# Contents

Preface XI List of Symbols XIII

- 1 Introduction 1
- 1.1 Why Study Drying Mud? 2
- 1.2 Objectives and Organization of the Book 4
- 1.3 Approach and Scope 6 References 7

#### 2 Elasticity 9

- 2.1 On Springs 9
- 2.2 Deformation, Displacement and Strain 11
- 2.3 Transformations of Strains, Principal Strains and Volumetric

v

- Strain 15
- 2.4 Stress 17
- 2.5 Thermodynamics and the Work of Deformation *21*
- 2.6 Linear Elasticity 23
- 2.7 Different Formulations of Linear Elasticity 26
- 2.8 Plane Elasticity 29
- 2.8.1 Plane Strain and Plane Stress 29
- 2.8.2 Airy Stress Function *31*
- 2.9 Summary 32
- 2.10 Further Reading *33* 
  - References 33

# **3 Fracture Mechanics** 35

- 3.1 Griffith and Fracture Energy 35
- 3.2 Stress Concentration 40
- 3.3 Stress Intensity Factors 41
- 3.4 Fracture Toughness and the Relationship Between  $\mathcal{K}$  and G 44
- 3.5 Summary of the Critical Conditions for Fracture 46

VI	Contents	
	3.6	An Example: Thin-Film Fracture 47
	3.7	Nonlinear and Dissipative Effects of Fracture 53
	3.7.1	A Plastic Zone Model of Fracture 54
	3.7.2	A Mesoscopic View – The Path-Independent <i>J</i> -integral 56
	3.7.3	Dynamic Elasticity and Dynamic Fracture 58
	3.8	Crack Path Selection 60
	3.9	Summary and Further Reading 64
		References 65
	4	Poroelasticity 69
	4.1	Pressure and Stress in a Two-component System 70
	4.1.1	Fick's Laws 73
	4.1.2	Darcy's Law 74
	4.1.3	Network and Total Stress 74
	4.2	Linear Poroelasticity 75
	4.2.1	Poroelastic Energy Density 77
	4.2.2	Poroelastic Constitutive Relations 78
	4.2.3	Different Formulations of Poroelasticity 82
	4.3	Relationship Between Poroelasticity and Thermoelasticity 84
	4.4	Worked Examples of Poroelastic Deformation 86
	4.5	Poroelasticity and a Driving Force for Fracture 88
	4.6	Summary and Further Reading 92
		References 94
	5	Colloids and Clays 97
	5.1	DLVO Theory 98
	5.1.1	van der Waals Potential 98
	5.1.2	Electrostatic Potential 101
	5.1.3	DLVO Theory and its Limitations 106
	5.2	Clays 110
	5.3	Summary and Further Reading 114
		References 115
	6	Desiccation 117
	6.1	Surface Tension and Capillary Pressure 118
	6.1.1	Contact Lines and Capillary Rise 120
	6.2	Solidification Through Evaporation 122
	6.2.1	Skin Formation 122
	6.2.2	Crystals and Cages 124
	6.2.3	Aggregation 126
	6.3	Pore-Scale Processes 128
	6.3.1	Structure of a Drying Soil 130
	6.3.2	Dynamics of a Drying Soil 131
	6.4	Continuum Models of Drying 135
	641	Saufa as During 127

6.4.1 Surface Drying *135* 

Contents VII

6.4.2 6.5	Internal Transport: Carman–Kozeny 139 Further Reading 140
	References 141
7	Patterns of Crack Networks in Homogeneous Media 145
7.1	Introduction 145
7.2	Experimental Observations 146
7.2.1	Sequential Fragmentation and Length Scale Selection 148
7.2.2	Scaling of Crack Width 152
7.2.3	Distribution of Angles Between Cracks 153
7.3	Directional Drying 154
7.4	Characterizing the Crack Pattern: 2D View 155
7.4.1	Scale Invariance in Crack Patterns: Self-Similar and Self-Affine
	Structures 155
7.4.1.1	Scale Invariant Crack Width Distribution 156
7.4.1.2	Fractal Dimension of the Crack Edge 157
7.4.1.3	Self-Affinity of the Fracture Surface 158
7.4.1.4	Fractal Fracture Mechanics 160
7.4.2	Topology and Connectivity of the Crack Network 161
7.4.2.1	Minkowski Numbers and Densities 165
7.4.2.2	Network Theory Approach: Mapping onto an Equivalent
	Network 167
7.4.3	Percolation 169
7.5	Instabilities: Spirals and Wavy Cracks, En Echelon/En Passant
	Cracks, Star Cracks, and Wing Cracks 173
7.5.1	En Echelon Cracks 174
7.5.2	En Passant Cracks 174
7.5.3	Spiral Cracks 175
7.5.4	Wavy Cracks 177
7.5.5	Star Bursts and More Patterns 178
7.6	Crack Dynamics and Branching Cracks 179
7.7	Transition Between Different Modes of Instability and
	Fracture 182
7.7.1	Dendrite to Fracture 182
7.7.2	Viscous Fingering to Fracture 184
7.7.3	Invasion Percolation to Fracture 185
7.8	Towards Three Dimensions: Geological Formations, Drying Soil
	and Peeling 188
7.8.1	Obreimoff's Experiment 188
7.8.2	Natural Mud Cracks in Quasi-2D 189
7.9	Simulation of Quasi-2D Patterns 190
7.9.1	2D Modelling of Fracture: The Fibre Bundle Model 191
7.9.2	Random Fuse Model 192
7.9.3	Spring Network Model 192
7.9.4	Other Models 196

Contents	
7.10	Summary 197
7.11	Further Reading 197
	References 198
8	The Effects of Plasticity on Crack Formation 207
8.1	Introduction to Rheology 207
8.1.1	Elastic Material and Fluid 208
8.1.2	Linear Viscoelasticity 211
8.1.3	Bingham Model 214
8.2	Elastoplasticity for Slow Deformation Processes 216
8.2.1	Decomposition of Elastic and Plastic Deformation 216
8.2.2	Thermodynamics of Elastoplasticity 218
8.2.3	Yield Conditions and the Normality Law 219
8.2.4	Yield Conditions of Paste-Like Materials 222
8.3	Crack Propagation in a Layer of Wet Paste 223
8.3.1	Plumose Structure in Crack Surfaces 223
8.3.2	Microscopic Observation of Plastic Deformation 224
8.3.3	Measurements of the Speed of Crack Growth in a Uniform Paste
	Layer 227
8.4	Theoretical Approaches for Crack Velocities 230
8.4.1	Viscoelastic Effect on Crack Propagation: 1D Lattice Model of
	Rheological Elements 231
8.4.2	Competition of Global Plastic Relaxation and
	Crack Growth 233
8.5	Memory Effect of Paste Due to Its Plasticity 238
8.5.1	Memory of Vibration and Its Visualization as Desiccation Crack
	Pattern 239
8.5.1.1	Memory of Vibration and Lamellar Crack Pattern 239
8.5.1.2	Plasticity of Paste 241
8.5.1.3	Condition for the Memory Effect of Vibration: Experimental
	Results 243
8.5.2	Residual Tension Theory to Explain Memory Effect of
	Vibration 244
8.5.2.1	Quasi-linear Analysis 245
8.5.2.2	Governing Equations for Non-linear Analysis 249
8.5.2.3	Non-linear Analysis 251
8.5.2.4	Condition for the Memory Effect of Vibration: Theoretical
	Explanation 253
8.5.3	Position Control of Cracks by Memory Effect and Faraday
	Waves 254
8.5.4	Memory of Flow and a Role of Interaction Between Colloidal
	Particles 258
	Further Reading 262
	References 263

Contents IX

- 9Special Topics2679.1Tailoring Crack Patterns2679.1.1Effect of Electric Fields on Desiccation Cracks2689.1.1.1Effects of a Direct Field (DC)2689.1.1.2Effect of an Alternating Field (AC)2709.1.1.3DC Field Effect in Drying Droplets2719.1.2Effect of a Magnetic Field on Designation Cracks268
- 9.1.2 Effect of a Magnetic Field on Desiccation Cracks 274
- 9.1.3 Patterning Cracks Through Micro-Technology 276
- 9.2 Designing Crack-Resistant Materials and Composites 279
- 9.2.1 Composites of Soft and Hard Particles 284
- 9.2.1.1 Employing Heterogeneous Material 285
- 9.2.2 Crack Reduction with 'Liquid Particles' 288
- 9.3 Crack Patterns in Drying Droplets of Biofluids 290
- 9.3.1 Human Blood Droplets and Drying Dynamics 291
- 9.3.2 Effect of Relative Humidity on Drying Droplets 295
- 9.3.3 Substrate Effect on Drying Droplets of Blood 296
- 9.4 Evolving Crack Networks 297
- 9.4.1 Columnar Joints 298
- 9.4.2 Evolving Mud Cracks 304
- 9.4.3 Other Crack Patterns 308
- 9.5 Further Reading 310
  - References 311

### Appendix A: A Primer on Vectors and Tensors 317

- A.1 Tensor Notation *317*
- A.2 Tensor Multiplication 319
- A.3 Tensor Transformations 321
- A.4 Tensor Differentiation 323

#### Appendix B: Fractals: Self-Similar and Self-Affine Systems 327

- B.1 Self-Similarity and Fractal Dimension 327
- B.2 Self-Affine Systems 331
- B.3 Further Reading 332
- References 333
- Appendix C: Formulation of Elastoplasticity Based on Dissipation Functions 335 References 336
- Appendix D: Steady Propagating Solution of Langer Model 337
- Appendix E: Stress Expression in Finite Deformation Theory 339 References 341

Index 343