## Contents

1 Solutions to the Exercises of Chapter I (Complement KI).Waves and Particles. Introduction to the Fundamental Ideasof Quantum Mechanics1
1.1 Interference and Diffraction with a Beam of Neutrons ..... 1
1.2 Bound State of a Particle in a "Delta Function Potential" ..... 3
1.3 Transmission of a "Delta Function" Potential Barrier ..... 9
1.4 Bound State of a Particle in a "Delta Function Potential", Fourier Analysis ..... 13
1.5 Well Consisting of Two Delta Functions ..... 24
1.6 Bound State in a Square Potential ..... 38
1.7 The Piecewise Constant Lennard-Jones Potential ..... 43
1.8 Two-Dimensional Potential ..... 47
2 Solutions to the Exercises of Chapter II (Complement $\mathrm{H}_{\text {II }}$ ). The Mathematical Tools of Quantum Mechanics ..... 55
Dirac Notation. Commutators. Eigenvectors and Eigenvalues ..... 55
2.1 A First Approach ..... 55
2.2 Diagonalization, Orthonormal Basis, Closure Relation ..... 58
2.3 Superposition of States ..... 64
2.4 A Ket-Bra Operator ..... 66
2.5 Orthogonal Projector ..... 67
2.6 $\sigma_{x}$ Matrix ..... 68
$2.7 \quad \sigma_{y}$ Matrix ..... 70
2.8 Hamiltonian $H$ of a Particle in a One-Dimensional Problem ..... 72
2.9 Toward the Virial Theorem in Quantum Mechanics ..... 74
$2.10 \quad$ Operators $X$ and $P$ ..... 77
Complete Sets of Commuting Observables, C.S.C.O. ..... 78
2.11 A C.S.C.O. of a Three-State System ..... 78
2.12 A C.S.C.O. of Two Operators ..... 81
3 Solutions to the Exercises of Chapter III (Complement $L_{\text {III }}$ ). The Postulates of Quantum Mechanics ..... 85
3.1 Analysis of a One-Dimensional Wave Function ..... 85
3.2 Probability and One-Dimensional Wave Function ..... 88
$3.3 \quad$ Wave Function Defined Using Momenta ..... 90
3.4 Spreading of a Free Wave Packet ..... 96
3.5 Particle Subjected to a Constant Force ..... 100
3.6 Three-Dimensional Wave Function ..... 103
3.7 Generic Three-Dimensional Wave Function ..... 106
$3.8 \quad$ Probability Current ..... 108
3.9 Complete Description of a Quantum State Using the Probability Density and Probability Current ..... 112
$3.10 \quad$ Virial Theorem ..... 115
3.11 Two-Particle Wave Function ..... 119
3.12 Infinite One-Dimensional Well ..... 123
3.13 Infinite Two-Dimensional Well (cf. Complement $\mathrm{G}_{\mathrm{II}}$ ) ..... 126
3.14 Temporal Evolution Within a Coupled Three-Level System ..... 133
3.15 Interaction Picture ..... 138
3.16 Correlations Between Two Particles ..... 143
3.17 Introduction to the Density Matrix (or Density Operator) ..... 157
3.18 Temporal Evolution of the Density Matrix ..... 160
3.19 Two-Particle Density Matrix ..... 161
References ..... 164
4 Solutions to the Exercises of Chapter IV (Complement $\mathrm{J}_{\mathrm{IV}}$ ). Application of the Postulates to Simple Cases: Spin 1/2 and Two-Level Systems ..... 165
4.1 First Approach to Spin States and Quantum Precession ..... 165
4.2 Continuation of the First Approach with a Nonstationary Magnetic Field ..... 170
4.3 Continuation of the First Approach with a Magnetic Field with Two Components ..... 174
4.4 Density Matrix and Spin Measurements ..... 178
4.5 Evolution Operator of a Spin $1 / 2\left(c f\right.$. Complement $\left.\mathrm{F}_{\mathrm{III}}\right)$ ..... 186
4.6 Study of the Spin State of Two Particles Described by a Single Wave Function ..... 190
4.7 Continuation of the Study of the Two-Particle Spin State Described by a Single Wave Function ..... 196
4.8 Linear Triatomic Molecule ..... 202
4.9 Hexagonal Molecule ..... 209
Reference ..... 217
5 Solutions to the Exercises of Chapter V (Complement $\mathrm{M}_{\mathrm{V}}$ ). The One-Dimensional Harmonic Oscillator ..... 219
5.1 One-Dimensional Harmonic Oscillator ..... 220
5.2 Anisotropic Three-Dimensional Harmonic Oscillator ..... 225
5.3 Harmonic Oscillator: Two Particles, Part I ..... 232
5.4 Harmonic Oscillator: Two Particles, Part II ..... 240
5.5 Harmonic Oscillator: Two Particles, Part III ..... 249
5.6 Charged Harmonic Oscillator in a Variable Electric Field ..... 255
5.7 A Fourier-Like Operator Applied to a One-Dimensional Harmonic Oscillator ..... 266
5.8 The Time Evolution Operator Applied to a One-Dimensional Harmonic Oscillator ..... 269
6 Solutions to the Exercises of Chapter VI (Complement $\mathrm{F}_{\mathrm{VI}}$ ). General Properties of Angular Momentum in Quantum Mechanics ..... 283
6.1 Mean Value of a Magnetic Moment for a Given State ..... 283
6.2 Magnetic Moment Measurement in a Four-Dimensional Space ..... 286
6.3 Link Between the Classical Angular Momentum and the Corresponding Quantum Operator ..... 293
6.4 Rotation of a Polyatomic Molecule ..... 295
6.5 Study of the Angular Part of a Wave Function ..... 310
6.6 An Electric Quadrupole in an Electric Field Gradient ..... 314
6.7 On Rotational Matrices ..... 322
6.8 Rotation and Angular Momentum ..... 326
6.9 Fluctuations and Angular Momentum Measurements ..... 330
6.10 Heisenberg-Type Relations for Angular Momenta ..... 340
6.11 State Minimizing Angular Momentum Fluctuations ..... 344
Reference ..... 353
7 Solutions to the Exercises of Chapter VII (Complement $\mathrm{G}_{\mathrm{VII}}$ ). Particle in a Central Potential. The Hydrogen Atom ..... 355
7.1 Particle in a Cylindrically Symmetric Potential ..... 355
7.2 Three-Dimensional Harmonic Oscillator in a Uniform Magnetic Field ..... 361
Bibliography ..... 377

