Contents

1 Basic Physics **1**
- 1.1 Fusion . . . 1
- 1.2 Plasma . . . 7
- 1.3 Coulomb Collisions . . . 10
- 1.4 Electromagnetic Theory . . . 17

2 Motion of Charged Particles **23**
- 2.1 Gyromotion and Drifts . . . 23
  - 2.1.1 Gyromotion . . . 23
  - 2.1.2 $\boldsymbol{E} \times \boldsymbol{B}$ Drift . . . 26
  - 2.1.3 Grad-$\boldsymbol{B}$ Drift . . . 27
  - 2.1.4 Polarization Drift . . . 29
  - 2.1.5 Curvature Drift . . . 30
- 2.2 Constants of the Motion . . . 33
  - 2.2.1 Magnetic Moment . . . 33
  - 2.2.2 Second Adiabatic Invariant* . . . 34
  - 2.2.3 Canonical Angular Momentum . . . 36
- 2.3 Diamagnetism* . . . 38

3 Magnetic Confinement **43**
- 3.1 Confinement in Mirror Fields . . . 43
  - 3.1.1 Simple Mirror . . . 43
  - 3.1.2 Tandem Mirrors* . . . 48
- 3.2 Closed Toroidal Confinement Systems . . . 51
  - 3.2.1 Confinement . . . 51
  - 3.2.2 Flux Surfaces . . . 55
  - 3.2.3 Trapped Particles . . . 57
  - 3.2.4 Transport Losses . . . 61

4 Kinetic Theory **67**
- 4.1 Boltzmann and Vlasov Equations . . . 68
- 4.2 Drift Kinetic Approximation . . . 68
- 4.3 Fokker–Planck Theory of Collisions . . . 71
- 4.4 Plasma Resistivity . . . 78
- 4.5 Coulomb Collisional Energy Transfer . . . 80
- 4.6 Krook Collision Operators* . . . 84

5 Fluid Theory **87**
5.1 Moments Equations . . . 87
5.2 One-Fluid Model . . . 91
5.3 Magnetohydrodynamic Model . . . 95
5.4 Anisotropic Pressure Tensor Model* . . . 98
5.5 Strong Field, Transport Time Scale Ordering . . . 100

6 Plasma Equilibria **105**
6.1 General Properties . . . 105
6.2 Axisymmetric Toroidal Equilibria . . . 107
6.3 Large Aspect Ratio Tokamak Equilibria . . . 113
6.4 Safety Factor . . . 119
6.5 Shafranov Shift* . . . 122
6.6 Beta* . . . 125
6.7 Magnetic Field Diffusion and Flux Surface Evolution* . . . 127
6.8 Anisotropic Pressure Equilibria* . . . 130
6.9 Elongated Equilibria* . . . 132
6.9.1 Geometry . . . 132
6.9.2 Flux surface average . . . 134
6.9.3 Equivalent toroidal models . . . 134
6.9.4 Interpretation of thermal diffusivities from measured temperature gradients . . . 136
6.9.5 Prediction of poloidal distribution of conductive heat flux . . . 137
6.9.6 Mapping radial gradients to different poloidal locations . . . 138

7 Waves **141**
7.1 Waves in an Unmagnetized Plasma . . . 141
7.1.1 Electromagnetic Waves . . . 141
7.1.2 Ion Sound Waves . . . 143
7.2 Waves in a Uniformly Magnetized Plasma . . . 144
7.2.1 Electromagnetic Waves . . . 144
7.2.2 Shear Alfven Wave . . . 147
7.3 Langmuir Waves and Landau Damping . . . 149
7.4 Vlasov Theory of Plasma Waves* . . . 152
7.5 Electrostatic Waves* . . . 158

8 Instabilities **165**
8.1 Hydromagnetic Instabilities . . . 168
8.1.1 MHD Theory . . . 169
8.1.2 Chew–Goldberger–Low Theory . . . 170
8.1.3 Guiding Center Theory . . . 172
8.2 Energy Principle . . . 175
8.3 Pinch and Kink Instabilities . . . 179
8.4 Interchange (Flute) Instabilities . . . 183

8.5 Ballooning Instabilities . . . 189
8.6 Drift Wave Instabilities . . . 193
8.7 Resistive Tearing Instabilities* . . . 196
8.7.1 Slab Model . . . 196
8.7.2 MHD Regions . . . 197
8.7.3 Resistive Layer . . . 199
8.7.4 Magnetic Islands . . . 200
8.8 Kinetic Instabilities* . . . 202
8.8.1 Electrostatic Instabilities . . . 202
8.8.2 Collisionless Drift Waves . . . 203
8.8.3 Electron Temperature Gradient Instabilities . . . 205
8.8.4 Ion Temperature Gradient Instabilities . . . 206
8.8.5 Loss–Cone and Drift–Cone Instabilities . . . 207
8.9 Sawtooth Oscillations* . . . 211

9 Neoclassical Transport 215
9.1 Collisional Transport Mechanisms . . . 215
9.1.1 Particle Fluxes . . . 215
9.1.2 Heat Fluxes . . . 217
9.1.3 Momentum Fluxes . . . 218
9.1.4 Friction Force . . . 220
9.1.5 Thermal Force . . . 220
9.2 Classical Transport . . . 222
9.3 Neoclassical Transport – Toroidal Effects in Fluid Theory . . . 225
9.4 Multifluid Transport Formalism* . . . 231
9.5 Closure of Fluid Transport Equations* . . . 234
9.5.1 Kinetic Equations for Ion–Electron Plasma . . . 234
9.5.2 Transport Parameters . . . 238
9.6 Neoclassical Transport – Trapped Particles . . . 241
9.7 Extended Neoclassical Transport – Fluid Theory* . . . 247
9.7.1 Radial Electric Field . . . 248
9.7.2 Toroidal Rotation . . . 249
9.7.3 Transport Fluxes . . . 249
9.8 Electrical Currents . . . 251
9.8.1 Bootstrap Current . . . 251
9.8.2 Total Current . . . 252
9.9 Orbit Distortion* . . . 253
9.9.1 Toroidal Electric Field – Ware Pinch . . . 253
9.9.2 Potato Orbits . . . 254
9.9.3 Orbit Squeezing . . . 255
9.10 Neoclassical Ion Thermal Diffusivity . . . 256
9.11 Paleoclassical Electron Thermal Diffusivity . . . 258
9.12 Transport in a Partially Ionized Gas* . . . 259

10 Plasma Rotation* 263
10.1 Neoclassical Viscosity . . . 263
10.1.1 Rate-of-Strain Tensor in Toroidal Geometry . . . 263
10.1.2 Viscous Stress Tensor . . . 264
10.1.3 Toroidal Viscous Force . . . 265
10.1.4 Parallel Viscous Force . . . 269
10.1.5 Neoclassical Viscosity Coefficients . . . 270
10.2 Rotation Calculations . . . 272
10.2.1 Poloidal Rotation and Density Asymmetries . . . 272
10.2.2 Shaing-Sigmar-Stacey Parallel Viscosity Model . . . 275
10.2.3 Stacey-Sigmar Poloidal Rotation Model . . . 276
10.2.4 Radial Electric Field and Toroidal Rotation Velocities . . . 280
10.3 Momentum Confinement Times . . . 281
10.3.1 Theoretical . . . 281
10.3.2 Experimental . . . 282
10.4 Rotation and Transport in Elongated Geometry . . . 283
10.4.1 Flux surface coordinate system . . . 283
10.4.2 Flux surface average . . . 285
10.4.3 Differential Operators in Generalized Geometry . . . 285
10.4.4 Fluid Equations in Miller Elongated Flux Surface Coordinates 286

11 Turbulent Transport 293
11.1 Electrostatic Drift Waves . . . 293
11.1.1 General . . . 293
11.1.2 Ion Temperature Gradient Drift Waves . . . 296
11.1.3 Quasilinear Transport Analysis . . . 296
11.1.4 Saturated Fluctuation Levels . . . 298
11.2 Magnetic Fluctuations . . . 299
11.3 Wave–Wave Interactions* . . . 301
11.3.1 Mode Coupling . . . 301
11.3.2 Direct Interaction Approximation . . . 302
11.4 Drift Wave Eigenmodes* . . . 304
11.5 Microinstability thermal diffusivity models* . . . 306
11.5.1 Ion transport . . . 307
11.5.2 Electron transport . . . 312
11.6 Gyrokinetic and Gyrofluid Theory* . . . 315
11.6.1 Gyrokinetic Theory of Turbulent Transport . . . 316
11.6.2 Gyrofluid Theory of Turbulent Transport . . . 318
11.7 Zonal Flows* . . . 321

12 Heating and Current Drive 323
12.1 Inductive . . . 323
12.2 Adiabatic Compression* . . . 326
12.3 Fast Ions . . . 329

12.3.1 Neutral Beam Injection . . . 329
12.3.2 Fast Ion Energy Loss . . . 331
12.3.3 Fast Ion Distribution* . . . 334
12.3.4 Neutral Beam Current Drive . . . 336
12.3.5 Toroidal Alfven Instabilities . . . 337
12.4 Electromagnetic Waves . . . 339
12.4.1 Wave Propagation . . . 339
12.4.2 Wave Heating Physics . . . 342
12.4.3 Ion Cyclotron Resonance Heating . . . 346
12.4.4 Lower Hybrid Resonance Heating . . . 347
12.4.5 Electron Cyclotron Resonance Heating . . . 348
12.4.6 Current Drive . . . 349

13 Plasma–Material Interaction 355
13.1 Sheath . . . 355
13.2 Recycling . . . 358
13.3 Atomic and Molecular Processes . . . 359
13.4 Penetration of Recycling Neutrals . . . 364
13.5 Sputtering . . . 365
13.6 Impurity Radiation . . . 367

14 Divertors 373
14.1 Configuration, Nomenclature and Physical Processes . . . 373
14.2 Simple Divertor Model . . . 376
14.2.1 Strip Geometry . . . 376
14.2.2 Radial Transport and Widths . . . 376
14.2.3 Parallel Transport . . . 378
14.2.4 Solution of Plasma Equations . . . 379
14.2.5 Two-Point Model . . . 380
14.3 Divertor Operating Regimes* . . . 382
14.3.1 Sheath-Limited Regime . . . 382
14.3.2 Detached Regime . . . 383
14.3.3 High Recycling Regime . . . 383
14.3.4 Parameter Scaling . . . 384
14.3.5 Experimental Results . . . 385
14.4 Impurity Retention . . . 385
14.5 Thermal Instability* . . . 388
14.6 2D Fluid Plasma Calculation* . . . 391
14.7 Drifts . . . 393
14.7.1 Basic Drifts in the SOL and Divertor . . . 393
14.7.2 Poloidal and Radial $\boldsymbol{E} \times \boldsymbol{B}$ Drifts . . . 394
14.8 Thermoelectric Currents . . . 396
14.8.1 Simple Current Model . . . 396
14.8.2 Relaxation of Simplifying Assumptions . . . 398

14.9 Detachment . . . 400
14.10 Effect of Drifts on Divertor and SOL Plasma Properties* . . . 402
14.10.1 Geometric Model . . . 402
14.10.2 Radial Transport . . . 403
14.10.3 Temperature, Density and Velocity Distributions . . . 404
14.10.4 Electrostatic Potential . . . 406
14.10.5 Parallel Current . . . 407
14.10.6 Grad-B and Curvature Drifts . . . 408
14.10.7 Solution for Currents and Potentials at Divertor Plates . . . 410
14.10.8 $E \times B$ Drifts . . . 411
14.10.9 Total Parallel Ion Flux . . . 413
14.10.10 Impurities . . . 413
14.10.11 Geometric Invariance . . . 415
14.10.12 Model Problem Calculation: Effect of $\boldsymbol{B}_{\boldsymbol{\phi}}$ Direction on SOL-Divertor Parameters . . . 416
14.11 Blob Transport* . . . 422

15 Plasma Edge **425**
15.1 H-Mode Edge Plasma . . . 425
15.2 Transport in the Plasma Edge . . . 426
15.2.1 Fluid Theory . . . 426
15.2.2 Multi-Fluid Theory* . . . 430
15.2.3 Torque Representation* . . . 431
15.2.4 Kinetic Corrections for Non-Diffusive Ion Transport . . . 433
15.3 Differences Between L-Mode and H-Mode Plasma Edges . . . 439
15.4 Effect of Recycling Neutrals . . . 443
15.5 $\boldsymbol{E} \times \boldsymbol{B}$ Shear Stabilization of Turbulence . . . 444
15.5.1 $\boldsymbol{E} \times \boldsymbol{B}$ Shear Stabilization Physics . . . 445
15.5.2 Comparison with Experiment . . . 447
15.5.3 Possible "Trigger" Mechanism for the L–H Transition . . . 448
15.6 Thermal Instabilities . . . 449
15.6.1 Temperature Perturbations in the Plasma Edge . . . 449
15.6.2 Coupled Two-Dimensional Density–Velocity–Temperature Perturbations* . . . 453
15.6.3 Spontaneous Edge Pressure Pedestal Formation . . . 458
15.7 Poloidal Velocity Spin-Up* . . . 461
15.7.1 Neoclassical Spin-Up . . . 463
15.7.2 Fluid Momentum Balance Calculation of Poloidal Velocity Spin-Up . . . 463
15.7.3 Poloidal Velocity Spin-Up Due to Poloidal Asymmetries . . . 464
15.7.4 Bifurcation of the Poloidal Velocity Spin-Up . . . 466
15.8 ELM Stability Limits on Edge Pressure Gradients . . . 467
15.8.1 MHD Instability Theory of Peeling Modes* . . . 468

15.8.2 MHD Instability Theory of Coupled Ballooning-Peeling Modes* . . . 470
15.8.3 MHD Instability Analysis of ELMs . . . 472
15.9 MARFEs . . . 476
15.10 Radiative Mantle . . . 480
15.11 Edge Operation Boundaries . . . 482

16 Neutral Particle Transport 485
16.1 Fundamentals* . . . 485
16.1.1 1D Boltzmann Transport Equation . . . 485
16.1.2 Legendre Polynomials . . . 486
16.1.3 Charge Exchange Model . . . 487
16.1.4 Elastic Scattering Model . . . 488
16.1.5 Recombination Model . . . 491
16.1.6 First Collision Source . . . 491
16.2 P_N Transport and Diffusion Theory* . . . 493
16.2.1 P_N Equations . . . 493
16.2.2 Extended Diffusion Theories . . . 496
16.3 Multidimensional Neutral Transport* . . . 500
16.3.1 Formulation of Transport Equation . . . 500
16.3.2 Boundary Conditions . . . 502
16.3.3 Scalar Flux and Current . . . 502
16.3.4 Partial Currents . . . 504
16.4 Integral Transport Theory* . . . 504
16.4.1 Isotropic Point Source . . . 505
16.4.2 Isotropic Plane Source . . . 506
16.4.3 Anisotropic Plane Source . . . 507
16.4.4 Transmission Probabilities . . . 509
16.4.5 Escape Probabilities . . . 509
16.4.6 Inclusion of Isotropic Scattering and Charge Exchange . . . 510
16.4.7 Distributed Volumetric Sources in Arbitrary Geometry . . . 511
16.4.8 Flux from a Line Isotropic Source . . . 511
16.4.9 Bickley Functions . . . 512
16.4.10 Probability of Traveling a Distance t from a Line, Isotropic Source without a Collision . . . 513
16.5 Collision Probability Methods* . . . 514
16.5.1 Reciprocity among Transmission and Collision Probabilities 514
16.5.2 Collision Probabilities for Slab Geometry . . . 515
16.5.3 Collision Probabilities in Two-Dimensional Geometry . . . 515
16.6 Interface Current Balance Methods . . . 517
16.6.1 Formulation . . . 517
16.6.2 Transmission and Escape Probabilities . . . 517
16.6.3 2D Transmission/Escape Probabilities (TEP) Method . . . 519
16.6.4 1D Slab Method . . . 524

16.7 Extended Transmission-Escape Probabilities Method* 525
16.7.1 Basic TEP Method 525
16.7.2 Anisotropic Angular Fluxes 526
16.7.3 Extended Directional Escape Probabilities 528
16.7.4 Average Neutral Energy Approximation 531
16.8 Discrete Ordinates Methods* 533
16.8.1 $\boldsymbol{P}_L$ and $\boldsymbol{D}$–$\boldsymbol{P}_L$ Ordinates 534
16.9 Monte Carlo Methods* 536
16.9.1 Probability Distribution Functions 537
16.9.2 Analog Simulation of Neutral Particle Transport 537
16.9.3 Statistical Estimation 539
16.10 Navier–Stokes Fluid Model* 541
16.11 Tokamak Plasma Refueling by Neutral Atom Recycling 542

17 Power Balance **549**
17.1 Energy Confinement Time 549
17.1.1 Definition 549
17.1.2 Experimental Energy Confinement Times 550
17.1.3 Empirical Correlations 551
17.2 Radiation 554
17.2.1 Radiation Fields 554
17.2.2 Bremsstrahlung 556
17.2.3 Cyclotron Radiation 557
17.3 Impurities 559
17.4 Burning Plasma Dynamics 561

18 Operational Limits **565**
18.1 Disruptions 565
18.1.1 Physics of Disruptions 565
18.1.2 Causes of Disruptions 567
18.2 Disruption Density Limit 567
18.2.1 Radial Temperature Instabilities 569
18.2.2 Spatial Averaging* 571
18.2.3 Coupled Radial Temperature–Density Instabilities* 573
18.3 Nondisruptive Density Limits 576
18.3.1 MARFEs 576
18.3.2 Confinement Degradation 577
18.3.3 Thermal Collapse of Divertor Plasma 580
18.4 Empirical Density Limit 581
18.5 MHD Instability Limits 581
18.5.1 β-Limits 581
18.5.2 Kink Mode Limits on $q(a)/q(0)$ 584

19 Fusion Reactors and Neutron Sources **587**
19.1 Plasma Physics and Engineering Constraints 587
19.1.1 Confinement 587
19.1.2 Density Limit 588
19.1.3 *Beta* Limit 589
19.1.4 Kink Stability Limit 590
19.1.5 Start-Up Inductive Volt-Seconds 590
19.1.6 Noninductive Current Drive 591
19.1.7 Bootstrap Current 592
19.1.8 Toroidal Field Magnets 592
19.1.9 Blanket and Shield 593
19.1.10 Plasma Facing Component Heat Fluxes 593
19.1.11 Radiation Damage to Plasma Facing Components 596
19.2 International Tokamak Program 597
19.3 Fusion Beyond ITER 600
19.4 Fusion-Fission Hybrids? 603

Appendices
A Frequently Used Physical Constants 611
B Dimensions and Units 613
C Vector Calculus 617
D Curvilinear Coordinates 619
E Plasma Formulas 627
F Further Reading 629
G Attributions 633

Subject Index **641**