1 Introduction

1.1 Classification of Rotor Systems

In general, rotating machinery consists of disks of various shapes, shafts whose diameters change depending on their longitudinal position, and bearings situated at various positions.[1] In vibration analyses, such a complex rotor system is simplified and a suitable mathematical model is adopted. In this modeling process, we must know which parameters are important for the system.

Rotating machines are classified according to their characteristics as follows: If the deformation of the rotating shaft is negligible in the operating speed range, it is called a *rigid rotor*. If the shaft deforms appreciably at some rotational speed in the operating speed range, it is called a *flexible rotor*. We cannot determine to which of these categories the rotor system belongs by considering only its dimensions. In rotordynamics, the rotating speeds that produce resonance responses due to mass eccentricity are called *critical speeds*. The deformation of a rotor becomes highest in the vicinity of the critical speed. Therefore, the range of the rated speed relative to these critical speeds determines whether the rotor is rigid or flexible.

Figure 1.1 called a *critical speed diagram* shows variations of critical speeds and vibration modes versus the stiffness of the supports for a symmetrical rotor. The left part of this figure represents values for rotors that are supported softly. In the first and second modes, the rotors do not deform appreciably but the supporting parts deflect. In this case, the rotor is considered to be a rigid rotor. As the stiffness of the supports decreases, the natural frequency of these modes approaches zero. In the third mode, the rotor deforms and it is considered to be a flexible rotor. Depending on the type of mode to be discussed, the same system may be considered as a rigid or a flexible rotor. On the right part of this figure, deformation occurs in all three modes and therefore the rotor is considered to be flexible in every mode.

In some models, disks are considered to be rigid and the distributed mass of an elastic shaft is concentrated at the disk positions. Such a model is called a

1) Rotor is often used as the general term for the rotating part of a rotating machine. The opposite term is *stator*, which means the static part of the machine.

Linear and Nonlinear Rotordynamics: A Modern Treatment with Applications, Second Edition.
Yukio Ishida and Toshio Yamamoto.

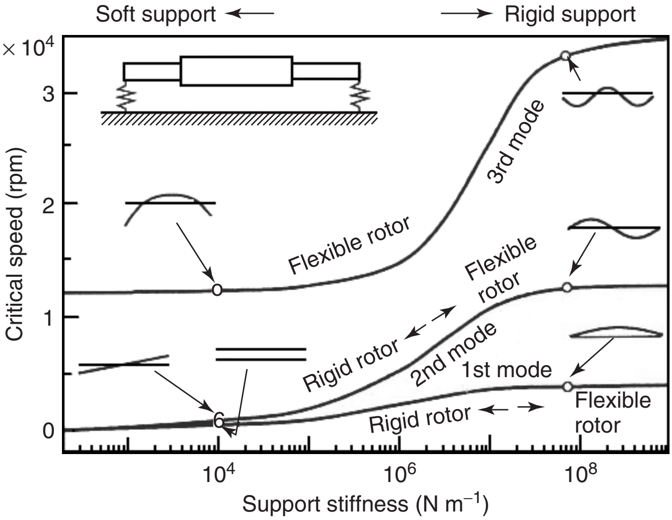


Figure 1.1 Critical speeds and mode shape versus the stiffness of the bearing support.

lumped-parameter system. If a flexible rotor with distributed mass and stiffness is considered, this model is called a *distributed-parameter system* or *continuous rotor system*. The mathematical treatment of the latter is more difficult than that of the former because it is governed by the partial differential equations.

Rotors are sometimes classified into vertical shaft systems and horizontal shaft systems. We mainly discuss the former model, however, the latter model tends to be considered in cases in which we must clarify the effect of gravity.

Rotors are sometimes classified as *high-speed rotors* or *low-speed rotors*. In this case, speed means the angular velocity or the peripheral velocity. Since high angular velocity often causes vibration, we use the term in association with the first definition in this book. The boundary between high and low is not clear and it differs depending on the situation. In some cases, the major critical speed is considered as the boundary. In ball bearing engineering, the term refers to the latter definition because it determines heating due to friction. The dimensionless parameter called *DN value* is used as an index related to the peripheral velocity. This value is defined as the product of the shaft diameter (mm) and the rotational speed (rpm). However, the unit symbol (mm · rpm) is omitted from the result of the calculation. Concerning high peripheral velocity, ball bearings and roller bearings of a main shaft of an aircraft engine have attained velocities of as much as $DN = 3 \times 10^6$ in a laboratory setting, though in practice these bearings are generally used at approximately 2.2×10^6 (Zaretsky, 1998). With regard to high angular velocity bearings, the spindle of a drill for dental use operates at approximately 50×10^4 rpm. However, since its shaft diameter is small, its DN value is consequently relatively small. For example, a bearing with an inner diameter of 3.175 mm used in dental drilling operates at approximately $DN = 1.6 \times 10^6$.

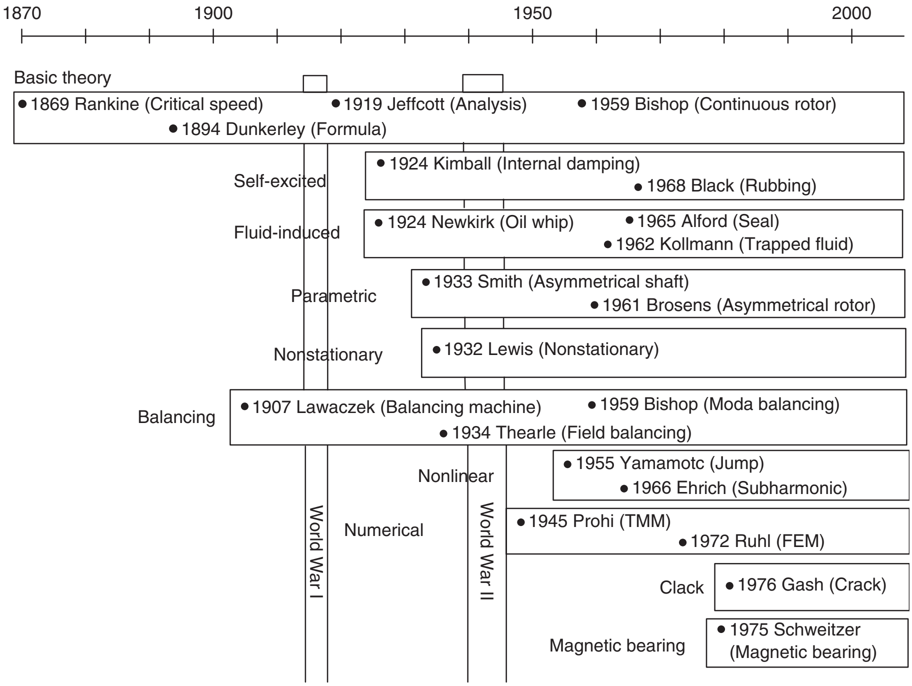


Figure 1.2 History of rotordynamics.

1.2 Historical Perspective

The evolution of research in the field of rotordynamics is shown in Figure 1.2. Research on rotordynamics spans at least a 140-year history, starting with Rankine's paper on whirling motions of a rotor in 1869. Rankine discussed the relationship between centrifugal and restoring forces and concluded that operation above a certain rotational speed is impossible. Although this conclusion was wrong, his paper (refer to the "Topic: The First Paper on Rotordynamics") is important as the first publication on rotordynamics. The research progressed significantly at the end of the nineteenth century with contributions by de Laval and others. De Laval, an engineer in Sweden, invented a one-stage steam turbine and succeeded in its operation. He first used a rigid rotor, but later used a flexible rotor and showed that it was possible to operate above the critical speed by operating at a rotational speed about seven times the critical speed (Stodola, 1924).

In the early days, the major concern for researchers and designers was to predict the critical speed, because the first thing that had to be done in designing rotating machinery was to avoid resonance. Dunkerley (1894) derived an empirical formula that gave the lowest critical speed for a multirotor system. He was the first to use the term "*critical speed*" for the resonance rotational speed. The word "*critical*," which refers to a state or a value at which an abrupt change in a quality or state occurs, was coined possibly based on Rankine's conclusion mentioned above. Holzer (1921) proposed an approximate method to calculate the natural frequencies and mode shapes of torsional vibrations.

The first recorded fundamental theory of rotordynamics can be found in a paper written by Jeffcott (1919) . We can appreciate Jeffcott's great contributions if we recall that a shaft with a disk at the midspan is called the *Jeffcott rotor*, especially among researchers in the United States. This simplified fundamental rotor system is also called the *Laval rotor*, named after de Laval.

The developments made in rotordynamics up to the beginning of the twentieth century are detailed in the masterpiece written by Stodola (1924). This superb book explains nearly the entire field related to steam turbines. Among other things, this book includes the dynamics of elastic shafts with disks, the dynamics of continuous rotors without considering the gyroscopic moment, the balancing of rigid rotors, and methods for determining approximate values of critical speeds of rotors with variable cross sections.

Thereafter, the center of research shifted from Europe to the United States, and the scope of rotordynamics expanded to consider various other phenomena. Campbell (1924) at General Electric investigated vibrations of steam turbines in detail. His diagram, representing critical speed in relation to the cross points of natural frequency curves and the straight lines proportional to the rotational speed, is now widely used and referred to as the *Campbell diagram*. As the rotational speed increased above the first critical speed, the occurrence of self-excited vibrations became a serious problem. In the 1920s, Newkirk (1924) and Kimball (1924) first recognized that internal friction of shaft materials could cause an unstable whirling motion. These phenomena, in which friction that ordinarily dampens vibration causes self-excited vibration, attracted the attention of many researchers. Newkirk and Taylor (1925) investigated an unstable vibration called *oil whip*, which was due to an oil film in the journal bearings. Rotor is generally surrounded by a stator such as seals with a small clearance. Newkirk (1926) showed a forward whirl induced by a hot spot on the rotor surface, which was generated by the contact of the rotor and the surroundings. This hot spot instability is called the *Newkirk effect*.

About a decade later, the study of asymmetrical shaft systems and asymmetrical rotor systems began. The former are systems with a directional difference in shaft stiffness, and the latter are those with a directional difference in rotor inertia. Two pole generator rotors and propeller rotors are examples of such systems. As these directional differences rotate with the shaft, terms with time-varying coefficients appear in the governing equations. These systems therefore fall into the category of parametrically excited systems. The most characteristic property of asymmetrical systems is the appearance of unstable vibrations in some rotational speed ranges. Smith (1933)'s report is a pioneering work on this topic. Various phenomena related to the asymmetries of rotors were investigated actively in the middle of the twentieth century by Taylor (1940) and Foote, Poritsky, and Slade (1943), Brosens and Crandall (1961), and Yamamoto and Ota (1963a, 1963b, 1964).

Nonstationary phenomena during passage through critical speeds have been studied since Lewis reported his investigation on the Jeffcott rotor in 1932.

Numerous reports on this topic are classified into two groups. One group classifies nonstationary phenomena that occur in a process with a constant acceleration and the other classifies phenomena that occur with a limited driving torque. In the latter case, mutual interaction between the driving torque and the shaft vibration must be considered. As the theoretical analysis of such transition problems is far more difficult than that of stationary oscillations, many researchers adopted numerical integrations. The asymptotic method developed by the Russian school of Krylov and Bogoliubov (1947) and Bogoliubov and Mitropol'skii (1958) considerably boosted the research on this subject.

The vibrations of rotors with continuously distributed mass were also studied. The simplest continuous rotor model corresponding to the Euler beam was first studied in the book by Stodola (1924). In the 1950s and 1960s, Bishop (1959), Bishop and Gladwell (1959), and Bishop and Parkinson (1965) reported a series of papers on the unbalance response and the balancing of a continuous rotor. Eshleman and Eubanks (1969) derived more general equations of motion considering the effects of rotary inertia, shear deformation, and gyroscopic moment, and investigated these effects.

The most important and fundamental procedure to reduce unfavorable vibrations is to eliminate geometric imbalance in the rotor. The balancing technique for a rigid rotor was established relatively early. A practical balancing machine based on this technique was invented by Lawaczeck in 1907 (Miwa and Shimomura, 1976). In 1925, Suehiro invented a balancing machine that conducts balancing at a speed in the postcritical speed range (Miwa and Shimomura, 1976). And in 1934, Thearle developed the two-plane balancing (Thearle, 1934). The arrival of high-speed rotating machines made it necessary to develop a balancing technique for flexible rotors. Two representative theories were proposed. One was *the modal balancing method* proposed in the 1950s by Federn (1957) and Bishop and Gladwell (1959). The other was *the influence coefficient method* proposed in the early 1960s and developed mainly in the United States alone with the progress of computers. Goodman (1964) improved this method by taking into the least square methods.

In the latter half of the twentieth century, various vibrations due to fluid were studied. The above-mentioned oil whip is a representative flow-induced vibration of rotors. In the middle of the twentieth century, Hori (1959) succeeded in explaining various fundamental characteristics of oil whip by investigating the stability of shaft motion and considering pressure forces due to oil films. At almost the same time, other types of flow-induced vibrations attracted the attention of many researchers. Seals that are used to reduce the leakage of working fluids through the interface between rotors and stators sometimes induce unstable vibrations. In 1964, Alford reported accidents due to labyrinth seals. Another one was a self-excited vibration called *the steam whirl*. The mechanism of this vibration in turbines was explained by Thomas (1958) and that in compressors was explained by Alford (1965). These phenomena are still attracting the interest of many researchers for practical importance. The vibration of a hollow rotor containing fluid is a relatively new problem of flow-induced vibrations. In 1967, Ehrich reported that fluid trapped in engine shafts induced asynchronous vibrations. A noteworthy paper on this phenomenon is that of Wolf (1968). He succeeded in explaining the appearance of an unstable speed range in a postcritical region of a rotor system containing inviscid fluid.

As rotors became lighter and their operational speeds higher, the occurrence of nonlinear resonances such as subharmonic resonances became a serious problem. Yamamoto (1955, 1957a) studied various kinds of nonlinear resonances after he reported on subharmonic resonances due to ball bearings, in 1955. He discussed systems with weak nonlinearity that can be expressed by a power series of low order. Aside from subharmonic resonances, he also investigated combination resonances (he named them summed-and-differential harmonic oscillations) and combination tones. In the 1960s, Tondl (1965) studied nonlinear resonances due to oil films in journal bearings. Ehrich (1966) reported subharmonic resonances observed in an aircraft gas turbine with squeeze-film damper bearings. The cause of strong nonlinearity in aircraft gas turbines is the radial clearance of squeeze-film damper bearings. Later, Ehrich (1988, 1991) reported the occurrence of various types of subharmonic resonances up to a very high order and also chaotic vibrations in practical engines.

In the practical design of rotating machinery, it is necessary to know accurately the natural frequencies, modes, and forced responses to unbalances in complex-shaped rotor systems. The representative techniques used for this purpose are the transfer matrix method and the finite-element method. Prohl (1945) used the transfer matrix method in the analysis of a rotor system by expanding the method originally developed by Myklestad (1944). This analytical method is particularly useful for multirotor-bearing systems and has developed rapidly since the 1960s by the contribution of many researchers such as Lund and Orcutt (1967) and Lund (1974). The finite-element method was first developed in structural dynamics and then used in various technological fields. The first application of the finite-element method to a rotor system was made by Ruhl and Booker (1972). Then, Nelson and McVaugh (1976) generalized it by considering rotating inertia, gyroscopic moment, and axial force.

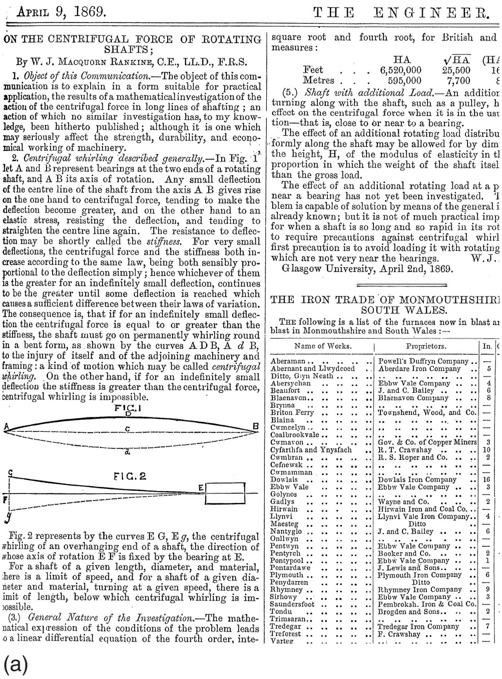
From the 1950s, cracks were found in rotors of some steam turbines (Ishida, 2008). To prevent serious accidents and to develop a vibration diagnosis system for detecting cracks, research on vibrations of cracked shafts began. In the 1970s, Gasch (1976) and Henry and Okah-Avae (1976) investigated vibrations, giving consideration to nonlinearity in stiffness due to open–close mechanisms. They showed that an unstable region appeared or disappeared at the major critical speed, depending on the direction of the unbalance. The research is still being developed and various monitoring systems have been proposed.

The latest topics in rotordynamics are magnetic bearings that support a rotor without contacting it and active control. This study has received considerable attention since Schweitzer (1975) reported his work in 1975. Nonami (1985) suppressed an unbalance response of a rotor controlling the bearing support actively using the optimal regulator theory.

In this chapter, the history of rotordynamics has been summarized briefly. The authors recommend the readers to read excellent introductions on the history of rotordynamics by Miwa (1991), Dimarogonas (1992), and Nelson (1994, 1998). The following are representative books on rotordynamics, some of which have detailed bibliographies: Stodola (1924), Kimball (1932), Biezeno and Grammel (1939), Dimentberg (1961), Tondl (1965), Gunter (1966), Roewy and Piarulli (1969), Eshleman, Shapiro, and Rumabarger (1972), Gasch and Pfutzner (1975), Miwa and Shimomura (1976), Dimarogonas and Paipetis (1983), Vance (1988), Darlow (1989), Lalanne and Ferraris (1990), Zhang (1990), Rao (1991), Ehrich (1992), Lee (1993), Childs (1993), Kramer (1993), Gasch, Nordmann, and Pfützner (2002), Genta (2005), Muszynska (2005), and Bachschmid, Pennacchi, and Tanzi (2010).

Topic: The First Paper on Rotordynamics

The first paper on rotordynamics is attributed to William John Macquorn Rankine (1820–1872), a professor at the University of Glasgow. He is well known as a scientist and engineer, particularly for his development of the Rankine cycle in the theory of heat engines. However, his interests were extremely varied. In 1869, he wrote an article entitled "On the Centrifugal Force of Rotating Shafts" in "The Engineer." Figure 1.3 shows a part of this article. He published his analysis of spinning shafts and wrote as follows: "For a shaft of a given length, a diameter, and material, there is a limit of


APRIL 9, 1869. THE ENGINEER.

ON THE CENTRIFUGAL FORCE OF ROTATING SHAFTS;

By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.S.

1. *Object of this Communication.*—The object of this communication is to explain in a form suitable for practical application, the results of a mathematical investigation of the action of the centrifugal force in long lines of shafting; an action of which no similar investigation has, to my knowledge, been hitherto published; although it is one which may seriously affect the strength, durability, and economical working of machinery.

2. *Centrifugal whirling described generally.*—In Fig. 1 let A and B represent bearings at the two ends of a rotating shaft, and A B its axis of rotation. Any small deflection of the centre line of the shaft from the axis A B gives rise on the one hand to centrifugal force, tending to make the deflection become greater, and on the other hand to an elastic stress, resisting the deflection, and tending to straighten the centre line again. The resistance to deflection may be shortly called the *stiffness*. For very small deflections, the centrifugal force and the stiffness both increase according to the same law, being both sensibly proportional to the deflection simply; hence whichever of them is the greater for an indefinitely small deflection, continues to be the greater until some deflection is reached which causes a sufficient difference between their laws of variation. The consequence is, that if for an indefinitely small deflection the centrifugal force is equal to or greater than the stiffness, the shaft must go on permanently whirling round in a bent form, as shown by the curves A D B, A *d* B, to the injury of itself and of the adjoining machinery and framing: a kind of motion which may be called *centrifugal whirling*. On the other hand, if for an indefinitely small deflection the stiffness is greater than the centrifugal force, centrifugal whirling is impossible.

FIG. 1

FIG. 2

Fig. 2 represents by the curves E G, E *g*, the centrifugal whirling of an overhanging end of a shaft, the direction of whose axis of rotation E F is fixed by the bearing at E.

For a shaft of a given length, diameter, and material, there is a limit of speed, and for a shaft of a given diameter and material, turning at a given speed, there is a limit of length, below which centrifugal whirling is impossible.

(3.) *General Nature of the Investigation.*—The mathematical expression of the conditions of the problem leads to a linear differential equation of the fourth order, inte-

square root and fourth root, for British and measures:

	HA	√HA	(HA
Feet . . .	6,520,000	25,500	16
Metres . . .	595,000	7,700	8

(5.) *Shaft with additional Load.*—An additior turning along with the shaft, such as a pulley, h effect on the centrifugal force when it is in the usu tion—that is, close to or near to a bearing.

The effect of an additional rotating load distribu formly along the shaft may be allowed for by dim the height, H, of the modulus of elasticity in tl proportion in which the weight of the shaft itsel than the gross load.

The effect of an additional rotating load at a p near a bearing has not yet been investigated. T blem is capable of solution by means of the general i already known; but it is not of much practical imp for when a shaft is so long and so rapid in its rot to require precautions against centrifugal whirl first precaution is to avoid loading it with rotating which are not very near the bearings. W. J.

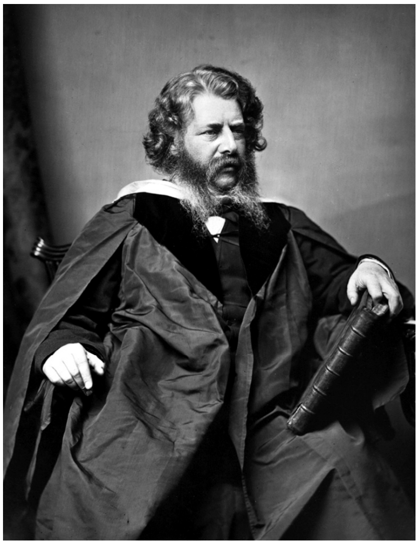
Glasgow University, April 2nd, 1869.

THE IRON TRADE OF MONMOUTHSHIRE SOUTH WALES.

THE following is a list of the furnaces now in blast ar blast in Monmouthshire and South Wales:—

Name of Works.	Proprietors.	In.
Aberaman	Powell's Duffryn Company	—
Abernant and Llwydcoed	Aberdare Iron Company	5
Ditto, Glyn Neath		—
Abersychan	Ebbw Vale Company	4
Beaufort	J. and C. Bailey	6
Blaenavon	Blaenavon Company	8
Brynna		—
Briton Ferry	Townshend, Wood, and Co.	—
Blaina		—
Cwmcelyn		—
Coalbrookvale		—
Cwmavon	Gov. & Co. of Copper Miners	3
Cyfarthfa and Ynysfach	R. T. Crawshay	10
Cwmbran	R. S. Roper and Co.	2
Cefnewsk		—
Cwmamman		—
Dowlais	Dowlais Iron Company	16
Ebbw Vale	Ebbw Vale Company	3
Golynos		—
Gadlys	Wayne and Co.	2
Hirwain	Hirwain Iron and Coal Co.	—
Llynvi	Llynvi Vale Iron Company	4
Maesteg	Ditto	—
Nantyglo	J. and C. Bailey	6
Onllwyn		—
Pentwyn	Ebbw Vale Company	—
Pentyrch	Booker and Co.	2
Pontypool	Ebbw Vale Company	1
Pontardawe	J. Lewis and Sons	—
Plymouth	Plymouth Iron Company	6
Penydarren	Ditto	—
Rhymney	Rhymney Iron Company	9
Sirhowy	Ebbw Vale Company	3
Saundersfoot	Pembroksh. Iron & Coal Co.	—
Tondu	Brogden and Sons	2
Trimsaran		—
Tredegar	Tredegar Iron Company	7
Treforest	F. Crawshay	—
Varteg		—

(a)



(b)

Figure 1.3 Rankine and his paper. (a) The first article on rotordynamics (Rankine, (1869)). (b) Portrait of Rankine. (Courtesy of the University of Glasgow.)

speed, and for a shaft of a given diameter and material, turning a given speed, there is ***a limit of length***, below which centrifugal whirling is impossible.'' This limit of length corresponds to the critical speed and he gave the correct formula calculating critical speed. Although his prediction that the supercritical operation is impossible is not correct, his analysis is still worth mentioning.

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