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#### **Book Series: Statistical Physics of Fracture and Breakdown**

Editors: Bikas K. Chakrabarti and Purusattam Ray

Why does a bridge collapse, an aircraft or a ship break apart? When does a dielectric insulation fail or a circuit fuse, even in microelectronic systems? How does an earthquake occur? Are there precursors to these failures? These remain important questions, even more so as our civilization depends increasingly on structures and services where such failure can be catastrophic. How can we predict and prevent such failures? Can we analyze the precursory signals sufficiently in advance to take appropriate measures, such as the timely evacuation of structures or localities, or the shutdown of facilities such as nuclear power plants?

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