1.1 Introduction

All energy resources on earth have come from the sun, including the fossil fuel deposits that power our civilization at present. Plants grew by photosynthesis starting in the carboniferous era, about 300 million years ago, and the decay of some of these, instead of oxidizing back into the atmosphere, occurred underground in oxygen-free zones. These anaerobic decays did not release the carbon, but reduced some of the oxygen, leading to the present deposits of oil, gas, and coal. These deposits are now being depleted on a 100-year timescale, and will not be replaced. Once these accumulated deposits are depleted, no quick replenishment is possible. The energy usage will have to reduce to what will be available in the absence of the huge deposits. The words sustainable and renewable apply to this vision of the future.

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There is clear evidence that the amount of available oil is limited, and is distributed only to depths of a few miles. The geology of oil very clearly indicates limited supplies. It is agreed that the continental U.S. oil supplies have mostly been depleted. Deffeyes (Deffeyes, K. (2001) "Hubbert's Peak" (Princeton Univ. Press, Princeton) authoritatively and clearly" explains that liquid oil was formed over geologic time in favored locations and only in a "window" of depths between 7500 and 15 000 feet, roughly 1.5–3 miles. (At depths more than 3 miles the temperature is too high to form liquid oil from biological residues, and natural gas forms). The limited depth and the extremely long time needed to form oil from decaying organic matter (it only occurs in particular anaerobic, oxygen-free locations, otherwise the carbon is released as gaseous carbon dioxide), support the nearly obvious conclusion that the world's accessible oil is going to run out, certainly on a timescale of 100 years.

Furthermore, scientists increasingly agree that accelerated oxidation of the coal and oil that remain, as implied by the present energy use trajectory of advanced and emerging economies, is fouling the atmosphere. Increased combustion contributes to changes in the composition of the rather slim atmosphere of the earth in a way that will alter the energy balance and raise the temperature on the earth's surface. Dramatic loss of glaciers is widely noted, in Switzerland, in the Andes Mountains, and in the polar icecaps, which relates to sea-level rises.

New sources of energy to replace depleting oil and gas are needed. The new energy sources will stimulate changes in related technology. An increasing premium will probably be placed on new sources and methods of use that limit emission of gases that tend to trap heat in the earth's atmosphere. New emphasis is surely to be placed on efficiency in areas of energy generation and use. Conservation and efficiency are admired goals that are being reaffirmed.

All energy comes from the sun, from the direct radiation, from the indirectly resulting winds and related hydroelectric and wave energy possibilities. These sources are considered renewable, always available. Fuels resulting from long eras of sunlight, including deposits of coal, oil, and natural gas, are nonrenewable. These resources are depleting on time scales of decades to centuries. Solar radiation is the renewable energy source that is most obviously an opportunity at present to fill the shortfall in energy.

Solar energy, while the basic source of all energy on earth, presently provides only a tiny fraction of utilized energy supply. Global energy usage (global power consumption from all sources) has been estimated as available from the solar radiation falling on 1% of the earth's desert areas. Hence, from a rational and technical point of view there need never be a lack of energy. In recent years, the oil price has been on the order of \$100 per barrel, with predictions of prices much higher than the recent peak of \$147 per barrel in the span of several years. From the geological point of view, the world's supply of oil is finite, and there is some consensus that in the past 100 years nearly half of it has been used. A long-term energy perspective must be based on long-term resources, and oil is not a long-term resource on a 100-year basis.

Solar energy conversion has aspects in which electronic processes are important, and for that reason this is a major topic in our book. Direct photovoltaic conversion of light photons into electron–hole pairs and into electrons traversing an external circuit is one topic of interest. The second topic, direct absorption of photons to split water into hydrogen and oxygen, will be discussed. Other permanent energy sources, which are by-products of solar energy, for instance, windpower, hydropower, and power extracted from ocean waves, do not depend in any strong way on the microscopic and nanoscopic physical processes that are the focus of our book. A key part of our book along this vein is on nuclear fusion energy, a proven resource on the sun, whose reactions are well understood. We will look carefully at several approaches to using the effectively infinite supply of deuterium in the ocean. We need technology on earth to convert the deuterium to helium as occurs on the sun, the supply of deuterium if converted to energy would supply the energy needs of our civilization for millions of years.

There are some who raise alarm at the "dangerous" suggestions that our energydependent civilization could be reorganized to run only on the renewable forms of energy. These observers overlap those who deny that the existing supplies of oil and coal are strictly limited, and who refuse to address the future beyond such depletions.

The strong basis for such a fear is the overwhelming dependence at present on the fossil fuels, oil, coal, and natural gas, with small amounts of hydroelectric power and nuclear power. On charts, the present consumption levels from solar power,

windpower, geothermal power, wave and tidal power, are too small to be seen on the same scales.

Energy can be expressed as power times time, one kWh (kilowatt hour) is $1000 \times 3600 = 3.6 \times 10^6 \text{ J} = 3.6 \times 10^6 \text{ Ws}$. The BTU, British thermal unit, is 1054 J, and the less familiar "Quad" = 10^{15} BTU is thus 1.054×10^{18} J. It is stated below that the U.S. energy consumption was 94.82 Quads in 2009. In terms of average power, since a year is $365 \times 24 \times 3600 \text{ s} = 3.15 \times 10^7 \text{ s}$, this 3.17 TW. (This amounts to about 21.6% of global power, while one may note that U.S. population of 311 million is only 4.4% of the global population at 7 billion).

According to the BP Statistical Review of World Energy June 2010, the world's equivalent total power consumption in 2008 was 14.7 TW (see Figure 1.1). The largest sources in order are oil, coal, and natural gas, with hydroelectric accounting for 1.1 TW and nuclear about 0.7 TW, about 7.3 and 4.5%, respectively. Renewable power such as solar and wind are not tabulated by BP, but are clearly almost negligible on the present scale of fossil fuel power consumptions.

More details of the 2009 power consumption in the United States, breaking out the renewable energy portions, are shown in Figure 1.2.

Although the renewable energy portions are at present small, they are clearly in rapid growth. To get an idea of the growth, we find from reasonable sources



Figure 1.1 Global consumed power (based on BP Statistical Review of World Energy June 2010). The smallest band is nuclear, about 0.66 TW, and next smallest is hydroelectric, about 1.07 TW. (This is also referred to as TPES, total primary energy supply.) The largest in order are oil, coal, and natural gas, accounting for

about 88.2% of all energy consumption. Astute observers agree that the three leading sources shown here are likely to significantly decrease in the next century, as prices rise due to depletion of easily available sources. 3



Figure 1.2 Energy consumed in United States in 2009 totals to 94.82 Quads = 9.99×10^{19} J. Of this figure, 8.16% (7.745 Quads) is classified as renewable, as broken out on the right. In the renewable category, wind accounts for 9%, thus

only 0.7% of the total U.S. power consumption. (U.S. Energy Information Administration/ Renewable Energy Consumption and Electricity, Preliminary Statistics, 2009).

("Renewables 2011: Global Status Report" http://www.ren21.net/Portals/97/documents/GSR/GSR2011_Master18.pdf, see also http://www.aps.org/units/gera/meetings/march10/upload/CarlsonAPS3-14-10.pdf and "Global Trends in Renewable Energy Investment 2011" (Bloomberg New Energy Finance) available at http:// fs-unep-centre.org/publications/global-trends-renewable-energy-investment-2011.) estimates that in 2010 installed windpower capacity worldwide is 198 GW and growing at 30% per year. If this rate continues (which is not assured), it will be less than 20 years from 2010 until windpower reaches 5 TW, the present power from coal. This can thus be crudely extrapolated to happen by 2030. In a similar vein, in 2010 installed photovoltaic PV capacity is 40 GW and increasing at 43% per year. On this basis, it will take 13.5 years from 2010 to reach 5 TW, thus estimated in 2024.

These are long extrapolations, inherently uncertain in their accuracy. One may question that a 5 TW level from windpower is attainable from the point of view of land area and suitable sites, apart from capital investment, grid linkage and storage issues. The limiting capacities are not easy to estimate. However, one detailed study of China [1], based on windspeed data, predicted that installation of 1.5 MW turbines on mainland China could provide up to 24.7 PWh of electricity annually, which works out to an average power of 2.82 TW. This suggests that 5 TW wind capacity worldwide may be achievable. On the other hand, the *New York Times* [2] has recently published an analysis of power investment in China and finds that coal is by far the largest and most rapidly growing source of energy, and that windpower capacity is scarcely increasing.

Estimates of the power potentially available from direct photovoltaic conversion are straightforward. To reach 5 TW, assuming an average power density of 205 W/m² with 10% efficient solar cells requires an area $(5 \times 10^{12}/20.5) \text{ m}^2 = 2.44 \times 10^{11} \text{ m}^2$

that would be 493.8 km on a side. This area, compared to the area of the Sahara desert, 9×10^6 km², is 2.7%.

A detailed plan for providing renewable power to Europe has been given by Czisch. This comprehensive plan finds that transmission lines are essential to a plan that can power all of Europe at similar to present rates, without coal or oil as source (http://www.iset.uni-kassel.de/abt/w3-w/projekte/WWEC2004.pdf Dr. G. Czisch, "Low cost but totally renewable electricity supply for a huge supply area: a european/transeuropean example" (http://www2.fz-juelich.de/ief/ief-ste//datapool/steforum/Czisch-Text.pdf).).

The data in Figures 1.1 and 1.2 should be regarded as accurate numbers, and this total consumption is reasonably extrapolated to double by 2050 and triple by 2100. To make a difference in the global energy pattern, any new source has to be on the scale of 1–5 TW, on a long timescale. The total geothermal power at the earth's surface is estimated as 12 TW, only a small portion extractable. It is said that total untapped hydroelectric capacity is 0.5 TW and total power from waves and tides is less than 2 TW. These latter estimates are not so certain. See "Basic Research Needs for Solar Energy Utilization," Report of the Basic Energy Sciences Workshop on Solar Energy Utilization, April 18–21, 2005, U.S. Department of Energy.

An overview of the potential renewable energy sources in the global environment has been offered by Richter. The numbers in Table 1.1 are totals and do not indicate what fractions may be extractable.

These numbers do not reflect any estimate of what portion may be extractable. Thus, Figure 1.1 indicates 1.07 TW global hydroelectric power, which is far short of 7 TW in this table for river flow energy, and elsewhere it is estimated that untapped hydroelectric power is only 0.5 TW. Such an estimate probably does not consider the potential for water turbines, analogous to wind turbines, in worldwide rivers (based on Table 8.1, Richter [3]).

Our interest is in the science and technology of long-term solutions to energy production, with emphasis on the aspects that are addressed by nanophysics, or quantum physics. Quantum physics is needed to understand the energy release in the sun and in nuclear fusion reactors such as Tokamaks on earth, and also to understand photovoltaic cells and related devices. It seems sensible to describe these

14.7
30 500
840
56
3.5
32.2
91
7

 Table 1.1
 Global natural power sources in terawatts (adapted from Ref. [3]).

a) Solar input onto land area assuming 205 W/m².

processes as nanophysics, the physics that applies on the size scale of atoms and small nuclei, such as protons, deuterons, and ³He. Needed also are basic aspects of materials including plasmas and semiconductors. Our hope is to provide a basic picture based on Schrodinger's equation with enough details to account for nuclear fusion reactions in plasmas and photovoltaic cells in semiconductors. From our point of view, oil, gas, coal, and nuclear fission materials are not renewable sources of energy because of the short timescales for their depletion. We focus on the energy that comes from the sun, directly as radiation, and indirectly on earth in the form of winds, waves, and hydroelectric power.

Beyond this, we consider the vast amounts of deuterium in the oceans as a sustainable source of energy, once we learn how to make fusion reactors work on earth. The heat energy in the earth, geothermal energy, is renewable but its overlap with nanophysics is not large. In a similar vein, the energy of tidal motions, which is extracted from the orbital energy of the moon around the earth, is a long-term source, but it is not strongly related to nanophysics.

The main opportunities for nanophysics are in photovoltaic cells and related devices, aspects of energy storage, and in various approaches toward fusion based on deuterium and possibly lithium. We want to learn about the nanophysical nuclear fusion energy generation in the sun for its own importance, as an existence proof for fusion, and also as a guide to how controlled fusion might be accomplished on earth.

1.1.1

Direct Solar Influx

The primary energy source for earth over billions of years has been the radiation from the sun. The properties of the sun, including its composition and energy generation mechanisms, are now known, as a result of years of research. Our purpose here is to summarize modern knowledge of the sun, with the intention of showing how the energy production of the sun requires a quantum mechanical view of the interactions of particles such as protons and neutrons at small distance scales. The Schrodinger equation, needed for understanding the rather simple tunneling processes that must occur in the sun, will be used later to get a working understanding of atoms, molecules, and solids such as semiconductors.

1.1.1.1 Properties of the Sun

The mass of the sun is $M = 1.99 \times 10^{30}$ kg, its radius $R_{\rm s} = 0.696 \times 10^{6}$ km, at distance $D_{\rm es}$ about 93 million miles (1.496×10^{8} km) from earth. The sun's composition by mass is approximately 73.5% hydrogen and 24.9% helium, plus a distribution of light elements up to carbon. The sun's surface temperature is 5778–5973 K, while the sun's core temperature is estimated as 15.7×10^{6} K. (Much of the data for the sun have been taken from "Principles of Stellar Evolution and Nucleosynthesis" by Donald D. Clayton (University of Chicago, 1983) and "Sun Fact Sheet" by D. R. Williams (NASA, 2004)).

We are interested in the energy input to the earth by electromagnetic radiation, traveling at the speed of light, from the sun. A measurement is shown in Figure 1.3



Figure 1.3 Directly measured solar energy spectrum, from 200 to 2400 nm, from a satellite-carried spectrometer just above the earth's atmosphere. The units are related to energy, mW/m² nm, and the area under this curve

should be close to 1366 W/m². Note that the peak here is close to 486 nm, corresponding to a black body at 5973 K. The portion of this spectrum beyond about 700 nm cannot be seen, but represents infrared heat radiation [4].

obtained in the near vacuum above the earth's atmosphere. The curve closely fits the Planck radiation law,

$$u(\nu) = [8\pi h\nu^3/c^3] [\exp(h\nu/k_{\rm B}T) - 1]^{-1}, \qquad (1.1)$$

where $h = 6.6 \times 10^{-34}$ J s, $k_{\rm B} = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and the Kelvin temperature *T* is 5973 K. This is the Planck thermal *energy* density, units Joules per (Hz m³), describing the spectrum of black body radiation as a function of the frequency ν in Hertz. Equation 1.1 is the product of the number of electromagnetic modes per Hertz and per cubic meter at frequency ν , the energy per mode, and the chance that the mode is occupied. The *power* density is obtained by multiplying by c/4, where $c = 2.998 \times 10^8$ m/s is the speed of light. The Planck function is alternatively expressed in terms of wavelength through the relation $\nu = c/\lambda$.

Integrating this energy density over frequency and multiplying by c/4 leads to the Stefan–Boltzmann law for the radiation energy per unit time and per unit area from a surface at temperature *T*, which is

$$dU/dt = Uc/4 = \sigma_{\rm SB} T^4, \cdots \sigma_{\rm SB} = 2\pi^5 k_{\rm B}^4 / (15 \, h^3 \, c^2) = 5.67 \times 10^{-8} \, {\rm W/m^2 K^4}$$
(1.2)

The wavelength distribution of "black body radiation" peaks at wavelength λ_m such that $\lambda_m T = \text{constant} = 2.9 \text{ mm K}$. The value of $\lambda_m = 486 \text{ nm}$ for the solar spectrum

is in the visible corresponding to $T \approx 5973$ K. (The sharp dips seen in Figure 1.1 attest to the wavelength resolution of the measurement, but are not central to our question of the energy input to earth. These dips are atomic absorption lines presumably from simple atoms and ions in the atmosphere surrounding the sun).

A related aspect of the radiation is the pressure it exerts, which is $U/3 = (4/3 c) \sigma_{SB} T^4$. It is estimated that the temperature at the center of the sun is 1.5×10^7 K, which corresponds to radiation pressure $[4/(3 \times 3 \times 10^8)] \text{ s/m} 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 (1.5 \times 10^7 \text{ K})^4 = 0.126 \text{ Gbar}$, where 1 bar = 101 kPa. This is large but a small part of the total hydrostatic pressure of 340 Gbar at the center of the sun.

The area under this curve measured above the earth's atmosphere represents 1366 W/m² available at all times (and over billions of years). A fraction, α (the albedo, about $\alpha = 0.3$), of this is reflected back into space. However, if we take the radius of the earth as 6371 km, then the power intercepted, neglecting α , is 1.74×10^{17} W = 174 PW (petawatts). By comparison, the worldwide power consumption, for all purposes, in 2008 was 14.7 TW, and the average total electric power usage in the United Sates in 2004 was 460 GW [5], which is only 26 parts per million (ppm) of the solar energy flux! If there are 7 billion people on the earth, this power is 24,900 kW per person. On the basis of 460 GW and 294 million persons in the United States (in 2004), the electrical power usage per person works out as 14.7 TW/7 billion = 2.10 kW per person.

There is thus a vast flow of energy coming from space, even after we correct for the reflected light (albedo), and the absorption effects in the atmosphere. The question of whether it can be harvested for human consumption is related to its dilute nature. At ground level in the United States, an average solar power density is about 205 W/m². For example, an auto at 200 HP corresponds to 200×746 watts = 14 920 W, and would require a collection area 73 m², much bigger than a solar panel that could be put on the roof of the car. To supply the whole country, at a conversion efficiency of 20%, a surface area of dimension about 65 miles would provide 460 GW, leaving open questions of overnight storage of energy and distribution of the energy.

The challenge is to turn the incoming solar flux (and/or other secondary sources of sun-based energy, like the wind and hydroelectric power) into usable energy on the human level. In advanced societies, it represents energy for transportation, presently indicated by the price per gallon of gasoline, and the cost per kWh of electricity.

Our second interest, in a book that focuses on nanophysics or quantum physics, that applies to objects and devices on a size scale below 100 nm or so, is to learn something about how the sun releases its energy, and to think of ways we might create a similar energy generation on earth.

The spectrum in Figure 1.3 closely resembles the shape of the Planck black body radiation spectrum, plotted versus wavelength, for 5973 K. This spectrum was measured in vacuum above the earth's atmosphere, and directly measures the huge amount of energy perpetually falling on the earth from the sun, quoted as 1366 W/m². If we look at the plot, with units milliwatts/(m² nm), the area under the curve is the power density, W/m². To make a rough estimate, the area is the average value, about 700 mW/(m² nm), times the wavelength range, about 2000 nm. So this rough estimate gives 1400 W/m².

This spectrum (Figure 1.3) was measured by an automated spectrometer carried in a satellite well beyond the earth's atmosphere. The sharp dips in this spectrum are atomic absorption lines, the sort of feature that can be understood only within quantum mechanics. The atoms in question are presumably in the sun's atmosphere.

We are interested in the properties of the sun that is not only the source of all renewable energy, excluding the geothermal and tidal energies and including biofuels that are grown renewably by photosynthesis, but also serves as a model for fusion reactions that might be implemented on earth. The power density at the surface of the sun can be calculated from this measured power density shown in Figure 1.3. If the radiation power density just above the earth is measured as 1366 W/m², then the power density at the *surface of the sun* can be obtained as

$$P = 1366 \text{ W/m}^2 \times (D_{\rm es}/R_{\rm s})^2 = 6.312 \times 10^7 \text{ W/m}^2, \tag{1.3}$$

using the values above for the distance to the sun and the sun's radius, $D_{\rm es}$ and $R_{\rm s}$, respectively. Since we have a good estimate of the sun's surface temperature *T* from the peak position in Figure 1.3, we can use this power density to estimate the emissivity ε , using the relation $P = \varepsilon \sigma_{\rm SB} T^4$. This gives emissivity $\varepsilon = 0.998$, which seems reasonable.

Before we turn to an introductory discussion of how the sun stays hot, let us consider thermal radiation *from* the earth, raising the question of the energy balance for the earth itself. The earth's surface is 70% ocean, and it seems the average temperature T_E must be at least 273 K. Assuming this, the power radiated from the earth is

$$P = 4\pi R_{\rm E}^2 \sigma_{\rm SB} (T_{\rm E})^4.$$
(1.4)

Initially, we suppose that this power goes directly out into space. (A more accurate estimate of the earth's temperature is 288 K, see Ref. [3], p. 11.

Using $R_{\rm E} = 6173$ km and taking emissivity $\varepsilon = 1$, this is P = 160.6 PW. Let us compare this with an estimate of the absorbed power from the sun, being more realistic by taking the Albedo (fraction reflected) as 0.3. So power absorbed is 174 PW (1 - 0.3) = 121.8 PW. Since the earth maintains an approximately constant temperature, this comparison indicates that a net loss discrepancy of 38.8 PW, if we neglect any heat energy coming up from the core of the earth. (It is estimated that heat flow up from the earth's center is $Q = 4.43 \times 10^{13}$ W = 0.0443 PW, which is relatively small. Of this, 80% is from continuing radioactive heating and 20% from "secular cooling" of the initial heat. 44.3 TW is a large number (a bit larger than shown in Table 1.1), but on the scale of the solar influx it is not important in our approximate estimate. So, we will neglect this for the moment) [6].

Thus, a straightforward estimate of power radiated from earth exceeds the wellknown inflow. To resolve the discrepancy, it seems most plausible that the radiated energy does not all actually leave earth, but a portion is reflected back. A "greenhouse effect" reduces the black body radiation 160.6 PW down close to the 121.8 PW net radiation input from the sun (Figure 1.4). We can treat this as return radiation from a

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Figure 1.4 Earth as seen from space, NASA. The cloud cover is evident and is a factor both in the Albedo \approx 0.3 (the fraction of sunlight onto the earth that is reflected) and in the trapping of reradiated heat energy from the earth at 290 K (greenhouse effect). The accurate spherical shape comes from maximizing attractive gravitational energy, which caused the condensation of primordial dust into the compact, initially molten, earth. The

condensation energy is estimated (see text) as $U = -0.6 GM_E^2/R_E = -2.24 \times 10^{32}$ J, which is equal to (-1) times the present rate of global power usage times 5×10^{11} years. The power in the oceans' wave motions is estimated as 56 TW, see text. The radiation power intercepting the earth from the sun is 174 PW, which is 24.9 MW per person, on a 24 h, 7 day basis, counting 7 billion people.

"greenhouse" of temperature $T_{\rm G}$. So the modified energy balance is

$$P = 4\pi R_{\rm E}^2 \sigma_{\rm SB} [(T_{\rm E})^4 - (T_{\rm G})^4] = 121.8 \, {\rm PW}, \tag{1.5}$$

where we have taken the "greenhouse" temperature T_G as 191.3 K, in a simple analysis. According to Richter (op. cit., p. 13), the most important greenhouse gases are CO₂ and water vapor [3].

1.1.1.2 An Introduction to Fusion Reactions on the Sun

In the simplest terms, the power density $P = 63 \text{ MW/m}^2$ leaving the surface of the sun comes from nuclear fusion of protons, to create ⁴He, in the core of the sun. Let us find the total power radiated by the sun. This is $4\pi R_s^2 \times 63.12 \text{ MW} = 3.82 \times 10^{26} \text{ W}$, making use of $R_s = 0.696 \times 10^6 \text{ km}$. This $3.82 \times 10^{26} \text{ W}$ is such a large value, do we need fear the sun will soon be depleted? Fortunately, we can be reassured that the lifetime of the sun is still going to be long, by estimating its loss of mass from the

radiated energy. Using the energy-mass equivalence of Einstein,

$$\Delta M c^2 = \Delta E, \tag{1.6}$$

on a yearly basis, we have $\Delta E = 3.82 \times 10^{26} \text{ W} \times 3.15 \times 10^7 \text{ s/year} = 1.20 \times 10^{34} \text{ J/year}$. This is equivalent to $\Delta M = (1.20 \times 10^{34} \text{ J/year})/c^2 = 1.337 \times 10^{17} \text{ kg/year}$. Although ΔM is large, it is tiny in comparison to the much larger mass of the sun, $M = 1.99 \times 10^{30} \text{ kg}$. Thus, we find that the fractional loss of mass per year, $\Delta M/M$, for the sun is $1.337 \times 10^{17} \text{ kg/year} \div 1.99 \times 10^{30} \text{ kg} = 6.72 \times 10^{-14}/\text{year}$. This is tiny indeed, so the radiation is not seriously depleting the sun's mass. On a scale of 5.4 billion years, the accepted age of the earth, the fractional loss of mass of the sun, during the whole lifetime of earth, taking the simplest approach, has been only 0.036%.

Where does all this energy come from? It originates in the "strong force" of nucleons, which is large but of short range, a few femtometers. Chemical reactions deal with the covalent bonding force, nuclear reactions originate in the strong force, about a million times larger. The energy is from burning hydrogen to make helium, in principle similar to burning hydrogen to make water, but the energy scale is a million times larger.

In more detail, the composition of the sun is stated as 73.5% H and 24.9% He by mass, so the obvious candidate fusion reaction is the conversion of H into He. The basic proton–proton fusion cycle leading to helium in the *core* of the sun (out to about 0.25 of its radius) has several steps that can be summarized as

 $4p \rightarrow {}^{4}\text{He} + 2e^{+} + 2v_{e}.$ (1.7)

This says that four protons lead finally to an alpha particle (two protons and two neutrons, which forms the nucleus of the Helium atom), two positive electrons, and two neutrino particles.

This is a fusion reaction of some of the elementary particles of nature, which include, besides protons and neutrons, positive electrons (positrons) and neutrinos v_e . Positrons and neutrinos may be unfamiliar, but a danger is to become intimidated by unnecessary details, rather than, in an interdisciplinary field, to learn and make use of essential aspects. The important aspect here is that energy is released when particles combine to form products the sum of whose masses are less than the masses of the constituents. Furthermore, as we will learn, this reaction can proceed only when the source particles have high kinetic energy, to overcome Coulomb repulsion when the charged particles coalesce. In addition, the essential process of "quantum mechanical tunneling," an aspect of the wave nature of matter, allows the reaction to proceed when the interparticle energies are in the kiloelectron volt (keV) range, available at temperatures above 15 million K. From elementary physics, we recall that the average kinetic energy per degree of freedom in equilibrium at temperature *T* is

$$E_{\rm av} = \frac{1}{2} k_{\rm B} T,$$
 (1.8)

where Boltzmann's constant $k_{\rm B} = 1.38 \times 10^{-23}$ J/K. The energy units for atomic processes are conveniently expressed as electron volts, such that $1 \text{ eV} = 1.6 \times 10^{-19}$

 $J = 1.6 \times 10^{-19}$ W s. Chemical reactions release energy on the order of 1 eV per atom, while nuclear reactions release energies on the order of 1 MeV per atom, see Figure 1.5. A broad distribution of particle speed ν is allowed in the normalized Maxwell–Boltzmann speed distribution,

$$D(v) = (m/2\pi k_{\rm B}T)^{3/2} 4\pi v^2 \exp(-mv^2/2 k_{\rm B}T).$$
(1.9)

While one may have learned of this in connection with the speeds of oxygen molecules in air, it usefully applies to the motions of protons at 15 million K in the core of the sun.

The most probable speed is $(2 kT/m)^{1/2}$ that corresponds to a kinetic energy $E_k = 1/2 mv^2$ of kT. In connection with the probability of tunneling through the Coulomb barrier, which rises rapidly with rising interparticle energy (particle speed), one sees that the high-speed tail of the Maxwell–Boltzmann speed distribution is important. The overlap of the speed distribution, falling with energy, and the tunneling probability, rising with energy, typically as $\exp[-(E_G/E_k)^{1/2}]$ as we will learn later, leads to what is known as the "Gamow peak" for fusion reactions in the sun. (The sun's neutrino output has been measured on earth, and is now regarded as in satisfactory agreement with the p–p reaction rate in the core of the sun [9].)

The energy release of this reaction can be calculated from the change in the m_ic^2 terms. Using atomic mass units *u*, we go from 4×1.0078 to 4.0026 + 2 (1/1836) = 9.51×10^{-3} u, and using 935.1 MeV as uc^2 , we find 8.89 MeV per ⁴He, neglecting the neutrino energy. The atomic mass unit *u* is nearly the proton mass, but defined in fact as 1/12 the mass of the carbon 12 nucleus.

We should point out the large scale of the fusion energy release, here nearly 9 MeV on a single atom basis. This is about a million times larger than a typical chemical reaction, on a single molecule basis. The nuclear force that binds the protons and neutrons in the nuclei is indeed about a million times stronger than the typical



Figure 1.5 The sun's radiating power comes largely from nuclear fusion of protons p into ⁴He at 15 million K. Mass (nucleon) number A = Z + N. p, D, and T, are equivalent, respectively, to ¹H, ²H, and ³H. (reproduced from Ref. [8], Figure 1).

covalent bond energies in molecules and solids. This large size is, of course, a driving factor toward the use of fusion reactors on earth.

Returning to the sun, it is believed that the p–p cycle accounts for about 98% of the sun's energy output [10], all occurring in the core. The energy diffuses slowly out to the outer surface with attendant reductions in pressure and temperature, the latter from 15 million K to about 5800 K.

The first reaction in the proton-proton cycle at the sun's core is [11]

$$\mathbf{p} + \mathbf{p} \to \mathbf{D} + \mathbf{e}^+ + v_{\mathbf{e}},\tag{1.10}$$

where D is the deuteron, the bound state of the neutron and proton, which has mass 2.0136 *u*. (The mass unit, *u*, is defined as 1/12 of the mass of the 12 C nucleus. One *u* is about 1.67×10^{-27} kg). Here, the energy release is 1.44 MeV, which includes 0.27 MeV to the neutrino.

This first proton–proton reaction occurs very frequently in the sun, as the first step in the basic energy release process. But this reaction is impossible from the point of view of classical physics. It should not occur, from the following reasoning. Accepting the estimated temperature at the center of the sun as 1.5×10^7 K, the thermal energy in the center of mass motion of two protons would be $1/2 k_{\rm B}T = 1/2 1.38 \times 10^{-23} \times 1.5 \times 10^7$ J = 1.035×10^{-16} J = 646.9 eV.

(There will more realistically be a distribution of kinetic energies, and energies higher than 10 keV will frequently be available to colliding protons at 15 million K). But any such estimated energy is far short of the potential energy $k_{C}e^{2}/r$ that is required classically to put two protons in contact. (Here $k_{C} = 9 \times 10^{9}$ and $e = 1.6 \times 10^{-19}$ C). The radius of the proton has been measured and we will take it as 1.2×10^{-15} m. In this case, the Coulomb energy $k_{C}e^{2}/r$ in eV is $9 \times 10^{9} \times 1.6 \times 10^{-19}/(2 \times 1.2 \times 10^{-15}) = 0.6$ MeV. This energy is vastly higher than the kinetic energy (see Figure 1.6). Classically, this reaction will not occur because the two protons will never come into contact.

This fundamental discrepancy was resolved in the early years of the quantum mechanics, and in particular by George Gamow [12], an American physicist. The resolution is that the reaction proceeds by a process of "quantum mechanical tunneling," and the kinetic energies near the "solar Gamow peak" in the range 15–27 keV provide most of the reactions. We will return to this topic later. The process is now completely understood, and we will explore it in some detail because it is also central to experimental approaches to generating fusion energy on earth.

A later and important reaction in the p–p cycle, which we will come back to, is fusion of two deuterons. The result can be a triton Tplus a proton, ³He plus a neutron, or an α (⁴He) plus a gamma ray (photon). (A triton is one proton plus two neutrons, and forms tritium atoms similar to hydrogen and deuterium atoms. Tritium, as opposed to deuterium, does not occur in nature).

1.1.1.3 Distribution of Solar Influx for Conversion

The sun's energy density varies considerably with differing cloud cover characteristic of different parts of the world. A summary of this is shown in Figure 1.7a. The



Figure 1.6 Sketch of fusion by tunneling through Coulomb barrier. Even at 15 million K, the interparticle energy ε (kilovolts) is far below the Coulomb barrier V_{B} (megavolts). (reproduced from Ref. [8], Figure 2).

squares marked on this map represent about 0.16% of the earth area and are judged to be sources of all the world's power need, about 20 TW estimated for mid-century (see Figure 1.1), assuming the areas are covered with 10% efficient solar cells [8]. This is total power consumed, not just electric power!

The units in Figure 1.7b are effective hours of sunlight per year on a flat plate collector, including weather effects. The peak value 2100 h per year of sunlight works out an average W/m² value as $2100/(365 \times 24) 1000 \text{ W/m}^2 = 240 \text{ W/m}^2$. Note that in the Midwest portion of the United States where the effective hours per year are shown as around 1600, this corresponds to 1600/365 = 4.4 h per day, at around 1000 W/m^2 . This time span, 4.4 h, is roughly the duration of the peak electric demand, often about twice the night-time demand.

1.1.2 Secondary Solar-Driven Sources

Wind energy and river flow energy are indirect results of heating by the sun. A map of wind speed in the United States is shown in Figure 1.8. The peak values are in the range 8–9 m/s. The uneven distribution of the resource makes clear the need for a wide grid network or for conversion to a fuel such as hydrogen that could be piped or shipped in containers.

1.1.2.1 Flow Energy

The power that can be derived from wind or water flow is proportional to v^3 . To understand this result, consider an area $A = \pi R^2$ oriented perpendicular to a flow at



(a)



Figure 1.7 (a) Map of requirement of land to meet the world's total power demand (http://ethic-forum.unife.it/E602373e_ev-Balzani.pdf.) in mid-century solely by solar cells. (b) Map of solar intensity across the United States. The

units are described in the text. (Reproduced from Ref. [3], Figure 13.5, p. 159. Source U.S. Department of Energy, Energy Efficiency and Renewable Energy Division).

speed *v* of fluid of density *Q*. In one second, a length L = v containing mass M = AvQ will pass through the area. This represents a flow of kinetic energy $dK/dt = dM/dt v^2/2$, so that power $P = \eta \ dK/dt = \eta \ dM/dt \ v^2/2$ can be obtained if the efficiency of the turbine is η . Thus,

$$P(R) = \eta \pi R^2 \rho v^3 / 2. \tag{1.11}$$



Figure 1.8 Distribution of wind speeds across the United States (U.S. Department of Energy).

A turbine such as the one shown in Figure 1.9 with radius R = 63.5 m, and assuming v = 8 m/s, taking $\rho = 1.2$ kg/m³ for air at 20 °C yields

 $P = \eta \pi \, 63.5^2 \, 1.2 \, 8^3/2 = \eta \, 3.89 \, \text{MW}.$

The best efficiency in practice is about 0.4, giving 1.56 MW/turbine at the assumed 8 m/s, which is a favorable value, shown in the dark areas of the wind map, Figure 1.8, typically in the United States in a band running from Texas to Minnesota.

It is quite easy to show that the maximum efficiency is about 0.59 (Betz's law) (http://c21.phas.ubc.ca/article/wind-turbines-betz-law-explained.) by realizing that the speed ν' behind the turbine is reduced, and the average speed is $\nu_{av} = \frac{1}{2} (\nu + \nu')$. Thus, the corrected formula is

$$P(R) = \pi R^2 \rho v_{\rm av} (\nu^2 - {\nu'}^2)/2. \tag{1.12}$$

This formula provides a maximum power at most 0.59 of the unperturbed power $P_0(R) = \pi R^2 \rho v^3/2$. This corresponds to v = v/3, so one can see why the wind turbines are not longitudinally arranged because the exit air velocity is guite reduced.

Consider an array of such turbines, spaced by 10 *R*. Then the power per unit ground area delivered by the array of the designated turbines at 8 m/s is $1.56 \text{ MW}/(635 \text{ m})^2 = 3.86 \text{ W/m}^2$. A rough comparison with solar cells is that an average solar power at earth is 205 W/m² with an expected efficiency around 0.15, thus 30.75 W/m². The possibility exists of having both solar cells and wind turbines in the same area, plausible if the area is not cultivated. Questions of the installation costs are deferred, but the starting estimate of \$1/(peak installed watt) generally is useful.

We can ask how large a windfarm is needed to generate 500 GW, approximately the electricity used in the United States? If we take 3.86 W/m^2 , the answer is



Figure 1.9 Enercon model E-126 7.5 MW wind turbine. The hub height is 135 m. The specifications say that the machine can be set to cut off at a chosen wind speed in the range 28–34 m/s. From the text one would extrapolate

to a power from one device, at 28 m/s, of 66.9 MW. The specifications say the blades are epoxy resin with integrated lightning protection (http://www.enercon.de/en-en/66.htm.).

Area = $500 \times 10^9/3.86 = 12.95 \times 10^{10} \text{ m}^2$, or 360 km or 223 miles, on a side. This is comparable to the area of the state of Iowa, which is equivalent to 237 miles on a side! The positive aspect is that the turbines do not necessarily preclude the normal use of the land, for example, to grow wheat or corn. But there is no escape from the reality that both wind energy and solar energy are diffuse sources.

Or we may ask how many wind turbines at 1.56 MW per turbine? That number is $N = 500 \times 10^9/1.56 \times 10^6 = 320513$ turbines. At a spacing of 635 m = 0.394 miles per turbine or 2.53 turbines per mile, we could imagine turbines along 126 684 miles of highway. The total mileage in the Interstate Highway system is 46751 miles, while the total U.S. highways extend 162 156 miles. The cost of the turbines at \$1 per Watt is \$500 billion. The cost of the U.S. Interstate Highway system is said to be \$425 billion in 2006 dollars (http://en.wikipedia.org/wiki/Interstate_Highway_System.). \$500 billion is approximately equal to 0.07 of the U.S. military budget for a period of 10 years.

The same kinetic energy extraction analysis applies to water flow in a river, which benefits immediately from the factor 1000/1.2 = 833 increase in density. A recent measurement of Mississippi water flow (http://blog.gulflive.com/mississippi-pressnews/2011/05/mississippi_river_flooding_vic.html.) recorded 11 mph velocity and 16 million gallons per second flow under a bridge near Vicksburg, MS. With conversions 1 mph = 0.447 m/s and $1 \text{ U.S. gallon} = 4.404 \times 10^{-3} \text{ m}^3$, we have 4.92 m/s and $d \text{ V/d}t = 7.06 \times 10^4 \text{ m}^3/\text{s}$ for water flow at this location. The power is then $(dM/dt)(v^2/2) = 1000 \text{ kg/m}^3 (7.06 \times 10^4 \text{ m}^3/\text{s})(4.92 \text{ m/s})^2/2 = 94.8 \text{ MW}.$

The efficiency can be at most 0.59, corresponding to loss of speed by 2/3, and the resulting disruption of the river flow if the full cross section were filled with rotor blades would be prohibitive. Still it seems that tens of MW could be extracted from such a flow if it were continuous and if the installations could be sited to avoid blocking of commerce.

The size of a 1 MW river-flow or tidal-flow turbine is much smaller than a 1 MW wind turbine because of the 1000-fold increase in water density. Probably, this means the water turbine would be cheaper. Water turbines, highly developed for hydroelectric installations, in smaller forms for river-flow applications are not as well established commercially as are wind turbines.

1.1.2.2 Hydroelectric Power

Water running through turbines is used to generate electricity, with a typical efficiency of 90%. It is evident from Figure 1.2 that hydroelectric power is at present by far the largest renewable energy source, amounting to about 1.07 TW worldwide in 2008, or about 7.3%. These are extremely large projects typically, and the easiest sites are already utilized (see Figure 1.10). The situation, often, for a large installation is that it is close to a copper mine or an aluminum smelting facility, which has supported the capital investment. The availability of efficient DC power transmission lines may make the benefit of these large installations more widely available.

Similar large facilities are at Niagara Falls in the United States and the Akosombo Dam in Ghana, Africa. The Three Gorges dam in China at completion has a capacity of 22.5 GW. The planned Grand Inga Dam in Congo is projected as 39 GW. The Belo



Figure 1.10 Grand Coulee Dam is a hydroelectric gravity dam on the Columbia River in the U.S. state of Washington. The dam supplies four power stations with an installed capacity of 6.81 GW. It is the largest electric

power-producing facility in the United States (http://en.wikipedia.org/wiki/File: Grand_Coulee_Dam.jpg.).

Monte Dam on the Xingu, a tributary of the Amazon, has been approved (http://www.bbc.co.uk/news/world-latin-america-13614684.) by Brazil. The dam would be 3.7 mi long and the power would be 11 GW. The Itaipu Dam between Brazil and Paraguay is rated at 14 GW. From 20 0.7 GW generators, two 600 kV HVDC lines, each about 800 km long, carry the DC power to Sao Paolo, where terminal equipment converts to 60 Hz. It provides 90% of electric power in Paraguay and 19% of power in Brazil (http://en.wikipedia.org/wiki/Itaipu_Dam.).

Turbines can be made with the capacity to be reversed and to pump water back to the reservoir when demand is low. This storage capability is called "pumped hydro" and efficiency in the pumping mode can be 80%. Capacity on the American and Canadian sides of the Niagara River totals 5.03 GW, of which 0.374 GW is pumped storage/power producing units (pumped hydro) such as shown in the next figure. The pumped storage facility Carters Dam in Georgia provides a maximum power output of 500 MW during peak demand conditions. Figure 1.11 shows the generators and power distribution from this large water reservoir created by an earthen dam (http://en.wikipedia.org/wiki/File:U.S.ACE_Carters_Dam_powerhoU.S.e.jpg (http://www.niagarafrontier.com/power.html).).



Figure 1.11 Illustration of a 500 MW pumped hydroelectric energy storage facility. The turbines can be reversed to pump water back into the reservoir. This form of energy storage in the electric grid is of larger capacity and lower cost than any known form of battery. Carters Dam in Georgia, U.S. (http://en.wikipedia.org/ wiki/File:U.S.ACE_Carters_Dam_powerhoU.S. e.jpg (http://www.niagarafrontier.com/power. html).).

1.1.2.3 Ocean Waves

According to Table 1.1, the power available in all the oceans' waves is 56 TW, about 3.8 times the global energy consumption at present. Since the area of ocean is $139.4 \times 10^6 \text{ mi}^2 = 3.61 \times 10^{14} \text{ m}^2$, mi = 1609 m, the power per unit area from this estimate is 0.155 W/m^2 . This seems small, but of course ocean waves are really a secondary result of winds, which are themselves a secondary result of the sun's heating.

To check such an arbitrary number, a scientist or technologist should be skeptical and might seek to test it against his own rough estimate.

Estimate of Wave Energy Suppose the average ocean wave amplitude *D* is 1 m and the average frequency $\omega = 2\pi f$ of the oscillation is 0.1 rad/s. So a wave passes a given location every 1/f = 62.8 s. (These are guesses on the average depth and frequency of ocean waves). In the simplest model of an ocean wave, the water moves vertically in simple harmonic motion, $y = D \sin \omega t$. The speed dy/dt is thus $-D\omega \cos \omega t$, with maximum speed $D\omega$. The energy of this oscillation is, thinking of *M* as the mass of all the ocean to a depth 1 m,

$$E = 1/2M(D\omega)^2 \tag{1.13}$$

so that the power dE/dt is

$$P = 1/2M(D\omega)^2\omega. \tag{1.14}$$

Thus, $P = 1/2 M (1 \times 0.1)^2 0.1$, where $M = A D \times 1000$ kg, where A is the ocean area and 1000 is the density of water in kg/m³. Thus,

$$P/A = 0.5 \times 1000 \times 10^{-3} = 0.5 \text{ W/m}^2$$
,

compared to 0.155 W/m^2 from Table 1.1. We predict the total power in ocean waves is 181 TW, on this crude estimate, compared to 56 TW from Table 1.1.

This crude estimate is closer than one might have expected! (It is likely that the typical frequency is higher, and the typical amplitude is smaller). The estimate also helps us understand that the power is proportional to the square of the wave height and the cube of the wave frequency. In fact, the trajectory of water particles as the wave passes is not vertical but circular, and the wave is mathematically a "trochoidal" wave rather than a sinusoidal wave. If one imagines a disk of radius *R* rolling, a point on the radius r = R experiences sinusoidal motion but a point at radius r < R executes trochoidal motion. The model of sinusoidal motion is still useful (http:// hyperphysics.phy-astr.gsu.edu/hbase/waves/watwav2.html.).

The designs of devices, termed wave energy converters, WEC, to extract the wave energy, are naturally adapted to a particular situation, such as at a given depth of water beyond a shoreline, where waves are approaching land. The wave amplitude and speed increase as the open water wave approaches land. Water depths in the range 40–100 m are typical of present installations [13].

The potential extractable wave energy from the Pacific west coast of the United States is estimated [13] as 255 TWh per year, and in Europe about 280 TWh per year. These numbers are equivalent to powers of 0.029 TW and 0.032 TW, respectively (29 GW is an appreciable fraction, about 0.06, of electric power consumption in the United States). It is not clear what the capital and operational costs of such extraction would be, but at least one commercial device, the Pelamis, has been subsidized by the government of Portugal and put into service.

A plausible estimate of available wave power along a coastline is in terms of power per unit length of the coastline. On the Atlantic coast of Great Britain this is estimated [14] as 40 kW/m of exposed coastline. This estimate depends on the height of the waves, which is a function of the windspeed and the unimpeded span of water facing the coast over which the waves can collect energy from the wind. This estimate might be compared to the estimate above for the Pacific coast of the United States. If that coastline is 1000 miles or 1.6 Mm, then we get, at 40 kW/m, the estimate 64 GW, fairly close in agreement.

As waves approach land at depth *d*, the wave speed is

$$V = [(g\lambda/2\pi)\tan h(2\pi d/\lambda)]^{1/2}.$$
 (1.15)

The Pelamis (the word means "water snake") device is a linear array of four linked pontoons, each 30 m long, oriented perpendicular to the waves. The flexing motion occurring at the linking joints with wave passage is used to create electricity. Pelamis devices totaling 2.25 MW capacity have been installed in the sea near Portugal. Vertically bobbing buoy devices anchored at modest depths are also practical. Devices may also be based on trapping water from the tops of waves, extracting energy as that water falls back into the sea.

While the potential seems appreciable for tapping wave energy in coastal regions, the much larger potential power at the open sea seems in practice unavailable, by virtue of its remoteness.

On the other hand, one might conceive of "ocean stations," large floating facilities, which need not be close to land. Such stations might be used, for example, as a basis for desalinization of seawater, for extraction of deuterium from the sea, or for electrolytic hydrogen generation. Possible "ocean stations" for hydrogen production could also harvest wind and solar power. Schemes for delivery by tanker, analogous to the shipping of oil and liquid natural gas, might evolve.

An "ocean station" seems more practical than a "space station" for the human future, let alone facilities discussed (in the United States) for colonization of the moon. We will return in Chapter 5 to an estimate of the cost of a satellite system to send solar energy to earth from space.

An economically sound and competent city might launch its own ocean station, to capture energy for its sphere of influence, and thus reduce dependence on its surrounding grid. This scenario might extend to viable coastal cities worldwide, perhaps Dhaka or Mumbai, beyond New York City.

1.1.3

Earth-Based Long-Term Energy Resources

Some of the long-term energy that is available is stored in the earth, or is the result of the orbital motion of the moon around the earth. In addition, the composition of the ocean contains enough deuterium, present from the beginning of the earth, to constitute a long-term resource.

1.1.3.1 Lunar Ocean Tidal Motion

Tides are caused by the motion of the moon around the earth, in large part. In "funnel" locations like the Bay of Fundy, the flows can be large and rapid. Harvesting tidal flows can be similar to harvesting the flow energy of a river. In some cases, all of the flow can be funneled into a single set of turbines, a situation more like that at Niagara Falls. This is suggested by the artificial tidepool shown here in Figure 1.12.

Famous optimum locations, such as the Bay of Fundy, which has a tidal range of 17 m, are at least partly exploited. At present, the 20 MW tidal power plant at the Bay of Fundy is the only such plant in operation. However, there is scope for more energy to be tapped in this category.

An example of a much larger potential is shown in Figure 1.13, based on tidal flows in the British Isles. In the analogy to the tidal basin, the North Sea roughly plays that role in the example of the British Isles as the gateway between the Atlantic and the North Sea. The energy flow can be taxed on the intake and the exhaust of the cycle.

Calculations of the available power, up to 190 GW, are indicated on the diagram.

On smaller scales, the common occurrence of sandbars parallel to a beach suggests many locations that could be utilized. In the Atlantic coast of the United States, the Outer Banks of North Carolina enclose Pamlico Sound, an area about 2000 square miles, or $5.18 \times 10^9 \text{ m}^2$. The tidal excursion at Cape Hatteras, on the ocean side, is



Figure 1.12 An artificial tide pool [15]. As shown the pool is filled by the high tide at an earlier time, and is now able to discharge water through a turbine generating electricity.

3.6 feet, while the tidal excursion on the inside, for example, at Rodanth, on Pamlico Sound is only 0.72 feet. So it appears that the interior, Pamlico Sound, is decoupled from the tidal excursion on the Atlantic side, by the relatively small openings, through the Outer Banks, between the Sound and the open Atlantic Ocean, where the tides are over 3 feet. Nonetheless, the energy exchanged every 12 h, U = Mgh, where M is the mass of the water in Pamlico Sound to a depth of h = 0.72 feet, we can estimate to be



Figure 1.13 Map suggesting locations of optimal tidal energy flows from the Atlantic Coasts of the British Isles [16].

quite large. Namely, $U = (5.18 \times 10^9 \text{ m}^2 \times 0.22 \text{ m}) \times 1000 \times 9.8 \times 0.22 = 2.46 \times 10^{12} \text{ J}$. In terms of an average power P = dU/dt, this is 57 MW. The annual market value of the entirety of this potential power, at \$0.14/kWh, would be $57 \times 10^6 \times 3.15 \times 10^7 \times (3.6 \times 10^6)^{-1} \times 0.14 = \69.8×10^6 . In an age of governments needing to raise taxes, this might be an incentive to install water turbines.

This situation is present in numerous smaller scale examples. In New York City, the TV host will speak of the danger, on a given day, at a particular beach, of "rip currents," to swimmers. "Rip currents" are tidal flows of water through such constrictions (between open sea and a tidal pool) as we have discussed. There are many locations where sandbars or "keys" are located just off the mainland.

It would seem that constructing artificial entrapments of this sort, for example, a sandbar ("key") extended by levees (dams) to trap tidal flows, augmented with water turbines and grid connections, could be a new activity for the illustrious U.S. Army Corp of Engineers, which has installed numerous bridges, levees, and other water-related engineering projects in the United States.

1.1.3.2 Geothermal Energy

Geothermal potential according to Table 1.1 is 32.2 TW with a higher value, 44.3 TW, from a different source [17]. The core of the earth is molten, and heat leaks out to the surface. The energy release actually comes from two sources. One is radioactive decay of elements like uranium and thorium in the outer layers of the earth. The second is the heat from the earth's core that remains molten, at a much higher temperature. While the trend is of cooling of the core from its primordial high temperature, it has been mentioned that some heat input, a continual heating of the earth's core, comes from the motion of the moon, which continually distorts the shape of the earth, as well as driving the tides.

From a physics point of view, the condensation energy in forming the earth from a dispersed cloud of dust to a condensed sphere of radius *R*,

$$E = -3/5 \ GM^2/R, \tag{1.16}$$

is a benchmark value, easily calculated. Here *G* is the universal gravitation constant, $G = 6.67 \times 10^{-11}$, so that, with $M = 5.97 \times 10^{24}$ kg, and $R = 6.37 \times 10^{6}$ m, we find

$$E = -2.24 \times 10^{32}$$
 J.

This energy, released as kinetic energy, is vast, comparable to the present rate of consumption extended for 4.84×10^{11} years! It is clear that most of this energy has already been lost, mostly by radiation shortly after the condensation. If we were to attribute this full energy to heating of the earth, we can estimate what the temperature would have been. In a simple model of a solid or liquid, the thermal energy is

$$U = 3Nk_{\rm B}T. \tag{1.17}$$

If we attribute all the mass $M = 5.97 \times 10^{24}$ kg to iron atoms, atomic mass $55.85 \times 1.67 \times 10^{-27}$ kg, then $N = 6.4 \times 10^{49}$ atoms, and $T = U/3Nk_{\rm B} = 84.5 \times 10^{3}$ K. The radiation power from the surface of the early earth at that temperature would be

 $P = 4\pi R^2 \sigma_{SB} T^4$, where the Stefan–Boltzmann constant $\sigma_{SB} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$. This is evaluated as

$$P = 1.47 \times 10^{27}$$
 W.

In the simplest view, this suggests that the original heat energy could be radiated away in about 42 h, since $42 \times 3600 \times P = U$. But the radiative cooling quickly slows as the temperature falls, and the linear approach fails. At present, the inner core temperature has been estimated as 5700 K, while lava (magma) at temperatures \sim 1500 K is present at some locations as close as 10 km to the earth's surface. The remaining heat energy in the earth's core is, of course, enormous and certainly can be regarded as a renewable resource.

Practical extraction of the earth's heat is accomplished at locations where molten lava extends close to the surface, providing regions of hot rock that are used to heat injected water to produce steam. U.S. capacity of this type is 3.09 GW, with the largest facility at The Geysers field (http://www.gwpc.org/meetings/forum/2007/proceedings/Papers/Khan,%20Ali%20Paper.pdf.) in CA. Iceland has exploited its geothermal energy to a great extent. A map (http://www.magma-power.com/pages/magma_power_plant.html.) of locations in the United States where magma exists within 10 km of the surface reveals sites concentrated in western states and along the Aleutian Islands. While plant designs have been offered for tapping directly into a lava field, this has not been accomplished.

1.1.3.3 The Earth's Deuterium and its Potential

Fusion of light elements to release energy is the heating mechanism of the sun. A good starting point for fusion is the deuteron, two of which can fuse to make ⁴He with release of nearly 24 MeV of energy. The most likely products for DD fusion are actually a triton plus a proton, with 4 MeV; or ³He plus a neutron, with 3.27 MeV, so that the average energy release per DD fusion is 3.7 MeV. The deuteron fusion reactions are considered important because D particles, Deuterons, are present on earth, notably in seawater. Wherever protons occur, there is about 1/6400 chance of finding instead a Deuteron. Heavy water HDO, therefore, occurs as 1/3200 = 0.031% of all water. There is enough in the ocean that this is considered a sustainable or renewable energy source. The problem is that at present there is no practical process using Deuterons to actually release energy by the fusion reactions.

If we take the ocean mass as 1.37×10^{21} kg, comparing it with the mass per water molecule, $18 \times 1.67 \times 10^{-27}$ kg, we find that there are $N = 4.6 \times 10^{46}$ water molecules in the ocean. This means there are 9.2×10^{46} H atoms, and therefore there are 1.42×10^{43} deuterons. The energy release, if all of these deuterons were fused at the average energy release of 3.7 MeV, is therefore $1.42 \times 10^{43} \times 3.27 \times 10^{6} \times 1.6 \times 10^{-19}$ J = 7.45×10^{30} J. If the present energy consumption is 14.7 TW, so that one year's energy consumption is 4.63×10^{20} J, the deuteron-based energy would last for 1.6×10^{10} years. So, we may say that the deuterium in the ocean, if it can be converted, is a renewable resource. In Chapter 4 of the book, we will look into the possibilities for achieving this release of energy.

1.1.4 Plan of This Book

The underlying physics of solar energy, with a fairly detailed account of how the sun delivers its energy to earth are treated in Chapter 2. To prepare the reader for nanophysics-based energy conversion devices, principally solar cells of various types, background is provided in Chapter 3. Chapter 4 explains three methods that are known to release fusion energy in laboratory situations on earth. The power output from a Tokamak-type fusion reactor is analyzed and numerically estimated by scaling the simplified reaction model, shown in Chapter 2, to predict the sun's output, to the Tokamak realm of parameters. The topics then turn, in Chapter 5, to exploiting the solar radiation input to earth, converting some of the energy to electricity. The physics of solar thermal energy conversion is compared to that of photovoltaic conversion, and a survey of solar cell types is presented. Chapters 6–8 deal in more detail with types of solar cells, including prospects for developing new cells with higher efficiency and possibly at lower cost. Chapter 9 deals with aspects of producing hydrogen gas by photocatalytic cells, as well as practical possibilities for making hydrogen a storage medium for energy produced by wind or solar power. Chapter 10 deals with manufacturing and economic aspects of solar power, with attention to processes that might be scalable to large volume and low cost to replace a significant fraction of the power now obtained from oil, natural gas, and coal. Finally, Chapter 11 deals with the future of renewable energy, as a part of the global energy future.