## Contents

1	Solutions to the Exercises of Chapter I (Complement K <sub>1</sub> ).
	Waves and Particles. Introduction to the Fundamental Ideas
	of Quantum Mechanics 1

- 1.1 Interference and Diffraction with a Beam of Neutrons 1
- 1.2 Bound State of a Particle in a "Delta Function Potential" *3*
- 1.3 Transmission of a "Delta Function" Potential Barrier 9
- 1.4 Bound State of a Particle in a "Delta Function Potential", Fourier Analysis *13*
- 1.5 Well Consisting of Two Delta Functions 24
- 1.6 Bound State in a Square Potential 38
- 1.7 The Piecewise Constant Lennard–Jones Potential 43
- 1.8 Two-Dimensional Potential 47
- 2 Solutions to the Exercises of Chapter II (Complement H<sub>II</sub>). The Mathematical Tools of Quantum Mechanics 55

Dirac Notation. Commutators. Eigenvectors and Eigenvalues 55

- 2.1 A First Approach 55
- 2.2 Diagonalization, Orthonormal Basis, Closure Relation 58
- 2.3 Superposition of States 64
- 2.4 A Ket-Bra Operator 66
- 2.5 Orthogonal Projector 67
- 2.6  $\sigma_x$  Matrix 68
- 2.7  $\sigma_v$  Matrix 70
- 2.8 Hamiltonian *H* of a Particle in a One-Dimensional Problem 72
- 2.9 Toward the Virial Theorem in Quantum Mechanics 74
- 2.10 Operators *X* and *P* 77 Complete Sets of Commuting Observables, C.S.C.O. 78
- 2.11 A C.S.C.O. of a Three-State System 78
- 2.12 A C.S.C.O. of Two Operators 81

iv Contents

3	Solutions to the Exercises of Chapter III (Complement $L_{III}$ ).
	The Postulates of Quantum Mechanics 85
3.1	Analysis of a One-Dimensional Wave Function 85
3.2	Probability and One-Dimensional Wave Function 88
3.3	Wave Function Defined Using Momenta 90
3.4	Spreading of a Free Wave Packet 96
3.5	Particle Subjected to a Constant Force 100
3.6	Three-Dimensional Wave Function 103
3.7	Generic Three-Dimensional Wave Function 106
3.8	Probability Current 108
3.9	Complete Description of a Quantum State Using the Probability
	Density and Probability Current 112
3.10	Virial Theorem 115
3.11	Two-Particle Wave Function 119
3.12	Infinite One-Dimensional Well 123
3.13	Infinite Two-Dimensional Well (cf. Complement $G_{II}$ ) 126
3.14	Temporal Evolution Within a Coupled Three-Level System 133
3.15	Interaction Picture 138
3.16	Correlations Between Two Particles 143
3.17	Introduction to the Density Matrix (or Density Operator) 157
3.18	Temporal Evolution of the Density Matrix 160
3.19	Two-Particle Density Matrix 161
	References 164
4	Solutions to the Exercises of Chapter IV (Complement $J_{IV}$ ).
	Application of the Postulates to Simple Cases: Spin 1/2 and
	Two-Level Systems 165
4.1	First Approach to Spin States and Quantum Precession 165
4.2	Continuation of the First Approach with a Nonstationary Magnetic
	Field 170
4.3	Continuation of the First Approach with a Magnetic Field with Two
	Components 174
4.4	Density Matrix and Spin Measurements 178
4.5	Evolution Operator of a Spin $1/2$ ( <i>cf</i> . Complement F <sub>III</sub> ) 186
4.6	Study of the Spin State of Two Particles Described by a Single Wave
	Function 190
4.7	Continuation of the Study of the Two-Particle Spin State Described
	by a Single Wave Function 196
4.8	Linear Triatomic Molecule 202
1.0	Henry and Malesula 200

Hexagonal Molecule 209 4.9 Reference 217

5 Solutions to the Exercises of Chapter V (Complement  $M_V$ ).

## The One-Dimensional Harmonic Oscillator 219

- 5.1 One-Dimensional Harmonic Oscillator 220
- 5.2 Anisotropic Three-Dimensional Harmonic Oscillator 225
- 5.3 Harmonic Oscillator: Two Particles, Part I 232
- 5.4 Harmonic Oscillator: Two Particles, Part II 240
- 5.5 Harmonic Oscillator: Two Particles, Part III 249
- 5.6 Charged Harmonic Oscillator in a Variable Electric Field 255
- 5.7 A Fourier-Like Operator Applied to a One-Dimensional Harmonic Oscillator *266*
- 5.8 The Time Evolution Operator Applied to a One-Dimensional Harmonic Oscillator *269*
- 6 Solutions to the Exercises of Chapter VI (Complement F<sub>VI</sub>). General Properties of Angular Momentum in Quantum Mechanics 283
- 6.1 Mean Value of a Magnetic Moment for a Given State 283
- 6.2 Magnetic Moment Measurement in a Four-Dimensional Space 286
- 6.3 Link Between the Classical Angular Momentum and the Corresponding Quantum Operator 293
- 6.4 Rotation of a Polyatomic Molecule 295
- 6.5 Study of the Angular Part of a Wave Function *310*
- 6.6 An Electric Quadrupole in an Electric Field Gradient 314
- 6.7 On Rotational Matrices *322*
- 6.8 Rotation and Angular Momentum *326*
- 6.9 Fluctuations and Angular Momentum Measurements 330
- 6.10 Heisenberg-Type Relations for Angular Momenta 340
- 6.11 State Minimizing Angular Momentum Fluctuations 344 Reference 353
- 7 Solutions to the Exercises of Chapter VII (Complement G<sub>VII</sub>). Particle in a Central Potential. The Hydrogen Atom 355
- 7.1 Particle in a Cylindrically Symmetric Potential 355
- 7.2 Three-Dimensional Harmonic Oscillator in a Uniform Magnetic Field *361*

## Bibliography 377