

## Contents

**About the Author** *xi*

**Foreword** *xiii*

**Introduction** *xv*

- 1 The Asymptotic Perturbation Method for Nonlinear Oscillators** *1*
  - 1.1 Introduction *1*
  - 1.2 Nonlinear Dynamical Systems *3*
  - 1.3 The Approximate Solution *5*
  - 1.4 Comparison with the Results of the Numerical Integration *10*
  - 1.5 External Excitation in Resonance with the Oscillator *11*
  - 1.6 Conclusion *16*
  
- 2 The Asymptotic Perturbation Method for Remarkable Nonlinear Systems** *19*
  - 2.1 Introduction *19*
  - 2.2 Periodic Solutions and Their Stability *21*
  - 2.3 Global Analysis of the Model System *27*
  - 2.4 Infinite-period Symmetric Homoclinic Bifurcation *35*
  - 2.5 A Few Considerations *41*
  - 2.6 A Peculiar Quasiperiodic Attractor *42*
  - 2.7 Building an Approximate Solution *44*
  - 2.8 Results from Numerical Simulation *46*
  - 2.9 Conclusion *52*
  
- 3 The Asymptotic Perturbation Method for Vibration Control with Time-delay State Feedback** *53*
  - 3.1 Introduction *53*
  - 3.2 Time-delay State Feedback *53*
  - 3.3 The Perturbation Method *56*
  - 3.4 Stability Analysis and Parametric Resonance Control *59*
    - 3.4.1 The Frequency–Response Curve Is *62*

- 3.5 Suppression of the Two-period Quasiperiodic Motion 63
- 3.6 Vibration Control for Other Nonlinear Systems 68
  
- 4 The Asymptotic Perturbation Method for Vibration Control by Nonlocal Dynamics 69**
  - 4.1 Introduction 69
  - 4.2 Vibration Control for the van der Pol Equation 72
  - 4.3 Stability Analysis and Parametric Resonance Control 74
  - 4.4 Suppression of the Two-period Quasiperiodic Motion 79
  - 4.5 Conclusion 82
  
- 5 The Asymptotic Perturbation Method for Nonlinear Continuous Systems 83**
  - 5.1 Introduction 83
  - 5.2 The Approximate Solution for the Primary Resonance of the  $n$ th Mode 86
  - 5.3 The Approximate Solution for the Subharmonic Resonance of Order One-half of the  $n$ th Mode 91
  - 5.4 Conclusion 93
  
- 6 The Asymptotic Perturbation Method for Dispersive Nonlinear Partial Differential Equations 95**
  - 6.1 Introduction 95
  - 6.2 Model Nonlinear PDES Obtained from the Kadomtsev–Petviashvili Equation 97
  - 6.3 The Lax Pair for the Model Nonlinear PDE 98
  - 6.4 A Few Remarks 100
  - 6.5 A Generalized Hirota Equation in  $2 + 1$  Dimensions 100
  - 6.6 Model Nonlinear PDEs Obtained from the KP Equation 101
  - 6.7 The Lax Pair for the Hirota–Maccari Equation 103
  - 6.8 Conclusion 105
  
- 7 The Asymptotic Perturbation Method for Physics Problems 107**
  - 7.1 Introduction 107
  - 7.2 Derivation of the Model System 108
  - 7.3 Integrability of the Model System of Equations 111
  - 7.4 Exact Solutions for the C-integrable Model Equation 112
    - 7.4.1 Nonlinear Wave 112
    - 7.4.2 Solitons 112
    - 7.4.3 Dromions 113
    - 7.4.4 Lumps 116
    - 7.4.5 Ring Solitons 116
    - 7.4.6 Instantons 117

- 7.4.7 Moving Breather-Like Structures 117
- 7.5 Conclusion 120

## **8 The Asymptotic Perturbation Model for Elementary Particle Physics 121**

- 8.1 Introduction 121
- 8.2 Derivation of the Model System 122
- 8.3 Integrability of the Model System of Equations 124
- 8.4 Exact Solutions for the  $C$ -integrable Model Equation 125
  - 8.4.1 Nonlinear Wave 125
  - 8.4.2 Solitons 126
  - 8.4.3 Dromions 126
  - 8.4.4 Lumps 127
  - 8.4.5 Ring Solitons 127
  - 8.4.6 Instantons 129
  - 8.4.7 Moving Breather-like Structures 129
- 8.5 A Few Considerations 130
- 8.6 Hidden Symmetry Models 130
- 8.7 Derivation of the Model System 133
- 8.8 Coherent Solutions 138
  - 8.8.1 Nonlinear Wave 138
  - 8.8.2 Solitons 138
  - 8.8.3 Dromions 139
  - 8.8.4 Lumps 139
  - 8.8.5 Ring Solitons 140
  - 8.8.6 Instantons 141
  - 8.8.7 Moving Breather-like Structures 142
- 8.9 Chaotic and Fractal Solutions 143
  - 8.9.1 Chaotic–Chaotic and Chaotic–Periodic Patterns 143
  - 8.9.2 Chaotic Line Soliton Solutions 145
  - 8.9.3 Chaotic Dromion and Lump Patterns 145
  - 8.9.4 Nonlocal Fractal Solutions 147
  - 8.9.5 Fractal Dromion and Lump Solutions 147
  - 8.9.6 Stochastic Fractal Dromion and Lump Excitations 148
- 8.10 Conclusion 150

## **9 The Asymptotic Perturbation Method for Rogue Waves 151**

- 9.1 Introduction 151
- 9.2 The Mathematical Framework 153
- 9.3 The Maccari System 154
- 9.4 Rogue Wave Physical Explanation According to the Maccari System and Blowing Solutions 156
- 9.5 Conclusion 158

<b>10</b>	<b>The Asymptotic Perturbation Method for Fractal and Chaotic Solutions</b>	<b>159</b>
10.1	Introduction	159
10.2	A New Integrable System from the Dispersive Long-wave Equation	161
10.3	Nonlinear Coherent Solutions	165
10.3.1	Nonlinear Wave	165
10.3.2	Solitons	165
10.3.3	Dromions	166
10.3.4	Lumps	166
10.3.5	Ring Solitons	167
10.3.6	Instantons	167
10.3.7	Moving Breather-Like Structures	168
10.4	Chaotic and Fractal Solutions	168
10.4.1	Chaotic–Chaotic and Chaotic–Periodic Patterns	168
10.4.2	Chaotic Line Soliton Solutions	168
10.4.3	Chaotic Dromion and Lump Patterns	169
10.4.4	Nonlocal Fractal Solutions	169
10.4.5	Fractal Dromion and Lump Solutions	169
10.4.6	Stochastic Fractal Excitations	170
10.4.7	Stochastic Fractal Dromion and Lump Excitations	170
10.5	Conclusion	171
<b>11</b>	<b>The Asymptotic Perturbation Method for Nonlinear Relativistic and Quantum Physics</b>	<b>173</b>
11.1	Introduction	173
11.2	The NLS Equation for $a_1 > 0$	174
11.3	The NLS Equation for $a_1 < 0$	176
11.4	A Possible Extension	178
11.5	The Nonrelativistic Case	180
11.6	The Relativistic Case	183
11.7	Conclusion	185
<b>12</b>	<b>Cosmology</b>	<b>187</b>
12.1	Introduction	187
12.2	A New Field Equation	188
12.3	Exact Solution in the Robertson–Walker Metrics	191
12.4	Entropy Production	195
12.5	Conclusion	197
<b>13</b>	<b>Confinement and Asymptotic Freedom in a Purely Geometric Framework</b>	<b>199</b>
13.1	Introduction	199
13.2	The Uncertainty Principle	201
13.3	Confinement and Asymptotic Freedom for the Strong Interaction	203

13.4	The Motion of a Light Ray Into a Hadron	207
13.5	Conclusion	208
<b>14</b>	<b>The Asymptotic Perturbation Method for a Reverse Infinite-Period Bifurcation in the Nonlinear Schrodinger Equation</b>	<b>209</b>
14.1	Introduction	209
14.2	Building an Approximate Solution	210
14.3	A Reverse Infinite-Period Bifurcation	212
14.4	Conclusion	215
	<b>Conclusion</b>	<b>217</b>
	<b>References</b>	<b>219</b>
	<b>Index</b>	<b>235</b>

