

Contents

	Preface of the Book Series	<i>xvii</i>
	Preface of the Second Volume	<i>xix</i>
	Acknowledgements	<i>xxi</i>
13	Aberration Expansion	1
13.1	Introduction	1
13.2	Basic Geometrical Optics	4
13.2.1	Paraxial Relations	4
13.2.2	Delano's Relation	5
13.2.3	Conjugation Parameter	5
13.2.4	Special Lens Parameters	6
13.2.5	Pupil Imaging	6
13.2.6	Notations Used in Aberration Theory	7
13.3	Fermat's Principle	9
13.3.1	Introduction	9
13.3.2	Geometrical Approximation of the Wave Equation	9
13.3.3	Lagrangian and Hamiltonian Formulation of Optics	10
13.3.4	Optical Path Length	10
13.3.5	Fermat's Principle	11
13.3.6	Malus-Dupin's Law	12
13.3.7	Stationary Phase Method	13
13.4	Eikonal and Characteristic Function	14
13.4.1	Basic Approach of Aberration Theory	14
13.4.2	Characteristic Functions	14
13.4.3	Bruns' Point-Point Eikonal	15
13.4.4	Point-Angle Eikonal	16
13.4.5	Angle-Point Eikonal	16
13.4.6	Angle-Angle Eikonal	17
13.4.7	Relations Between the Eikonals	17
13.4.8	Impossibility of Perfect Imaging Systems	18
13.5	Aberration Expansion	19
13.5.1	Point Eikonal of a Paraxial Single Refractive Surface	19
13.5.2	Third Order Aberration of a Single Refractive Surface	20
13.5.2.1	The Angle-Angle Eikonal of the Surface and its Expansion	20
13.5.2.2	Applying the Law of Refraction	21
13.5.2.3	Applying Fermat's Principle for Conjugated Points	21
13.5.2.4	Application of More Practical Parameters of the System Description	22
13.5.2.5	Sorting the Terms and Definition of Aberration Coefficients	22
13.5.2.6	Generalization from a Surface to a System	23
13.5.3	Expansion and Symmetry-Invariants	24
13.5.4	Seidel's Approach	27
13.5.5	Shack/Sasian Formulation	28
13.5.6	Relation Between Wave and Ray Aberrations	29
13.5.7	Aberration Expansion of Transverse Ray Aberrations	30
13.5.8	Reduced Aberration Expansion	32
13.5.9	Aberration Expansion in Case of Reduced Symmetry	33
13.6	Ray Aberrations	33
13.6.1	Longitudinal Aberrations	33
13.6.2	Transverse Aberrations	34
13.6.3	Angle Aberrations	36
13.6.4	Spot Diagram	36
13.6.4.1	Introduction	36
13.6.4.2	Quantitative Spot Size	37
13.6.4.3	Ray Sampling of the Full Pupil	38
13.6.4.4	Full Field Spot Diagram	42
13.6.5	Ray Sampling for Aberration Curves	42
13.6.6	Kingslake's Diagram	43
	References	45
14	Primary Aberrations	47
14.1	Phenomenology of Primary Monochromatic Aberrations	47
14.1.1	Introduction	47
14.1.2	Spherical Aberration	47
14.1.3	Astigmatism	51
14.1.4	Field Curvature	54
14.1.4.1	Introduction	54
14.1.4.2	Simple Derivation of the Petzval Theorem	57
14.1.5	Coma	58

14.1.6	Distortion	60	14.5.4.1	Mahajan's Notation	94
14.1.7	Spot Shapes of Primary Aberrations	61	14.5.4.2	Version of Haferkorn	96
14.2	Primary Chromatical Aberrations	63	14.5.4.3	Angle Related Coefficients of Slyusarev	97
14.2.1	Introduction	63	14.5.5	Transverse Aberration Representation	97
14.2.2	Axial Chromatical Aberration	64	14.5.5.1	Version of Welford	97
14.2.2.1	Introduction	64	14.5.5.2	Formulation of Berek	97
14.2.2.2	Seidel Term of Axial Chromatical Aberration	66	14.5.6	Afocal Systems	98
14.2.2.3	The Decomposition of Primary Colour Aberration After Münz/Sacks	66	14.6	Seidel Aberration of Thin Lenses	98
14.2.2.4	Conrady's d - D Method	67	14.6.1	Introduction	98
14.2.2.5	Equivalent Abbe Number	69	14.6.1.1	Spherical Aberration of Thin Lenses	99
14.2.2.6	Localization of Colour Correction	70	14.6.1.2	Alternative Formula for Spherical Aberration	102
14.2.3	Lateral Chromatical Aberration	70	14.6.2	Primary Aberrations for Thin Lenses	104
14.2.3.1	Introduction	70	14.6.2.1	Primary Aberrations of a Thin Lens in Different Media	105
14.2.3.2	Examples	72	14.6.2.2	Slyusarev's Representation	106
14.3	Special Properties of Seidel Aberrations	73	14.6.2.3	Simplified Seidel Lens Synthesis of Miks	106
14.3.1	Standard Representation for Seidel Aberrations	73	14.6.2.4	Axial Chromatical Aberration of a Thin Lens	107
14.3.2	Seidel Pegel Diagrams	74	14.6.2.5	Achromatic Condition	108
14.3.3	Seidel Aberrations of the Pupil Imaging	75	14.7	Seidel Aberrations of Thick Lenses	108
14.4	Seidel Shift Formulas	77	14.7.1	Introduction	108
14.4.1	Stop-Shift Formulae	77	14.7.1.1	Spherical Aberration Third Order Contribution of a Thick Lens	108
14.4.1.1	Eccentricity Parameter and Ray Bundle Separation	77	14.7.1.2	Field Curvature of a Thick Lens	109
14.4.1.2	Stop Shift	78	14.7.2	Aberrations Coefficients of a Thick Lens	109
14.4.1.3	Stop Shift Equations in Third Order	80	14.7.3	Axial Colour of a Thick Lens	110
14.4.1.4	Consequences of the Stop Shift Equations	81	14.7.3.1	Approximation of Achromatic Power for Thick Lenses	110
14.4.1.5	Stop Shift Formula in Wave Aberrations	82	14.7.3.2	Exact Axial Color of a Thick Lens	111
14.4.1.6	The Anastigmatism Condition of Zinken/Sommer	82	14.7.3.3	General Approach for a System	112
14.4.1.7	Kingslake's H' - L Plot	83	14.8	Special Seidel Formulae	113
14.4.2	Conjugate Shift Equations	84	14.8.1	Third Order Aberrations of a Spherical Mirror	113
14.4.2.1	Seidel Formula	84	14.8.1.1	Seidel Representation	113
14.4.2.2	Special Correction Options	85	14.8.1.2	Direct Taylor Expressions	114
14.4.2.3	Bow-Sutton Condition	85	14.8.1.3	Special Cases	114
14.4.2.4	z -Invariant Correction	86	14.8.1.4	Example	115
14.4.3	Seidel Coefficients for General Changed Conditions	86	14.8.2	Seidel Aberrations of a Plane Plate	115
14.5	Aberration Coefficients	88	14.8.3	Air Lenses	115
14.5.1	Introduction	88	14.8.3.1	Introduction	115
14.5.2	Conventions and Conversions	88	14.8.3.2	Seidel Aberrations for a General Air Lens	117
14.5.3	Structural Aberration Coefficients	89	14.8.3.3	Air Lens with Equal Media	117
14.5.3.1	Introduction	89	14.8.3.4	Axial Colour Aberration of an Air Lens	118
14.5.3.2	Structural Aberration Coefficients for Third Order Aberrations	90		References	119
14.5.3.3	Analytic Coefficients for Special Setups	91	15	Wave Aberrations	121
14.5.3.4	Application Range of Structural Coefficients	91	15.1	Introduction	121
14.5.3.5	Geary's Cauf Plot	91	15.1.1	Meaning of Wave Aberrations	121
14.5.3.6	Examples	91	15.1.2	Definition of Wave Aberrations	122
14.5.4	Alternative Notations of Coefficients	94	15.1.3	Wave Aberration Description of Shack/Thompson	123

- 15.1.4 Expansion Conventions 124
- 15.1.5 Expansion Terms 124
- 15.1.6 Ray Aberrations 125
- 15.1.7 Chromatical Wave Aberrations 126
- 15.1.8 Additivity of Wave Aberration 127
- 15.1.9 Wavefront Tilt 128
- 15.1.10 Defocusing 128
- 15.1.11 Primary Wave Aberrations 129
- 15.1.12 Balancing of Wave Aberrations 129
- 15.2 Relation Between Ray and Wave Aberrations 130
- 15.2.1 Introduction 130
- 15.2.2 Approximate Relation 131
- 15.2.3 Selection of the Reference Sphere 132
- 15.2.4 Coordinate Mapping of the Wave Aberrations 132
- 15.2.5 Exact Relation 133
- 15.2.6 Numerical Conversion Between Rays and Wave 136
- 15.3 Zernike Polynomials 136
- 15.3.1 Introduction 136
- 15.3.2 Definition of the Zernike Polynomials 137
- 15.3.3 Zernike Expansion 141
- 15.3.4 Pros and Cons of a Zernike Representation 143
- 15.3.5 Linear Indexing and Conventions 144
- 15.3.6 Standard Zernike Functions 145
- 15.3.7 Calculation of Coefficients 145
- 15.3.7.1 Classical Methods 145
- 15.3.7.2 Fourier Spectrum of Zernikes 147
- 15.3.7.3 Robust Calculation of Zernikes via Jacobi Polynomials for High Orders 149
- 15.3.7.4 Sampling for the Calculation of Zernike Coefficients 149
- 15.3.7.5 Gram-Schmidt Orthogonalization 151
- 15.3.8 Field Dependence of Wave Aberrations and Zernike Expansions 152
- 15.3.8.1 Introduction 152
- 15.3.8.2 One-Dimensional Field-Dependence 152
- 15.3.8.3 General Two-Dimensional Field Dependence as a Taylor Series 153
- 15.3.8.4 Double Zernike Expansion 154
- 15.3.8.5 Examples 155
- 15.3.9 Special Issues and Conditions 156
- 15.3.9.1 Primary Aberrations as Zernikes 156
- 15.3.9.2 Defocus and Field Curvature 158
- 15.3.9.3 High NA Spherical Wave 158
- 15.3.9.4 Coefficients for Large Field Angles 160
- 15.3.9.5 Zernike Spectrum and Filtering 162
- 15.3.9.6 Zernikes in the Case of Vignetting 162
- 15.3.9.7 Zernike Representations of Local Perturbations 162
- 15.3.9.8 Conversion Between Zernike Functions and Monomials 165
- 15.3.9.9 Coefficients for Reversed Systems 168
- 15.3.9.10 Derivatives of Zernike Polynomials 168
- 15.3.9.11 Vector Zernike Polynomials 170
- 15.3.10 Coefficients Under Changed Conditions 171
- 15.3.10.1 Introduction 171
- 15.3.10.2 Influence of the Normalization Radius on the Zernike Coefficients 171
- 15.3.10.3 Change of the Zernikes Coefficients for a Decentred Pupil 172
- 15.3.11 Alternative Polynomials for Non-Circular Areas 173
- 15.3.11.1 Introduction 173
- 15.3.11.2 Numerical Orthogonalization for Arbitrary Boundaries 173
- 15.3.11.3 Examples 175
- 15.4 Quantitative Performance Criteria for Wave Aberrations 177
- 15.4.1 PV and Rms Value 177
- 15.4.2 Rayleigh Criterion 178
- 15.4.3 Marechal Criterion 179
- 15.4.4 Performance Assessment by Wave Aberrations 179
- 15.4.5 Quality Assessment by Zernikes 180
- 15.4.6 Energetic Weighting of Phase Aberrations 182
- 15.5 Propagation of Wave Aberrations 183
- 15.5.1 Introduction 183
- 15.5.2 Numerical Propagation Changes of Zernike Coefficients 184
- 15.6 Random and High-Frequency Wave Aberrations 186
- 15.6.1 Mathematical Description of Random Surfaces 186
- 15.6.2 Power Spectral Density of a Surface 187
- 15.6.3 Impact of Mid-Spatial Frequency Errors 187
- 15.6.4 Spatial Frequency Scaling 189
- 15.6.5 Atmospheric Turbulence 190
- 15.6.6 Zernike Expansion of Random Wavefronts 190
- 15.7 Further Examples 190
- 15.7.1 Introduction 190
- 15.7.2 Classical Representations of the Wavefront 192
- 15.7.3 Wavefront Surface Contributions 194
- 15.7.4 Zernike Coefficient Vector 198
- 15.7.5 Zernike Surface Contributions 198
- References 199
- 16 Aberrations with Diffraction 205**
- 16.1 Point Spread Function 205
- 16.1.1 Introduction 205
- 16.1.2 Spherical Aberration 207
- 16.1.3 Astigmatism 208

16.1.4	Coma	210	16.4.2.3	Integral Quality Factors	255
16.1.5	Geometrical Spot vs Diffraction PSF	212	16.4.3	OTF as a Function of the Field of View	256
16.1.6	Real Airy Size for Anisotropic Pupil Shapes	215	16.4.4	OTF in the Case of Aberrations	256
16.1.7	Selection of the Reference Ray	216	16.4.4.1	Introduction	256
16.1.8	Polychromatic PSF	217	16.4.4.2	Small Frequency Approximation	259
16.1.9	PSF with Aberrations for High NA and Polarized Pupils	218	16.4.4.3	Approximated Hopkins Ratio	260
16.1.10	PSF for Segmented Pupils	220	16.4.4.4	Diffraction-Limited Criterion for OTF	260
16.1.10.1	Introduction	222	16.4.4.5	PTF for Non-Symmetrical PSFs	261
16.1.10.2	Fourier Theory of Regular Segmented Pattern	222	16.4.4.6	MTF for Primary Aberrations	261
16.1.10.3	Diffraction Effects of Geometry and Number of Segments	224	16.4.5	OTF for a Cubic Phase Mask	262
16.1.10.4	Tip-Tilt and Piston Phase Errors	225	16.4.6	Examples for Degradation of Images	263
16.1.11	General Three-Dimensional Caustic Point Spread Distribution	226	16.5	Miscellaneous	267
16.1.11.1	Ideal Case	226	16.5.1	Degradation in Case of Scattering	267
16.1.11.2	Systems with Aberrations	228	16.5.2	Encircled Energy	268
16.1.12	Phase of the PSF	228	16.5.3	Edge Spread Function	268
16.1.12.1	Phase in the Image Plane	230	16.5.4	Line Spread Function	271
16.1.12.2	Phase in Defocused Planes	231		References	271
16.2	Strehl Ratio	232	17	Pupil Aberrations and Sine Condition	275
16.2.1	Definition	232	17.1	Pupil Aberrations	275
16.2.2	Approximations of the Strehl Ratio	233	17.1.1	Pupil Imaging	275
16.2.3	Problems and Limitations of the Strehl Definition	235	17.1.2	Pupil Sphere for Aplanatic Systems	275
16.2.4	Strehl in the Case of Non-Uniform Pupils	236	17.1.2.1	Introduction	275
16.2.5	Simple Analytical Relations	237	17.1.2.2	Photometrical Aspects of a Spherical Pupil	277
16.2.6	Strehl for Aberrations with Defocus	239	17.1.3	Aberrations of Pupil Imaging	277
16.2.7	Strehl for Aberration Balancing	239	17.1.3.1	Introduction	277
16.2.8	Strehl Ratio as Phase Histogram	240	17.1.3.2	Seidel Aberrations of the Pupil Imaging	279
16.2.9	Strehl for Statistical Aberrations	242	17.1.3.3	Pupil Aberrations in the Wave Picture	280
16.2.10	Strehl Calculation Using the Wigner Function	242	17.1.3.4	Tangent Condition	281
16.2.11	Strehl for Composed Phase Perturbations	243	17.1.3.5	Pupil Coma and Pupil Magnification	283
16.3	Alternative PSF Criteria	243	17.1.3.6	Chromatical Pupil Aberrations	284
16.3.1	Introduction	243	17.1.4	Pupil Distortion	284
16.3.2	Intensity Moments and Related Quantities	245	17.1.4.1	Introduction	284
16.3.3	Entropy Based Diameter	247	17.1.4.2	Pupil Grid Distortion in the Wave Picture	284
16.3.4	Two-Point Resolution	249	17.1.4.3	Photometrical Impact of the Pupil Distortion	285
16.3.4.1	Rayleigh Criterion	249	17.1.4.4	Pupil Distortion and Ray Aiming	285
16.3.4.2	Sparrow Criterion	249	17.1.4.5	Zernike Calculation in Case of Grid Distortion	288
16.3.4.3	Comparison and Discussion	250	17.1.5	Examples	290
16.3.4.4	Point Resolution for Partial Coherent Illumination	252	17.1.5.1	Telecentricity Error in a Scan System	290
16.4	Optical Transfer Function	253	17.1.5.2	Eyepiece with Kidney Bean Effect	290
16.4.1	Polychromatic Ideal MTF	253	17.1.5.3	Entrance Pupil Sphere in a Microscope Objective Lens	291
16.4.2	MTF Quality Assessment of Optical Systems	254	17.1.5.4	Pupil Walk in a Fish-Eye Lens	293
16.4.2.1	Hopkins Ratio	254	17.1.5.5	Stigmatic Imaging by an Elliptical Mirror	293
16.4.2.2	Test Charts	255	17.2	Sine Condition	294
			17.2.1	Introduction	294
			17.2.2	The Optical Sine Theorem	295
			17.2.3	Alternative Definitions of the Sine Condition	296

17.2.4	Sine Condition and Principal Sphere	297	18.1.2	Higher Order Aberrations Field vs Aperture	337
17.2.5	Sine Condition in Afocal Systems	298	18.1.3	Classification of Higher Order Aberrations	338
17.2.6	Sine Condition and Coma	299	18.1.4	Problems with Higher Order Expansions	340
17.2.6.1	Introduction	299	18.1.4.1	Selection of the Expansion Variable	340
17.2.6.2	Derivation of the Sine Condition by Coma-Correction	300	18.1.4.2	Ray Height at a Lens – the Thin Lens Dilemma	343
17.2.7	Generalized Sine Condition	302	18.1.5	Classification of Fifth Order Aberrations	347
17.2.8	Sine Condition Test and Misalignment	303	18.1.6	Spot Shapes of the Fifth Order Aberrations	348
17.2.9	Offence Against the Sine Condition	305	18.1.7	Scaling and Propagation of Aberrations	350
17.2.9.1	Definition	305	18.1.7.1	Converting the Surface Coefficients	350
17.2.9.2	Surface Contribution of OSC	306	18.1.7.2	Kerber's Transfer Formula	350
17.2.9.3	Coma Number	307	18.1.8	Buchdahl's Quasi Invariants	353
17.2.9.4	Vectorial OSC Condition	307	18.1.9	Convergence of Aberration Expansions	354
17.2.9.5	Coma Wave Aberration and OSC	307	18.1.10	The Special Shack Vector Multiplication	355
17.2.10	Field Dependence of Aberrations	308	18.2	Fifth Order Monochromatic Aberration Coefficients	355
17.2.10.1	Overview	308	18.2.1	Introduction	355
17.2.10.2	Vanishing Second Order Field Dependent Aberrations	308	18.2.2	Expansion of Buchdahl/Rimmer	356
17.3	Aplanatism	310	18.2.3	Other Expansion Approaches	358
17.3.1	Definition	310	18.3	Induced Aberrations	359
17.3.2	Aplanatic Spherical Surfaces	310	18.3.1	Introduction	359
17.3.3	General Aplanatic Surface	311	18.3.2	Phenomenological Understanding of Induced Aberrations	359
17.3.4	Aplanatic Spherical Lenses	313	18.3.3	Estimation of Induced Spherical Aberration	361
17.3.5	Comparison Ideal vs Stigmatic vs Aplanatic Imaging	313	18.3.4	Induced Aberrations in the Wave Picture	363
17.3.6	Higher Order Aplanatism	314	18.3.5	The Impact of the Stop on Induced Aberrations	365
17.3.7	Aplanatic Optical Systems	315	18.3.6	Examples for Induced Aberrations	365
17.3.7.1	Introduction	315	18.3.6.1	Endoscope Lens Duct	365
17.3.7.2	Two-Surface Landscape	315	18.3.6.2	Microscope Objective Lenses	366
17.3.7.3	Aspherical and Freeform Systems	318	18.3.6.3	Photographic Lenses	367
17.4	Isoplanatism	318	18.3.6.4	Diverse Systems	367
17.4.1	Introduction	318	18.4	Wave Aberrations of Sixth Order	367
17.4.2	Condition of Staebler/Lihotzky	320	18.4.1	Introduction	367
17.4.3	Isoplanatism as Wave Aberrations	321	18.4.2	Intrinsic Aberrations of Sixth Order	373
17.4.4	Off-Axis Isoplanatism	321	18.4.3	Aberration Coefficients for Stop Shifting	373
17.4.5	Berek's Formulations	322	18.4.4	Connection Between Image and Pupil Coefficients	374
17.4.6	Isoplanatic Patch Size	322	18.4.5	Transverse Aberrations of Fifth Order	374
17.5	General Invariants	323	18.5	Exact Aberration Formulas	375
17.5.1	The General Invariant of Welford	323	18.5.1	Aldis' Theorem	375
17.5.2	Smith's Cosine Invariant	324	18.5.1.1	Introduction	375
17.5.3	Skew Invariant	325	18.5.1.2	Derivation of Aldis' Theorem	376
17.5.4	Abbe's Sine Condition	326	18.5.1.3	Brewer's Generalization of Aldis' Theorem	378
17.5.5	Herschel's Condition and Volume Imaging	326	18.5.1.4	Derivation of Brewer's Formula	378
17.5.6	Overview About Invariants and Conditions	328	18.5.1.5	Implementation Aspects	379
	References	329	18.5.1.6	Examples	381
18	Higher Order and Induced Aberrations	333	18.5.2	Approach of Andersen	383
18.1	Higher Order Aberrations	333	18.5.3	Exact Formulas for Spherical Aberration	384
18.1.1	Introduction	333	18.5.3.1	Introduction	384

18.5.3.2	Delano's Formula	384	18.9.4.1	Surface Resolved Transverse Colour of Higher Order	418
18.5.3.3	Conrady's Ray Formula	387	18.9.4.2	Lens Resolved Lateral Chromatical Aberration	418
18.5.3.4	Conrady's OPD-Formula	388	18.9.5	Wave-Optical Description of Colour Aberrations	419
18.5.3.5	Kerber's Representation	390	18.9.5.1	Induced Colour Aberrations	419
18.6	Fifths Order Spherical Aberration of a Surface	390	18.9.5.2	Wave-Based Axial Colour Aberration at High Aperture	419
18.6.1	Introduction	390	18.9.6	Sphero-Chromatism	420
18.6.2	Seidel Third Order Contribution	390	18.9.6.1	Introduction	420
18.6.3	Expansion by Fermat's Principle	390	18.9.6.2	Surface Contribution of Sphero-Chromatism in Third Order	421
18.6.4	Other Expansion Approaches	392	18.9.6.3	Approximation of Slyusarev	423
18.6.5	Spherical Aberration of a Mirror Surface	392	18.9.6.4	Example Split Achromate	424
18.7	Fifth Order Spherical Aberration of a Thin Lens	393	18.9.7	Chromatical Variation of Primary Aberrations	425
18.7.1	Introduction	393	18.9.8	Introduction	425
18.7.2	Thin Lens Expansion by Fermat's Principle	393	18.9.8.1	Ochse's Approach of Colour Variation of Seidel Aberrations	426
18.7.3	Spherical Aberration of Thin Lens Systems	394	18.9.8.2	Achromatized Petzval Condition	428
18.7.3.1	General	394	18.10	Structural Aberration Coefficients of Higher Order	429
18.7.3.2	System with Two Thin Lenses	395	18.11	System Reversibility	430
18.8	Examples for Monochromatic Higher Order Aberrations	396	18.11.1	Introduction	430
18.8.1	Merte Surface	396	18.11.2	Examples	431
18.8.2	Microscope Lens	398	18.11.2.1	Two Identical Unity Relays	431
18.8.3	Interferometer Lens	398	18.11.2.2	Two Unity Magnifications with Different Focal Lengths and Field Lens	433
18.8.4	Microscope Monochromate	399	18.11.2.3	Symmetrical Thick Lens	434
18.8.5	Spherical Aberration of an Achromate	399	18.11.3	Microscope Lens	435
18.8.6	System Corrected for Fourth and Sixth Order	403		References	436
18.9	Higher Order Colour Aberrations	403	19	Proximate Raytrace, Astigmatism and Field Curvature	439
18.9.1	Introduction	403	19.1	Introduction	439
18.9.1.1	Overview	403	19.1.1	Local Curvature Analysis of Surfaces	440
18.9.1.2	Wynne's Theory of Chromatical Aberrations	407	19.1.1.1	Differential Geometry of Surfaces	440
18.9.1.3	Definition of the Secondary Spectrum	407	19.1.1.2	S- and T-Curvature of Spheres	443
18.9.1.4	Lens Contribution of the Secondary Spectrum	409	19.1.2	Taylor Expansion Around a Ray	443
18.9.2	Higher Order Axial Chromatical Aberration	410	19.1.2.1	Introduction	443
18.9.2.1	Introduction	410	19.1.2.2	Numerical Calculation of the Derivatives	444
18.9.2.2	Surface Resolved Axial Colour of Second Order	410	19.1.2.3	Accuracy of Finite Differences	445
18.9.2.3	The Formulation of Münz/Sacks	411	19.2	Proximate Raytrace	449
18.9.2.4	Higher Order Axial Colour of a Lens Doublet	412	19.2.1	Introduction	449
18.9.3	Examples for Second Order Axial Colour Aberration	413	19.2.2	Proximate Raytrace in Two Dimensions	450
18.9.3.1	Gabor Telescope	413	19.2.2.1	Expansion Orders in Proximate Raytrace	450
18.9.3.2	Schupman Achromate	413	19.2.2.2	Expansion Variables and Induced Effects	452
18.9.3.3	Two-Lens Apochromate	415	19.2.2.3	Transition and Offset	453
18.9.3.4	Broadband Application	415	19.2.2.4	Refraction	453
18.9.4	Transverse Chromatical Aberration	418	19.2.2.5	Matrix Formulation	453

19.2.2.6	Using Proximate Raytrace for Aberration Calculations	454	19.3.5.8	Astigmatism-Removal by Two-Mirror with x - and y -Tilts	484
19.2.3	Proximate Raytrace in Three Dimensions	456	19.3.5.9	Mirror Systems with Freeform Surfaces	485
19.2.3.1	Introduction	456	19.4	Field Curvature	486
19.2.3.2	Workflow	458	19.4.1	Introduction and Repetition of the Seidel Representation	486
19.2.3.3	Shift Transition onto a Surface	460	19.4.2	Longitudinal Interpretation	487
19.2.3.4	Refraction	460	19.4.3	Zernikes Description of Field Curvature	488
19.2.3.5	Back-Shift into the Vertex Plane	460	19.4.4	Summary of Formulae for Astigmatism and Field Curvature	488
19.2.4	Analytical Matrix Elements for Special Systems	461	19.4.5	A Remark on Curved Object, Image, Symmetry and Paraxiality	489
19.2.4.1	Introduction	461	19.4.6	Higher Order Field Curvature	490
19.2.4.2	Transition	461	19.4.7	Example for a Curved Image Surface	491
19.2.4.3	Refraction at a Spherical Surface	461	19.4.8	Distortion and Vignetting for Curved Images	492
19.2.4.4	Polynomial Mirrors with x -Symmetry	461	19.4.9	Field Curvature in Freeform Systems	493
19.2.4.5	Block Structure of Matrices According to Caron-Bäumer	462		References	494
19.2.5	Araki's Formulation of Aberrations by Proximate Raytrace	462	20	Vectorial Aberration Theory	499
19.2.6	Examples with Proximate Raytrace	463	20.1	Fundamentals	499
19.2.6.1	Spherical Aberration Surface Contributions in Different Orders	463	20.1.1	Introduction	499
19.2.6.2	Geometrical Spot Diagram	463	20.1.2	Parabasal Aberration Reference	500
19.2.6.3	Wave Aberration	464	20.1.3	Expansion Options	504
19.3	Astigmatism	464	20.1.3.1	Introduction	504
19.3.1	Introduction	464	20.1.3.2	Vectorial Formulation	505
19.3.2	Coddington Equations	464	20.1.4	Non-Circular Pupils	506
19.3.2.1	Introduction	464	20.1.5	Full-Field Diagrams for Performance Evaluation	507
19.3.2.2	Sagittal Astigmatic Image Location	466	20.1.6	Phase-Space Representation	510
19.3.2.3	Tangential Astigmatic Image Location	466	20.1.6.1	Introduction	511
19.3.2.4	Coddington Equation for an Arbitrary Azimuth	469	20.1.6.2	Examples	512
19.3.3	Wave Picture in Three Dimensions	470	20.2	Low Orders of Non-Symmetric Systems	516
19.3.3.1	Introduction	470	20.2.1	Introduction	516
19.3.3.2	Coordinate Systems	470	20.2.2	Types of Wave Aberrations	517
19.3.3.3	Initial Parameter	471	20.2.3	Image Plane Tilt	518
19.3.3.4	Refraction of the Chief Ray	472	20.2.4	Curved Image Shells	519
19.3.3.5	Parabasal Refraction of a Wave	472	20.3	Wave Description by Zernike Polynomials	520
19.3.3.6	Transfer Between Surfaces	472	20.3.1	Introduction	520
19.3.3.7	Curvature Matrix Formulation	472	20.3.2	Field Dependence by Zernike Coefficients	521
19.3.3.8	Parabasal $ABCD$ -Matrix	474	20.3.3	Examples	522
19.3.3.9	Matrix Reconstruction by Rays	475	20.3.4	Zernike Surface Contributions by Tang's Approach	524
19.3.3.10	Extension to Higher Orders	477	20.3.4.1	Introduction	524
19.3.4	The Principal Imperfect Astigmatic Reference	477	20.3.4.2	Calculation Scheme	525
19.3.5	Astigmatism of Mirror Systems	478	20.3.4.3	Example	527
19.3.5.1	Introduction	478	20.4	Nodal Aberration Theory	527
19.3.5.2	Seidel Astigmatism at a Spherical Mirror	478	20.4.1	Introduction	527
19.3.5.3	Astigmatism of Skew Mirrors	479	20.4.2	Basic Idea of Nodal Points	530
19.3.5.4	Schiefspiegler Principle	480	20.4.3	Aberrations Due to General Geometry	531
19.3.5.5	Two-Mirror Schiefspiegler	481	20.4.4	Numerical Calculation of the Nodal Points	533
19.3.5.6	Linear Astigmatism of Confocal Conics	483	20.4.4.1	Introduction	533
19.3.5.7	Coma at Skew Spherical Mirror Systems	483			

20.4.4.2	Example	534	20.8.3	Ray-Based Lateral Chromatical Aberration	574
20.4.5	Nodal Points of Primary Aberrations	534	20.8.4	Numerical Wave Aberration Calculation	575
20.4.5.1	Introduction	534	20.8.5	Examples	575
20.4.5.2	Primary Coma	536		References	577
20.4.5.3	Primary Astigmatism	538	21	Special Aberration Topics	583
20.4.5.4	Distortion	540	21.1	Distortion	583
20.4.5.5	Focus and Field Curvature	541	21.1.1	Introduction	583
20.4.6	Nodal Properties of Higher Order and Non-Spherical Surfaces	541	21.1.2	Geometrical Definitions of Distortion	583
20.4.6.1	Introduction	541	21.1.2.1	Introduction	583
20.4.6.2	Spherical Aberration	542	21.1.2.2	Primary Distortion Revisited	584
20.4.6.3	Higher-Order Coma	542	21.1.2.3	f -Theta Distortion	586
20.4.6.4	Higher-Order Astigmatism	543	21.1.2.4	TV-Distortion	587
20.4.7	Nodal Points for Aspherical and Freeform Systems	544	21.1.2.5	Centroid Based Distortion – Vignetting and Ray Aiming	588
20.4.7.1	Introduction	544	21.1.2.6	Telecentricity Errors	589
20.4.7.2	Nodal Point Changes Due to Pupil Shift	544	21.1.3	Physical Based Modelling of Distortion	591
20.4.7.3	Aberrations of an Asphere in Collimated Light	545	21.1.3.1	Distortion as Wave Aberration	591
20.4.7.4	Aberrations in Case of General Surface Shapes	546	21.1.3.2	Distortion Definition by PSF	592
20.4.8	Nodal Points for Zernike Pupils	547	21.1.3.3	Coma with Defocus and Caustic Bending	593
20.4.8.1	Introduction	547	21.1.3.4	Distortion by the Optical Transfer Function	593
20.4.8.2	Nodal Points of Primary Astigmatism	548	21.1.3.5	Shibuya's General Definition of Distortion	595
20.4.8.3	Example	549	21.1.3.6	Distortion Due to Object Depth	595
20.4.9	Nodal Lines	549	21.1.4	Aberration Theory of Distortion	596
20.4.10	Aberration Balancing	550	21.1.4.1	Tangent Condition	596
20.4.11	Approximations and Limitations	551	21.1.4.2	Change of Distortion with Magnification and Spherical Pupil Aberration	596
20.5	Plane Symmetric Systems	553	21.1.4.3	Pseudo Surface Contributions of Distortion	598
20.5.1	Aberrations of Plane Symmetric Systems	553	21.1.4.4	Similarity with Chromatical Magnification Difference	599
20.5.2	Image Tilt, Reference and Concatenation	555	21.1.5	Generalized Geometries	599
20.5.3	Nodal Distributions for Plane Symmetric Systems	555	21.1.5.1	Introduction	599
20.5.4	Structural Aberration Coefficients	556	21.1.5.2	Polynomial Expansion of Distortion	601
20.5.5	Examples	558	21.1.5.3	Example Head-Mounted Device	601
20.5.6	Aberrations of Confocal Plane Symmetric Systems	558	21.1.6	Keystone Distortion	603
20.6	Anamorphic Systems	559	21.1.6.1	Scheimpflug Setup	603
20.6.1	Introduction	559	21.1.6.2	Digital Projection Lenses	604
20.6.2	Wave Aberrations	562	21.1.6.3	Smile and Keystone Distortion in Spectrometers	604
20.6.3	Aberration Coefficients for Biconic Surfaces	563	21.1.7	Lens Mapping Function	605
20.6.4	Nodal Points of Anamorphic Systems	565	21.1.7.1	Introduction	605
20.6.5	Examples	565	21.1.7.2	Simple Analytical Mapping Approaches	605
20.7	Aberrations Due to Misalignment	567	21.1.7.3	Digital Distortion Calibration	606
20.7.1	Introduction	567	21.1.7.4	Scheimpflug Camera Distortion Model	607
20.7.2	Principles of Misalignment-Induced FFD Changes	568	21.2	Symmetry Principle	608
20.7.3	Examples	570	21.2.1	Introduction	608
20.8	Chromatical Aberrations in Non-Symmetrical Systems	570	21.2.2	Point Symmetry Around the Stop	609
20.8.1	Introduction	571	21.2.3	Mirror-Symmetric Systems	609
20.8.2	Ray-Based Axial Chromatical Aberration	572	21.2.4	Examples	611
			21.2.5	Double-Pass Systems	611
			21.2.6	Sasian's Symmetry Parameter	614

21.3	Monocentric Systems	615	21.6.2.4	Visual Depth Criterion of Abbe and Berek	643
21.3.1	Introduction	615	21.6.2.5	Moment Diameter for Spherical Aberration	
21.3.2	Paraxial Parameter	617		Caustic	644
21.3.3	The Monocentric Invariant	617	21.6.2.6	Change of Depth of Focus with Field	
21.3.4	Raytracing	618		Position	646
21.3.5	Vignetting in Monocentric Ball Lenses	618		References	649
21.3.6	Aberrations of Monocentric Systems	619	22	Aberrations of Special	
21.3.6.1	Introduction	619		Components	653
21.3.6.2	Spherical Aberration	619	22.1	Introduction	653
21.3.6.3	Axial Chromatical Aberration of Monocentric		22.2	Aspheres	653
	Systems	620	22.2.1	Introduction	653
21.3.7	Examples	621	22.2.2	Seidel Contributions of Aspheres	654
21.3.7.1	Introduction	621	22.2.3	Sixth Order Aberrations of Aspheres	655
21.3.7.2	Layered Ball Lenses	622	22.2.4	Structural Aberration Coefficients of	
21.3.7.3	Bouwer's Monocentric Corrector			Aspheres	655
	Meniscus	623	22.2.5	Coddington Equations for Aspheres	656
21.3.8	Array Cameras	624	22.2.6	Reflective Aspheres	656
21.4	Aberrations of Slightly Misaligned		22.2.7	Examples	656
	Surfaces	624	22.2.8	The See-Saw Plate Diagram of Burch	657
21.4.1	Introduction	624	22.2.8.1	Introduction	657
21.4.2	Seidel Primary Aberration Approach	625	22.2.8.2	Example Cassegrain Telescope	660
21.4.3	Wave-Optical Model	626	22.3	Cylindrical and Toroidal Lenses	660
21.4.3.1	Perturbation Approach of Rimmer	626	22.3.1	Introduction	660
21.4.3.2	General Wave-Optical Treatment	627	22.3.2	Spherical Aberration of a Toric Lens	661
21.4.4	Field-Dependence of Coma and		22.3.3	Primary Aberrations of Cylindrical Lens	661
	Astigmatism	628	22.3.4	Anamorphic Aberration Coefficients of a	
21.4.5	Misaligned Single Surfaces	628		Cylindrical Lens	662
21.4.5.1	Misaligned Single Spherical Surface	628	22.4	Gracing Incidence Surfaces	662
21.4.5.2	Spherical Off-Axis Mirror	629	22.4.1	Introduction	662
21.4.5.3	Tilted Parabolic Mirror	630	22.4.2	Imaging by Wolter-Telescopes	663
21.4.6	Example	630	22.4.2.1	Fundamentals	663
21.5	Special Properties of Spherical Aberration	633	22.4.2.2	Primary Aberrations	664
21.5.1	Spherical Aberration of a Refractive		22.4.3	General Analysis of Aberrations	665
	Surface	633	22.5	Fresnel Lenses	666
21.5.2	Refractive vs Reflective Components	635	22.5.1	Introduction	666
21.5.3	Spot Size of a Plane-Convex Lens for Spherical		22.5.2	Refined Discretized Model	667
	Aberration	635	22.5.3	Primary Aberrations of Segmented Fresnel	
21.5.4	Stopping Down a Lens	637		Surface Zones	669
21.5.5	Best Focus Criteria	638	22.5.4	Seidel Aberrations of Fresnel Lenses	670
21.5.6	Spherical Aberration of a Plane Interface	638	22.6	Aberrations of Gratings	671
21.6	Focus Definition and Depth of Field	639	22.6.1	Introduction	671
21.6.1	Definition of the Focus Plane	639	22.6.2	Aberrations of a Plane Linear Grating	672
21.6.1.1	Introduction	639	22.6.3	Aberrations of Reflective Concave	
21.6.1.2	Geometrical Aberration Criterion	640		Gratings	673
21.6.1.3	Criteria Using the Wave Aberration	641	22.6.4	Spectroscopic Performance Description	676
21.6.1.4	Criteria with Diffraction	641	22.6.5	Examples	677
21.6.1.5	Examples	641	22.6.5.1	Rowland Spectrometer	677
21.6.2	Depth of Focus	642	22.6.5.2	Cerny-Turner Spectrometer	678
21.6.2.1	Geometrical Depth Estimation for a Given Pixel		22.7	Diffraction Elements	680
	Size	642	22.7.1	Introduction	680
21.6.2.2	Diffraction Caustic and Rayleigh Length	642	22.7.2	Monochromatic Seidel Aberrations of	
21.6.2.3	Combined	642		Diffraction Surfaces	681

22.7.2.1	Introduction	681	23.1.2	Distortion Impact on the Intensity Distribution	719
22.7.2.2	Diffraction Focusing Lens	682	23.1.3	Distortion of the Pupil	720
22.7.2.3	Diffraction Landscape Lens	682	23.1.4	Aberration Theory and Irradiance	722
22.7.3	Chromatical Aberrations	682	23.1.4.1	Introduction	722
22.7.3.1	Dispersion	682	23.1.4.2	Photometric Law and Aberrations	723
22.7.3.2	Chromatical Seidel Coefficients	683	23.1.4.3	Exit Pupil Irradiance	723
22.7.3.3	Achromatic Hybrid Lens	684	23.1.4.4	Image Plane Irradiance	723
22.7.3.4	Efficiency Achromatization	686	23.1.5	Geometrical Spot Intensities	724
22.7.3.5	Multi-Order Harmonic Diffraction Lenses	688	23.1.5.1	Introduction	724
22.7.4	Example	689	23.1.5.2	Numerical Examples	726
22.8	Meta-Surfaces	690	23.2	Caustics	727
22.8.1	Introduction	691	23.2.1	Introduction	727
22.8.2	Simple Meta Lens Solutions	692	23.2.2	Caustics as Ray Envelopes	729
22.8.3	Example Systems	692	23.2.3	Ray and Wave Picture of Caustics	729
22.8.4	Seidel Aberrations of a Negative-Index-Lens	693	23.2.4	Calculation of Ray Caustics	730
22.9	Gradient Index Lenses	695	23.2.4.1	Introduction	731
22.9.1	Introduction	695	23.2.4.2	Caustic Surfaces of Wavefront Aberrations	732
22.9.2	Monochromatic Seidel Aberrations of GRIN Lenses	696	23.2.4.3	Circular Symmetry	733
22.9.2.1	Introduction	696	23.2.5	Caustic Systematic as Catastrophe Types	735
22.9.2.2	Paraxial Reference	696	23.2.6	Caustic as a Legendre Transform of Transverse Ray Aberrations	737
22.9.2.3	Aberration Calculation Method	697	23.2.7	Caustic in Phase Space	737
22.9.2.4	Inhomogeneous Surface Terms	699	23.2.8	Diffraction at Caustics	739
22.9.2.5	Transfer Terms	701	23.3	Polarization Aberrations	740
22.9.3	Radial GRIN Lenses	702	23.3.1	Introduction	740
22.9.3.1	Astigmatism of Radial GRIN Lenses	702	23.3.1.1	Jones Representation	741
22.9.3.2	Field Curvature of Wood Lenses	703	23.3.1.2	Jones Matrix of a Ray	743
22.9.3.3	Chromatical Seidel Aberrations of Radial Gradient Lenses	705	23.3.1.3	Polarization Raytrace and Coordinate Transforms	743
22.9.4	Axial Gradients	706	23.3.1.4	Extraction and Decomposition of the Jones Matrix	745
22.9.4.1	Introduction	706	23.3.1.5	Diattenuation and Retardance	746
22.9.4.2	Monochromatic Seidel Aberrations of Axial GRIN Media	706	23.3.1.6	Fresnel Coefficient Behaviour	746
22.9.4.3	Linear Axial Gradient Lenses	706	23.3.2	Polarization Aberrations	747
22.9.4.4	Chromatical Seidel Aberrations of Axial Gradient Lenses	707	23.3.2.1	Jones Pupil	747
22.9.5	Generalized GRIN Approaches	708	23.3.2.2	Seidel Decomposition of Polarization Aberrations	749
22.9.5.1	Freeform GRIN Profiles	708	23.3.2.3	Surface Decomposition of the Jones Matrix	750
22.9.5.2	Chromatical Properties of Blended Composites	708	23.3.2.4	Vectorial Zernike Basis	751
22.9.6	Comparison of GRIN with Conventional Lenses	710	23.3.2.5	Orientators	752
22.9.7	Examples	710	23.3.2.6	Orientation Zernike Polynomials	753
22.9.7.1	One-Pitch-Lens	710	23.3.2.7	Amplitude Response Matrix and PSF	754
22.9.7.2	Wood or Rod Lenses	710	23.3.2.8	Performance Criteria	757
22.9.7.3	Seidel	712	23.3.3	Example Calculations	758
22.9.7.4	Example of Freeform GRIN Lens	713	23.3.3.1	Refractive System	758
	References	714	23.3.3.2	Reflective System	758
			23.3.3.3	Corner Cube Retroreflector	758
			23.3.3.4	PSF and ARM Example	761
				References	761
23	Special Physical Related Aberrations	719		Index	765
23.1	Irradiance Effects of Aberrations	719			
23.1.1	Introduction	719			