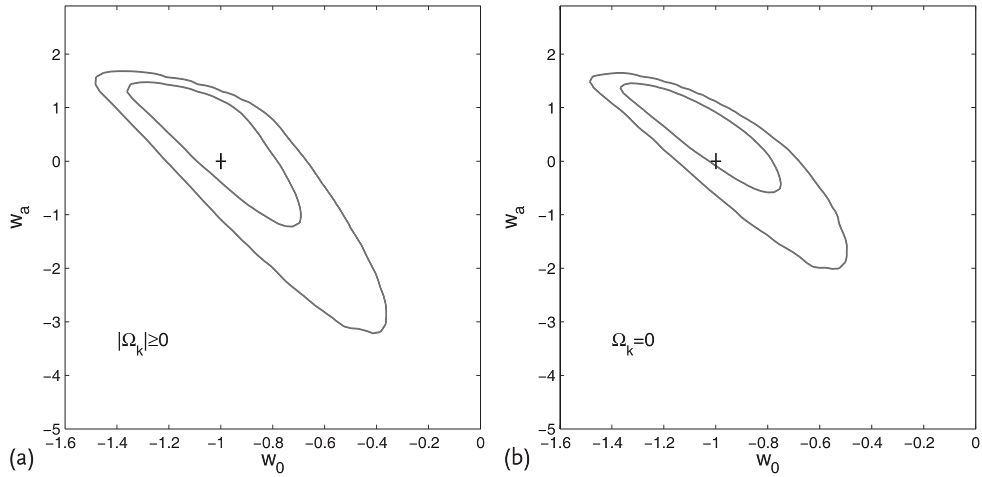

Figure 1.6 Constraints on the dark energy equation of state $w_X(a) = w_0 + w_a(1-a)$ obtained by Wang and Mukherjee (2007), using the same data as in Figure 1.5. A cosmological constant corresponds to $w_X(a) = -1$ (indicated by the cross in the figures). Panel (b) assumes a flat universe; panel (a) does not.

lowing references (Dark Energy Reviews; Copeland, Sami, and Tsujikawa, 2006; Caldwell and Kamionkowski, 2009) contain reviews with more complete lists of references of theoretical models. We discuss some of these models in Chapter 3.

The simplest explanation for the observed cosmic acceleration is that dark energy is a cosmological constant (although it is many orders smaller than expected based on known physics), and that gravity is not modified. Figure 1.6 shows constraints on the dark energy equation of state $w_X(a) = w_0 + w_a(1-a)$ (Chevallier and Polarski, 2001), obtained by Wang and Mukherjee (2007) using the same data as in Figure 1.5. Figures 1.5 and 1.6 show that a cosmological constant is consistent with current observational data, although uncertainties are large. Wang (2008a,c) found that this remains true from an analysis of more recent observational data. For complementary approaches to analyzing current data, see, for example, Wang and Tegmark (2005), and Current Data Results references.

1.2 Fundamental Questions about Cosmic Acceleration

Dark energy projects aim to solve the mystery of cosmic acceleration. In terms of observables, the **two fundamental questions** that need to be answered by dark energy searches are:

1. Is dark energy density constant in cosmic time?
2. Is gravity modified?

These questions can be answered by the precise and accurate measurement of the dark energy density $\rho_X(z)$ as a function of cosmic time (or the expansion history of the universe $H(z)$), and the growth history of cosmic large scale structure $f_g(z)$ from observational data. The answer to these questions will tell us whether cosmic acceleration is caused by dark energy or a modification of gravity, and if gravity is not modified, whether dark energy is a cosmological constant, or due to a dynamical field.

Dark energy is often parameterized by a linear equation of state (Chevallier and Polarski, 2001)

$$w_X(a) = w_0 + w_a(1-a) \,. \tag{1.17}$$

Because of our ignorance of the nature of dark energy, it is important to make model-independent constraints by measuring the dark energy density $\rho_X(z)$ (or the expansion history $H(z)$) as a free function of cosmic time (Wang and Garnavich, 2001; Tegmark, 2002; Daly and Djorgovski, 2003). Measuring $\rho_X(z)$ has advantages over measuring the dark energy equation of state $w_X(z)$ as a free function; $\rho_X(z)$ is more closely related to observables, hence is more tightly constrained for the same number of redshift bins used (Wang and Garnavich, 2001; Wang and Freese, 2006). Note that $\rho_X(z)$ is related to $w_X(z)$ as follows (Wang and Garnavich, 2001):

$$\frac{\rho_X(z)}{\rho_X(0)} = \exp\left\{\int_0^z \mathrm{d}z' \frac{3\left[1+w_X(z')\right]}{1+z'}\right\} . \tag{1.18}$$

Hence parametrizing dark energy with $w_X(z)$ implicitly assumes that $\rho_X(z)$ does not change sign in cosmic time. This precludes whole classes of dark energy models in which $\rho_X(z)$ becomes negative in the future ("Big Crunch" models, see Linde (1987); Wang *et al.* (2004) for an example) (Wang and Tegmark, 2004).

If the present cosmic acceleration is caused by dark energy, then

$$E(z) \equiv \frac{H(z)}{H_0} = \left[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_X X(z)\right]^{1/2} \,, \tag{1.19}$$

which generalizes Eq. (1.11) by replacing Ω_Λ with $\Omega_X X(z)$, with the dark energy density function $X(z) \equiv \rho_X(z)/\rho_X(0)$. For a cosmological constant, $X(z) = 1$. Once $E(z)$ is specified, the evolution of matter density perturbations on large scales, $\delta_m^{(1)}(\boldsymbol{x}, t) = D_1(t)\delta_m(\boldsymbol{x})$, is determined by solving the following equation (assuming that dark energy perturbation $\delta_X = 0$):

$$D_1'' + 2E(z)D_1' - \frac{3}{2}\Omega_m(1+z)^3 D_1 = 0 \,, \tag{1.20}$$

where $D_1 = \delta_m^{(1)}(\boldsymbol{x}, t)/\delta_m(\boldsymbol{x})$, and primes denote $\mathrm{d}/\mathrm{d}(H_0 t)$. Or, more conveniently:

$$a^2 E^2 D_1''(a) + \left[a^2 E \frac{\mathrm{d}E}{\mathrm{d}a} + 3aE^2\right] D_1'(a) - \frac{3}{2}\Omega_m \frac{D_1}{a^3} = 0 \,, \tag{1.21}$$

where primes denote $\mathrm{d}/\mathrm{d}a$. The usual initial condition is

$$D_1(a|a \to 0) = a \,. \tag{1.22}$$

The linear growth rate is defined as

$$f_g(z) \equiv \frac{\mathrm{d}\ln D_1}{\mathrm{d}\ln a} \,. \tag{1.23}$$

Note that we have assumed that dark energy and dark matter are separate (and that dark energy perturbations are negligible on scales of interest), which is true for the vast majority of dark energy models that have been studied in the literature. If dark energy and dark matter are coupled (a more complicated possibility), or if dark energy and dark matter are unified (unified dark matter models), Eq. (1.20) would need to be modified accordingly. Sandvik *et al.* (2004) found strong evidence for the separation of dark energy and dark matter by ruling out a broad class of so-called unified dark matter models. These models produce oscillations or exponential blowup of the dark matter power spectrum inconsistent with observations.

In the simplest alternatives to dark energy, the present cosmic acceleration is caused by a modification to general relativity. Such models can be tested by observational data, see for example, Modified Gravity (references with more details) for observational signatures of some modified gravity models. The only rigorously worked example is the DGP gravity model (Dvali, Gabadadze, and Porrati, 2000). The validity of the DGP model has been studied by Koyama (2007) and Song, Sawicki, and Hu (2007).

The DGP model can be described by a modified Friedmann equation:

$$H^2 - \frac{H}{r_0} = \frac{8\pi G\rho_m}{3} \,, \tag{1.24}$$

where $\rho_m(z) = \rho_m(0)(1+z)^3$. Solving the above equation gives

$$E(z) = \frac{1}{2}\left\{\frac{1}{H_0 r_0} + \sqrt{\frac{1}{(H_0 r_0)^2} + 4\Omega_m^0(1+z)^3}\right\} , \tag{1.25}$$

where Ω_m^0 and ρ_c^0 are defined by Eqs. (1.12) and (1.13), respectively. The added superscript "0" in Ω_m^0 denotes that this is the matter density fraction today in the DGP gravity model. Note that consistency at $z = 0$, $H(0) = H_0$ requires that

$$H_0 r_0 = \frac{1}{1 - \Omega_m^0} \,, \tag{1.26}$$

thus the DGP gravity model is parametrized by a single parameter, Ω_m^0.

For DGP gravity, the evolution of matter density perturbations are modified; this is a hallmark of modified gravity models. The linear growth factor in the DGP gravity model is given by Lue, Scoccimarro, and Starkman (2004), and Lue (2006):

$$D_1'' + 2E(z)D_1' - \frac{3}{2}\Omega_m(1+z)^3 D_1\left(1 + \frac{1}{3\alpha_{DGP}}\right) = 0 \,, \tag{1.27}$$

where primes denote $\mathrm{d}/\mathrm{d}(H_0 t)$, and

$$\alpha_{\mathrm{DGP}} = 1 - 2Hr_0\left(1 + \frac{\dot{H}}{3H^2}\right) = \frac{1 - 2H_0 r_0 + 2(H_0 r_0)^2}{1 - 2H_0 r_0} \,. \tag{1.28}$$

The dark energy model equivalent of the DGP gravity model is specified by requiring

$$\frac{8\pi G \rho_{\mathrm{de}}^{\mathrm{eff}}}{3} = \frac{H}{r_0} \,. \tag{1.29}$$

Equation (1.24) and the conservation of energy and momentum equation,

$$\dot{\rho}_{\mathrm{de}}^{\mathrm{eff}} + 3\left(\rho_{\mathrm{de}}^{\mathrm{eff}} + p_{\mathrm{de}}^{\mathrm{eff}}\right) H = 0 \,, \tag{1.30}$$

imply that (Lue, Scoccimarro, and Starkman, 2004; Lue, 2006)

$$w_{\mathrm{de}}^{\mathrm{eff}} = -\frac{1}{1 + \Omega_{\mathrm{m}}(a)} \,, \tag{1.31}$$

where

$$\Omega_{\mathrm{m}}(a) \equiv \frac{8\pi G \rho_{\mathrm{m}}(z)}{3H^2} = \frac{\Omega_{\mathrm{m}}^0 (1+z)^3}{E^2(z)} \,. \tag{1.32}$$

Note that

$$\Omega_{\mathrm{m}}(a|a \to 0) = 1 \,, \quad w_{\mathrm{de}}^{\mathrm{eff}}(a|a \to 0) = -0.5 \tag{1.33}$$

$$\Omega_{\mathrm{m}}(a|a \to 1) = \Omega_{\mathrm{m}}^0 \,, \quad w_{\mathrm{de}}^{\mathrm{eff}}(a|a \to 1) = -\frac{1}{1 + \Omega_{\mathrm{m}}^0} \,. \tag{1.34}$$

This means that the matter transfer function (which describes how the evolution of matter density perturbations depends on scale) for the dark energy model equivalent of a viable DGP gravity model ($\Omega_{\mathrm{m}}^0 < 0.3$ and $w \le -0.5$) is very close to that of the ΛCDM model at $k \gtrsim 0.001\,\mathrm{h\,Mpc}^{-1}$ (Ma *et al.*, 1999).

It is easy and straightforward to integrate Eqs. (1.20) and (1.27) to obtain $D_1(a)$, and thus $f_{\mathrm{g}}(z)$, for dark energy models and DGP gravity models, with the initial condition in Eq. (1.22), that is, $D_1(a|a \to 0) = a$ (which assumes that dark energy or modified gravity is negligible at sufficiently early times).

The measurement of $H(z)$ or $\rho_{\mathrm{X}}(z)$ allows us to determine whether dark energy is a cosmological constant. The measurement of $f_{\mathrm{g}}(z)$ allows us to determine whether gravity is modified. An ambitious SN Ia survey can provide measurement of $H(z)$ to a few percent in accuracy (Wang and Tegmark, 2005). An ambitious galaxy redshift survey can measure both $H(z)$ and $f_{\mathrm{g}}(z)$ to better than a few percent in accuracy (Wang, 2008b). An ambitious weak lensing survey can measure $r(z)$ (which gives $H(z)$ in an integral form) to a few percent in accuracy and the growth factor $G(z)$ [$G(z) \propto D_1(t)$] to several percent in accuracy (Knox, Song, and Tyson, 2006). All these surveys are feasible within the next decade if appropriate resources are made available.

We will discuss each of the major observational methods for probing dark energy (i.e., determining the cause of the observed recent cosmic acceleration) in detail in Chapters 4–8, the key instrumentation for dark energy experiments in Chapter 9, and the future prospects for probing dark energy in Chapter 10.