

## Answers of Problems

### Chapter 1

#### 1.1

- a)  $[ML^{-1}T^{-2}]$ , Pa  
 b)  $[ML^2T^{-3}]$ ,  $J s^{-1}$  (W)  
 c)  $[LT^{-2}\theta]$ ,  $J kg^{-1}K^{-1}$

#### 1.2.

$[ML^{-1}T^{-2}]/[L] = [ML^{-3}][LT^{-2}]$ . Therefore, it is dimensionally sound.

#### 1.3.

a)

$$\begin{aligned} 1 \text{ bar cm}^3 &= 10^5 \text{ Pa cm}^3 \\ &= 10^5 \text{ N cm}^3 \text{ m}^{-2} \\ &= 10^5 \times 10^{-6} \text{ N m} \\ &= 0.1 \text{ J} \end{aligned}$$

b)

$$\begin{aligned} 1 \text{ lb}_f \text{ in}^{-2} &= 4.44822 \text{ N in}^{-2} \\ &= (39.370)^2 \times 4.44822 \text{ Pa} \\ &= 6.895 \times 10^3 \text{ Pa} \end{aligned}$$

$$25.3 \times 6.895 \times 10^3 = 1.744 \times 10^5 \text{ Pa}$$

#### 1.4.

The weight of a body is the gravitational force acting on the body, that is,

*Biochemical Engineering: A Textbook for Engineers, Chemists and Biologists*, Second Edition.

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(mass)  $\times$  (gravitational acceleration,  $g$ ). Strictly speaking, weight will vary slightly with locations and altitudes on the Earth.

**1.5.**

Under low pressure we may assume ideal gas behavior for gas phase.

Assume  $x = 0.01$

pressure of  $\text{NH}_3 = 2.70 \times 0.01 = 0.027 \text{ atm}$

Because of the low mole fraction of  $\text{NH}_3$ , we can use the molar volume of pure water, that is  $18.01/998 = 0.01805 \text{ m}^3 \text{ kmol}^{-1}$ . The molar concentration of  $\text{NH}_3$  in water is  $0.01/0.01805 = 0.554 \text{ kmol m}^{-3}$ . Therefore,  $H = 0.027/0.554 = 0.0487 \text{ atm m}^3 \text{ kmol}^{-1}$ .

At 1 atm  $y = 0.027/1 = 0.027$

$m = y/x = 0.027/0.01 = 2.70$

(If  $m$  is defined as  $m = C_G/C_L$ ,  $C_G = n/V = p/(RT)$ )

$$= 12.18 p/T = 12.18 \times 0.027/293$$

$$= 1.122 \times 10^{-3}$$

$$m = 1.122 \times 10^{-3}/0.554 = 0.002026)$$

**1.6.**

Assuming the ideal gas law, the total mole of  $200 \text{ m}^3$  air + methane is

$$1(\text{atm}) \times 200 \text{ m}^3 = n0.08206 \times 293 \quad n = 8.318 \text{ kmol}$$

Mole number of methane to be removed is

$$8.318 \times 0.20 \times 0.99 = 1.647 \text{ kmol h}^{-1}$$

Mole fraction of methane in water at a partial pressure of 0.20 atm is

$$x = p/H = 0.20/(3.76 \times 10^4) = 5.32 \times 10^{-6}$$

Density of water at  $20^\circ \text{C}$  is  $998 \text{ kg m}^{-3}$ , and thus

$$\text{Mole number of water is } 998/18 = 55.4 \text{ kmol m}^{-3}$$

Mole number of methane dissolved in water is

$$55.4 \times 1000 \times 5.32 \times 10^{-6} = 0.295 \text{ mol m}^{-3}$$

A minimum amount of water needed is

$$1647/0.295 = 5580 \text{ m}^3 \text{ h}^{-1}$$

1.7.

a)  $PE = 1.0 \times 9.8 \times 10 = 98 \text{ J}$

b)  $KE = 1.0 \times u^2/2 = 98 \text{ J}$     $u = 14 \text{ m s}^{-1}$

1.8.

Heat to be removed is

$$204.3 \times 100 = 2.403 \times 10^4 \text{ kcal h}^{-1}$$

Specific heat capacity of water is  $1.0 \text{ kcal kg}^{-1} \text{ }^\circ\text{C}^{-1}$

Water needed is

$$2.403 \times 10^4 / [1.0(30 - 20)] = 2403 \text{ kg h}^{-1}$$

1.9.

$$\Delta H = Q = 0.950 \times 1.02 \times 1000 \times (135 - 95) = 38\,760 \text{ kcal h}^{-1}$$

$$\text{saved heat } (1 - 38\,760/58\,140) \times 100 = 33\%$$

## Chapter 2

2.1.

The heat transfer rate for the radial direction is  $Q = -\kappa A(dT/dr)$ .

For the tube length  $L$ , the area  $A$  at  $r = r$  is;  $A = 2\pi rL$

Thus,  $Q = 2\pi \kappa rL(dT/dr)$  and integration with the inside surface temperature  $T_i$  ( $^\circ\text{C}$ ) at  $r = r_i$  and outside temperature  $T_o$  ( $^\circ\text{C}$ ) at  $r = r_o$  gives

$$Q = 2\pi\kappa L(T_i - T_o) / \ln(r_o/r_i)$$

2.2.

By interpolation, the viscosity of water at  $37^\circ\text{C}$  is  $0.69 \times 10^{-3} \text{ Pa s}$

$$(2.1 \times 10^{-9} \times 1.00 \times 10^{-3})/293 = (D_{\text{at } 37^\circ\text{C}} \times 0.69 \times 10^{-3})/310$$

$$D_{\text{at } 37^\circ\text{C}} = 3.2 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$$

2.3.

From the force balance acting on an imaginary coaxial fluid cylinder of radius  $r$  and length  $L$ , the velocity distribution in laminar flow through a straight round tube is given as follows,

$$\pi r^2 \Delta P = -2\pi rL\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{1}{2} \frac{\Delta P}{L\mu} r$$

$$u = -\frac{\Delta P}{4L\mu} r^2 + C$$

$$\text{At } r = r_i \quad u = 0$$

$$u = \frac{\Delta P r_i^2}{4L\mu} \left( 1 - \frac{r^2}{r_i^2} \right)$$

Total volumetric flow rate is

$$F = \int_0^{r_i} 2\pi r u \, dr = \frac{\pi \Delta P r_i^4}{8L\mu}$$

Averaged flow rate  $v$

$$v = F/(\pi r_i^2) = \frac{\Delta P r_i^2}{8L\mu}$$

$$\text{Therefore, } \frac{u}{v} = 2 \left( 1 - \frac{r^2}{r_i^2} \right) \quad (2.8)$$

$$\text{and } v = F/(\pi r_i^2) = \frac{\Delta P r_i^2}{8L\mu} \quad (2.9)$$

#### 2.4.

$$(\text{Re}) = dv\rho/\mu = 0.03 \times 1.97 \times 800/2 = 23.6$$

Laminar flow; from Equations 2.10 and 2.12

$$\begin{aligned} \Delta P &= 2f\rho v^2(L/d) = 2 \times 16 \times 800 \times 1.97^2 \times 50/(23.6 \times 0.03) \\ &= 7.02 \times 10^3 \text{ kPa} = 69.3 \text{ atm} \end{aligned}$$

#### 2.5.

By use of the partial pressure difference

$$k_{Gp}(17.5/760 - 15/760) = 0.0041 \times 10/18.01$$

$$k_{Gp} = 0.692 \text{ kmol h}^{-1} \text{ m}^{-2} \text{ atm}^{-1}$$

From Example 2.4

$$k_{Gc} = 0.0821 \times (273 + 22.5) \times k_{Gp} = 16.8 \text{ m h}^{-1}$$

$$\begin{aligned} \Delta y_f &= D_{\text{H}_2\text{O}}/k_{Gc} = (0.25 \times 3600/10\,000)/16.8 = 5.36 \times 10^{-3} \text{ m} \\ &= 5.36 \text{ mm} \end{aligned}$$

2.6.

From Equation 2.17

$$h = \kappa / \Delta y_f \Delta y_f = \kappa / h = 0.55 / 2300 = 2.39 \times 10^{-4} \text{ m} \\ = 239 \mu\text{m}$$

2.7.

From Equation 2.6

$$\tau = K(du/dy)^n$$

At a shear rate of  $60 \text{ s}^{-1}$

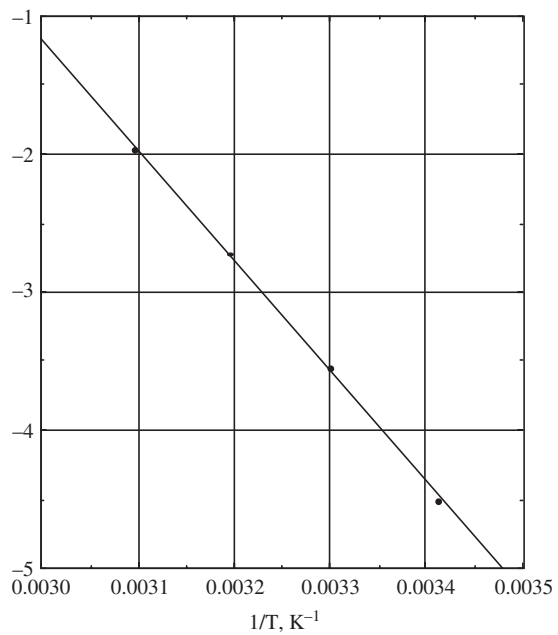
$$\tau = 0.415(60)^{1.23} = 63.9 \text{ Pa}$$

$$\mu_a = 63.9 / 60 = 1.07 \text{ Pa s}$$

### Chapter 3

3.1.

According to Equation 3.7  $\ln(k)$  is plotted against  $1/T$

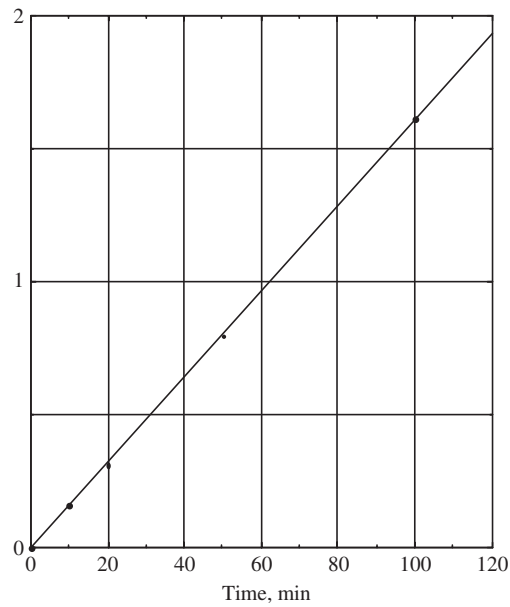


The slope of the straight line through the data points ( $-E_a/R$ ) is  $-8010$  K. Thus, the activation energy is given as:

$$\begin{aligned} E_a &= 8010 \times 8.31 \\ &= 6.66 \times 10^4 \text{ kJ kmol}^{-1} \end{aligned}$$

### 3.2.

By use of Equation 3.11  $\ln(C_{A0}/C_A)$  is plotted against. The slope of the straight line ( $k$ ) is  $0.0161 \text{ min}^{-1}$ . Thus, this is the first order reaction, and the rate constant is  $0.0161 \text{ min}^{-1}$ .



### 3.3.

If the reactants A and B are introduced in the stoichiometric ratio, the following equations hold, that is  $C_{A0} = C_{B0}$  and  $C_A = C_B$ .

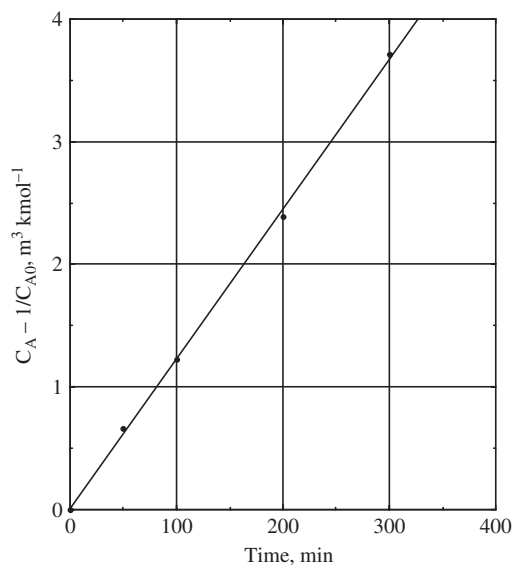
$$-r_A = -\frac{dC_A}{dt} = kC_A^2$$

By integration with the initial condition, at  $t=0$ ,  $C_A = C_{A0}$ .

$$\frac{1}{C_A} - \frac{1}{C_{A0}} = \frac{1}{C_{A0}} \frac{x_A}{1-x_A} = kt$$

**3.4.**

By plotting  $1/C_A - 1/C_{A0}$  against  $t$ , a straight line is obtained.



The slope of the line is 0.012, and thus the second-order rate constant is  $0.012 m^3 kmol^{-1} min^{-1}$ .

**3.5.**

The half time  $t_{1/2}$  is

$$\begin{aligned} t_{1/2} &= \ln 2 / kd \\ &= 0.693 / 0.001 = 690 \text{ s} = 11.5 \text{ min} \end{aligned}$$

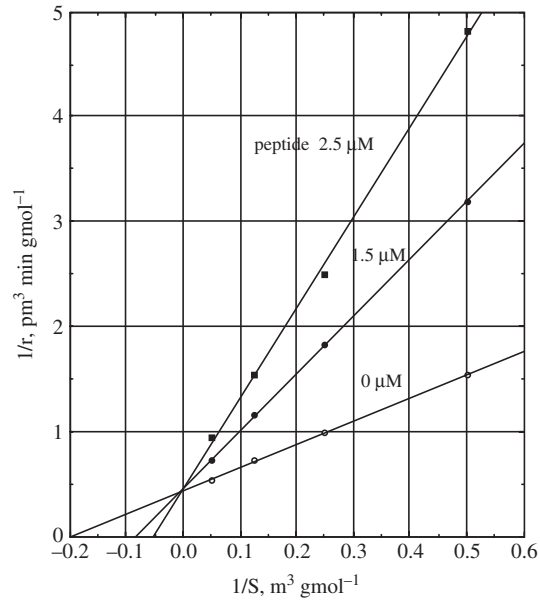
**3.6.**

- 1) Initial rate  $0.046 \text{ gmol m}^{-3} \text{ min}^{-1}$
- 2) Production of *p*-nitrophenol in  $\mu\text{mol cm}^{-3} \text{ min}^{-1}$  is 0.046. Thus, 0.046 unit  $\text{cm}^{-3}$ .

**3.7.**

By the lineweaver–Burk plot

$$\begin{aligned} K_m &= 2.5 \text{ g mol m}^{-3}, \\ V_{\max} &= 8.1 \times 10^{-6} \text{ g mol m}^{-3} \text{ s}^{-1} \end{aligned}$$



## 3.8

- 1) The Lineweaver–Burk plot shows that the mechanism is competitive inhibition. From the straight line for the data without the inhibitor peptide  $K_m$  and  $V_{max}$  are  $4.9 \text{ mmol l}^{-1}$  and  $2.20 \text{ mmol l}^{-1} \text{ min}^{-1}$ , respectively.
- 2) The slopes of the Lineweaver–Burk plot with the inhibitor of  $1.5$  and  $2.5 \mu\text{M}$  are  $5.5 \times 10^3$  and  $8.6 \times 10^3 \text{ min}$ , respectively. By use of Equation 3.40, the  $K_I$  values are  $1.0 \times 10^{-3}$  and  $0.86 \times 10^{-3} \text{ gmol m}^{-3}$ , respectively, and an averaged value is  $0.93 \times 10^{-3} \text{ gmol m}^{-3}$ .

## 3.9.

1)

$$C_{ES} = \frac{C_{E_0}}{\left(1 + \frac{C_S}{K_1}\right) + \frac{K_m}{C_S}}$$

$$\frac{C_E C_S}{C_{ES}} = K_m, \quad \frac{C_{ES} C_S}{C_{ES_2}} = K_I$$

From these equations

$$C_{E_0} = C_{ES} + \frac{C_{ES} C_S}{K_I} + C_E$$

$$\text{Thus, } C_{ES} = \frac{C_{E_0}}{\left(1 + \frac{C_S}{K_1}\right) + \frac{K_m}{C_S}}$$



$$r_p = k_2 C_{ES} = \frac{k_2 C_{E_0}}{\left(1 + \frac{C_S}{K_I}\right) + \frac{K_m}{C_S}}$$

$$= \frac{V_{\max} C_S}{C_S \left(1 + \frac{C_S}{K_I}\right) + K_m}$$

This is Equation 3.42.

- 2) By assuming a suitable value of  $K_m = K_I$  in the above equation, values of  $r_p$  are calculated. When  $K_m = 60 \text{ g mol m}^{-3}$  is assumed,  $V_{\max} = 1.0 \text{ g mol m}^{-3} \text{ min}^{-1}$  is obtained, and  $r_p$  values calculated agree with the experimental values.

## Chapter 4

### 4.1.

- 1) From Equation 4.3  $\mu = \ln 2 / t_d = 0.693 / 0.5 = 1.39 \text{ h}^{-1}$ .
- 2) From Equation 4.5  $\ln (C_x / C_{x0}) = \mu_{\max} t$

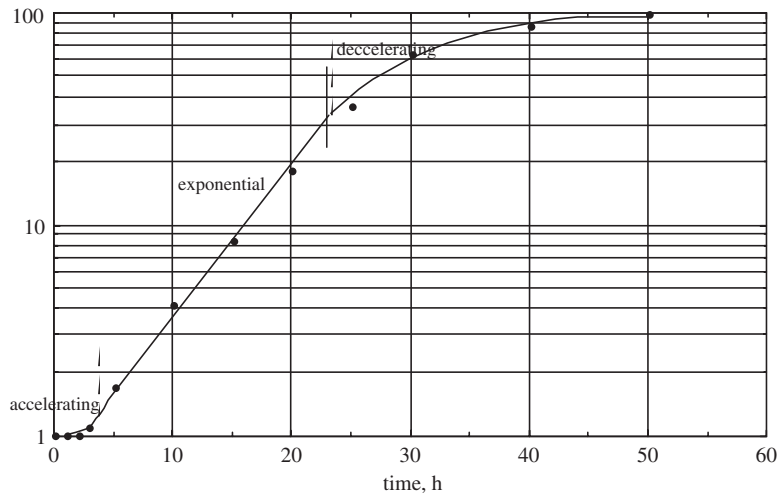
$$\text{Thus, } \ln(10/0.1) = 1.39 t, \quad t = 3.3 \text{ h}$$

### 4.2.

- 1)  $\mu_{\max} = \ln (C_x / C_{x0}) / t = \ln (0.5 / 0.1) / 1 = 1.61 \text{ h}^{-1}$
- 2)  $t_d = \ln 2 / \mu_{\max} = 0.693 / 1.61 = 0.43 \text{ h}$
- 3)  $t = \ln (1.0 / 0.1) / \mu_{\max} = 2.30 / 1.61 = 1.4 \text{ h}$ .

### 4.3.

1 and 2.



3. From the slope of the straight line for the exponential growth phase

$$\mu_{\max} = 0.16 \text{ h}^{-1}$$

4.4.

$$\mu_{\max} = 0.05 \text{ h}^{-1}$$

4.5.

From Equation 4.5

$$\begin{aligned} \mu_{\max} &= \ln(C_x/C_{x0})/t = \ln(54/19)/7 \\ &= 0.15 \text{ h}^{-1} \end{aligned}$$

From Equation 4.4  $Y_{xS} = (54 - 19)/81 = 0.43$  (kg dry cells formed)/(kg substrate consumed)

4.6.

At steady state  $C_x = (4.0 - 0.2) \times 0.42 = 1.6$  g dry cells  $\text{l}^{-1}$

From Equation 4.2  $r_x = \mu C_x$

$$1.6 \times 0.5 \text{ (g dry cell l}^{-1} \text{ h}^{-1}) = \mu \times 1.6$$

$$\mu = 0.5 \text{ h}^{-1}$$

Please see also Section 12.6.3.

## Chapter 5

5.1.

$$U_o = 188 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$$

5.2.

a) From heat balance  $10(40 - 20) = 8(80 - t_2)$

$$t_2 = 55^\circ\text{C}$$

b)

$$\begin{aligned} (\Delta t)_{\text{lm}} &= (\Delta t_2 - \Delta t_1) / \ln(\Delta t_2 / \Delta t_1) = [(80 - 40) - (55 - 20)] / \ln(40/35) \\ &= 37.4 \text{ }^\circ\text{C} \end{aligned}$$

$$900 \text{ W m}^{-2} \text{ K}^{-1} = 774.4 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ K}^{-1} = 3240 \text{ kJ h}^{-1} \text{ m}^{-2} \text{ K}^{-1}$$

$$\begin{aligned} A &= Q / [U(\Delta t)_{\text{lm}}] = 10 \times 992 \times 4.2(40 - 20) / (3240 \times 37.4) \\ &= 6.88 \text{ m}^2 \end{aligned}$$

**5.3.**

$$(\text{Re}) = du\rho/\mu = (0.01 \times 10 \times 1.26)/(1.75 \times 10^{-5}) = 7200$$

Under turbulent flow Equation 5.8a is used.

$$(\text{Pr}) = c_p\mu/\kappa = (1.006 \times 10^3 \times 1.75 \times 10^{-5})/0.0247 = 0.713$$

$$(\text{Nu}) = hd_i/\kappa = 0.023 \times (7200)^{0.8}(0.713)^{1/3} = 25.0$$

$$h = 25.0 \times 0.0247/0.01 = 61.8 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1} = 220 \text{ kJ h}^{-1} \text{ m}^{-2} \text{ K}^{-1}$$

**5.4.**

At 25 °C, values of the properties of water are as follows. Specific heat,  $c_p = 4,182 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ , density,  $\rho = 998.2 \text{ kg m}^{-3}$ , thermal conductivity,  $\kappa = 0.597 \text{ W m}^{-1} \text{ K}^{-1}$ , viscosity,  $\mu = 9.93 \times 10^{-4} \text{ Pa s}$

$$(\text{Re}) = du\rho/\mu = (0.025 \times 1.2 \times 998.2)/(9.93 \times 10^{-4}) = 3.02 \times 10^4$$

$$(\text{Pr}) = c_p\mu/\kappa = (4182 \times 9.93 \times 10^{-4})/0.597 = 6.96$$

$$(\text{Nu}) = h_i d_i/\kappa = 0.023 \times (3.02 \times 10^4)^{0.8}(6.96)^{1/3} = 168.5$$

Thus, the water-side coefficient of heat transfer is

$$h_i = 168.5 \times 0.597/0.025 = 4020 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1} = 3465 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ K}^{-1}$$

The steam-side coefficient of heat transfer is converted to  $h_{is}$  based on the inside tube surface by use of Equation 5.5.

$$h_{is} = 2000 \times 30/25 = 2400 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ K}^{-1}$$

We use thermal conductivity of stainless steel of  $14 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$  and Equation 5.4

$$\begin{aligned} 1/U_i &= 1/3465 + 1/5000 + 0.0025/14 + 1/2400 \\ &= 0.00109 \end{aligned}$$

$$U_i = 920 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ K}^{-1}$$

(Strictly the heat transfer resistance of the tube wall  $\ln(r_o/r_i)/(2\pi r_i)/(2\pi\kappa)$  should be used in the place  $2(r_o - r_i)/\kappa$ . However, the obtained value of  $U_i$  is almost the same.)

**5.5.**

Methanol side

$$(\text{Re}) = (0.046 \times 0.25 \times 780)/0.00042 = 2.13 \times 10^4$$

$$(\text{Pr}) = (0.62 \times 0.00042 \times 3600)/0.18 = 5.21$$

From Equation 5.8

$$h_{im} = (0.18/0.046) \times 0.023 \times (2.13 \times 10^4)^{0.8} \times (5.21)^{1/3} = 452 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$$

Water side

Flow rate of water

From heat balance

$$\begin{aligned} 0.62 \times 780 \times 0.25 \times \pi/4 \times (0.046)^2 \times 30 \\ = 1.0 \times 1000 \times u \times \pi/4 \times (0.08^2 - 0.05^2) \times 5 \end{aligned}$$

$$u = 0.39 \text{ m s}^{-1}$$

$$(\text{Re}) = [(0.08 - 0.05) \times 0.39 \times 1000]/0.0009 = 1.30 \times 10^4$$

$$(\text{Pr}) = (1.0 \times 0.009 \times 360)/0.52 = 6.23$$

$$\begin{aligned} h_{\text{ow}} &= (0.52/0.03) \times 0.023 \times (1.30 \times 10^4)^{0.8} \times (6.23)^{1/3} \\ &= 1434 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1} \end{aligned}$$

The overall heat transfer coefficient based on the outer surface of the inner tube  $U_o$

$$U_o = 1/[50/(452 \times 46) + 1/1434] = 320 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$$

The logarithmic mean temperature difference  $(\Delta t)_{\text{lm}}$  is

$$(\Delta t)_{\text{lm}} = [(60 - 25) - (30 - 20)]/\ln(35/10) = 20 \text{ }^\circ\text{C}$$

Required heat transfer area is

$$\begin{aligned} A_o &= [0.62 \times 780 \times 1.0 \times \pi/4 \times (0.046)^2 \times 30 \times 3600] / (320 \times 20) \\ &= 13.6 \text{ m}^2 \end{aligned}$$

Required total length of the tube

$$L = 13.6/(\pi \times 0.05) = 86 \text{ m}$$

## 5.6.

$$(\text{Re}) = (L^2 \dot{N} \rho / \mu) = (1.0 \times 1.67 \times 900)/0.09 = 1.67 \times 10^4$$

$$(\text{Pr}) = (0.468 \times 0.09 \times 3600)/0.109 = 1390$$

By use of Equation 5.13

$$h = 0.109 \times 0.36 \times (1.67 \times 10^4)^{2/3} \times (1390)^{1/3} = 290 \text{ kcal h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$$

## Chapter 6

### 6.1.

The solubility of  $\text{SO}_2$  at 76 mmHg is 1.27 kg- $\text{SO}_2$ /100 kg- $\text{H}_2\text{O}$

Assuming ideal gas behavior the mole number of 340  $\text{m}^3$  of the gas mixture is

$$n = 1 \times 340/(0.08206 \times 293.15) = 14.13 \text{ kmol}$$

The mole number of  $\text{SO}_2$  to be removed is

$$14.13 \times 0.10 \times 0.95 = 1.343 \text{ kmol} = 86.03 \text{ kg} - \text{SO}_2$$

The amount of water required is

$$(86.03/1.27) \times 100 = 6770 \text{ kg} - \text{H}_2\text{O h}^{-1}$$

### 6.2.

As shown in Example 2.4,

$$1k_{Gc} = RTk_{Gp}$$

At 293.15 K

$$1k_{Gc} = 0.08206 \times 293.15 \times k_{Gp} = 24.06k_{Gp}$$

When  $k_{Gp} = 8.50 \text{ kmol m}^{-2} \text{ h}^{-1} \text{ atm}^{-1}$ ,  $k_{Gc} = 204 \text{ m h}^{-1}$

### 6.3.

The Henry's law is expressed as  $p = H_m C_L$  or  $p = H_x x_L$ . The relation between  $H_m$  and  $H_x$  is given as  $H_x V_m = H_m$ , because  $x_L/V_m = C_L$ , where  $V_m$  is the molar volume of a liquid phase. From Table 6.1  $H_x$  of  $\text{NH}_3$  at  $20^\circ\text{C}$  is  $1/0.37 \text{ atm}$ . The molar volume of pure water  $0.018 \text{ m}^3 \text{ mol}^{-1}$  may be used, because the concentration of  $\text{NH}_3$  in liquid phase is low. Thus,

$$H_m = 1/0.37 \times 0.018 = 0.0486 \text{ atm m}^3 \text{ kmol}^{-1}$$

As shown in Example 6.1(a)

$$m = (H_m)/RT = 2.70 \times 0.018/(0.08206 \times 293.15) = 2.02 \times 10^{-3}$$

(This value is given in Example 6.2.)

a) From Equation 6.12

$$1/K_L = 1/(mk_G) + 1/k_L$$

$$\begin{aligned} 1/(mk_G) &= 1/K_L - 1/k_L = 1/(2.70 \times 10^{-5}) - 1/(1.44 \times 10^{-4}) \\ &= 1/(3.32 \times 10^{-5}) \end{aligned}$$

$$k_{Gc} = 1.64 \times 10^{-2} \text{ m s}^{-1}$$

b)

$$\begin{aligned} [(1/(mk_G))/(1/K_L)] &= (2.7 \times 10^{-5})/(3.32 \times 10^{-5}) = 0.81 \\ &81\% \end{aligned}$$

**6.4.**

Similar to Equation 6.11

$$\begin{aligned} 1/K_{Gp} &= 1/k_{Gp} + H_m/k_L = 1/1.10 + 0.0461/0.34 \\ &= 1.0447 \end{aligned}$$

$$K_{Gp} = 0.96 \text{ kmol m}^{-2} \text{ h}^{-1} \text{ atm}^{-1}$$

$$K_{Gc} = 0.96 \times 0.08206 \times 288.15 = 22.7 \text{ m h}^{-1}$$

$$\text{percentage of gas film resistance} = (0.96/1.1) \times 100 = 87\%$$

or  $k_{Gp}$  is converted to  $k_{Gc}$  as shown in Problem 6.2.

$$m = H_m/(RT) = 0.0461/(0.08206 \times 288.15) = 0.00195$$

$$1/K_{Gc} = 1/k_{Gc} + m/k_L$$

$$1/K_{Gc} = 1/(1.10 \times 23.6) + 0.00195/0.34 = 0.0447$$

$$K_{Gc} = 22.6 \text{ m h}^{-1}$$

$$22.6/(1.1 \times 23.6) \times 100 = 87\%$$

**6.5.**

Average contact time =  $0.4/20 = 0.02 \text{ s}$

From Equation 6.18

$$\begin{aligned} k_L &= 2(D/\pi t)^{1/2} = 2 \times [2.1 \times 10^{-9}/(\pi \times 0.02)]^{1/2} \\ &= 3.66 \times 10^{-4} \text{ m s}^{-1} \end{aligned}$$

**6.6.**

As stated in Section 6.7, if concentrations are defined per unit volume of inert carrier gas and solvent, their flow rates are constant. Thus, when the concentrations are expressed by  $X = x/(1-x)$  and  $Y = y/(1-y)$ , the operating line becomes a straight line.

From mass balance

$$U_G'(0.009 - 0.001) = U_L'(0.08 - 0)$$

$$U_G'/U_L' = 10$$

From Equation 6.51

$$H_{OG} = H_G + H_L m U_G'/U_L' = 0.45 + 0.06 \times 10 \times 0.3 = 0.63 \text{ m}$$

Since the equilibrium is given by  $Y = 0.06X$

$$\begin{aligned} N_{OG} &= (Y_B - Y_T)/(\Delta Y)_{lm} = (0.009 - 0.001)/(\Delta Y_T - \Delta Y_B)/\ln(\Delta Y_T/\Delta Y_B) \\ &= (0.009 - 0.001)/(0.001 - 0.0042)/\ln(0.001/0.0042) = 3.59 \end{aligned}$$

From Equation 6.47

$$Z = H_{OG} \times N_{OG} = 0.63 \times 3.59 = 2.26 \text{ m}$$

**6.7.**

From Equation 6.50

$$k_L a = U_L / H_L = (20\,000/1000)/0.30 = 66.7 \text{ h}^{-1}$$

**6.8.**

See Section 6.7.3 and Figure 6.5.

## Chapter 7

**7.1.**

$$\tau = V/F = 0.1/0.01 = 10 \text{ min}$$

For CSTR, by use of Equation 7.6

$$0.12 \times 10 = x_A / (1 - x_A) \quad x_A = 0.55$$

For PFR, by use of Equation 7.10

$$1.2 = -\ln(1 - x_A) \quad x_A = 0.70$$

**7.2.**

1) As for PFR, the fractional conversion is same, and  $x_A = 0.70$ .

For the first reactor of CSTR

$$t = V/F = 0.05/0.01 = 5 \text{ min}$$

$$0.12 \times 5 = x_A / (1 - x_A) \quad x_A = 0.375 \quad C_{A10} = 0.625 \text{ kmol m}^{-3}$$

For the second reactor  $x_A = 0.375$

$$(C_{A10} - C_{A20})/C_{A10} = 0.375 \quad C_{A20} = 0.390 \text{ kmol m}^{-3}$$

$$x_{A2} = 0.61$$

2) CSTR  $x_A = 0.55$ , PFR  $x_A = 0.70$

**7.3.**

$$E_f = 4\pi R^2 D_{\text{eff}} \left( \frac{dC_A}{dr} \right)_{r=R} / \frac{4}{3} \pi R^3 k C_{Ab} = \frac{1}{3\phi^2} \left( \frac{dC_A^*}{dr^*} \right)_{r^*=1}$$

$$dC_A / dr \text{ (at } r = 1) = 3\phi \coth 3\phi - 1$$

$$E_f = (3\phi \coth 3\phi - 1)/3\phi^2 \quad (7.20)$$

## 7.4.

a)

$$R = 0.2 \text{ cm}, k = 0.001 \text{ s}^{-1}, D_{\text{eff}} = 1.2 \times 10^{-6}$$

$$\phi = 0.2/3 \times [0.001/(1.2 \times 10^{-6})]^{0.5} = 1.92$$

$$E_f = (3\phi \coth 3\phi - 1)/3\phi^2 = 0.43 \text{ or from Figure 7.4.}$$

b) Decrease  $\phi$  by decreasing  $R$ , decreasing  $k$  (decreasing the amount of catalyst in a sphere) and/or increasing  $D_{\text{eff}}$ .

## 7.5.

a) From Equation 3.28

$$r_p = \frac{V_{\text{max}} C_A}{K_m + C_A} = \frac{2.0 \times 10^{-5} \times 0.1}{0.010 + 0.1} = 1.82 \times 10^{-5} \text{ kmol m}^{-3} \text{ s}^{-1}$$

After 10 min  $0.01 \text{ kmol m}^{-3}$  will be reacted, and thus the conversion

$$x_A = 0.11$$

b) If we use the same amount of the enzyme for immobilization,

$$\phi = 0.005/3 \times [2.0 \times 10^{-5}/(0.010 \times 1.0 \times 10^{-6} \times 10^{-4})]^{0.5} = 7.45$$

$$C_{\text{Ab}}/K_m = 0.10/0.010 = 10$$

From Figure 7.5  $E_f = 0.50$

The initial reaction rate is  $1.82 \times 10^{-5} \times 0.50 = 9.1 \times 10^{-6} \text{ kmol m}^{-3} \text{ s}^{-1}$

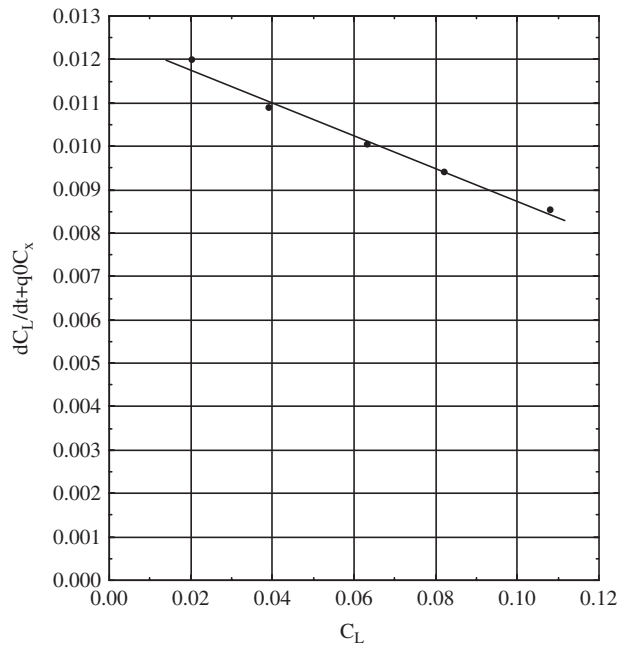
c) From Figure 7.5, the Thiele modulus  $\phi$  must be smaller than 3.0. Therefore,  $d = 2R$  should be smaller than  $0.004 \text{ m} = 0.4 \text{ cm}$ .

## 7.6.

From the slope of the first straight line in Figure 7.13

$$dC_L/dt = q_o C_x = 8.2 \times 10^{-3} \text{ mM l}^{-1} \text{ s}^{-1}$$





From the curve after re-start of aeration, we obtain the following table

time, s	$C_L$	$dC_L/dt$	$dC_L/dt + q_0 C_x$
30	0.020	$3.8 \times 10^{-3}$	$12 \times 10^{-3}$
35	0.039	$2.7 \times 10^{-3}$	$10.9 \times 10^{-3}$
45	0.063	$1.87 \times 10^{-3}$	$10.1 \times 10^{-3}$
55	0.082	$1.21 \times 10^{-3}$	$9.4 \times 10^{-3}$
75	0.108	$0.44 \times 10^{-3}$	$8.6 \times 10^{-3}$
100	0.119	—	—

According to Equation 7.29  $dC_L/dt + q_0 C_x$  is plotted against  $C_L$  as shown in this figure. The slope of the straight line is  $-K_L a$ . From this figure  $K_L a = 0.038 \text{ s}^{-1} = 137 \text{ h}^{-1}$

7.7.

1) mole fraction of oxygen in equilibrium with air

$$2.0 \times 10^{-5} \times 0.21 = 4.2 \times 10^{-6}$$

$$0.233 \text{ mol m}^{-3}$$

the oxygen transfer rate

$$= 80 \times 0.233$$

$$= 18.64 \text{ mol m}^{-3} \text{ h}^{-1}$$

2)  $7460 \text{ g dry cells m}^{-3}$

**7.8.**

$$(\text{Re}) = (2 \times 1010 \times 0.3^2) / (9.8 \times 10^{-4}) = 1.86 \times 10^5$$

From Figure 7.8 the power number  $N_p$  is 6.

By use of Equation 7.32

$$P_0 = 6 \times 1010 \times 2^3 \times 0.3^5 = 118 \text{ W}$$

By use of Equation 7.34

$$\begin{aligned} \log(P_G/P_0) &= -192 (1/3)^{4.38} (\text{Re})^{0.115} (0.3 \times 2^2 / 9.8)^{0.653} \\ &\quad \times [0.6 / (60 \times 2 \times 0.3^3)] \\ &= -192 \times 0.00813 \times 4.04 \times 0.254 \times 0.185 = -0.296 \end{aligned}$$

$$P_G/P_0 = 0.506, \quad P_G = 60 \text{ W}$$

The superficial gas velocity  $U_G$

$$U_G = 0.60 / (60 \times 0.45^2 \times \pi) = 0.0157 \text{ m s}^{-1}$$

By use Equation 7.36b

$$\begin{aligned} k_L a &= 0.002 (60/0.572)^{0.7} 0.0157^{0.2} \\ &= 0.002 \times 26.0 \times 0.436 = 0.023 \text{ s}^{-1} = 82 \text{ h}^{-1} \end{aligned}$$

(If Equation 7.37 is used

$$\begin{aligned} (k_L a d^2 / D_L) &= 0.060 (\text{Re})^{1.5} (0.3 \times 2^2 / 9.8)^{0.19} [9.8 \times 10^{-4} / (1010 \times 2.1 \times 10^{-9})]^{0.5} \\ &\quad (9.8 \times 10^{-4} \times 0.0157 / 7.0 \times 10^{-2})^{0.6} (2 \times 0.3 / 0.0157)^{0.32} \\ &= 0.060 \times 8.02 \times 10^7 \times 0.671 \times 21.5 \times 0.00639 \times 3.21 = 1.42 \times 10^6 \\ k_L a &= 1.42 \times 10^6 \times 2.10 \times 10^{-9} / 0.3^2 = 0.0331 \text{ s}^{-1} = 119 \text{ h}^{-1} \end{aligned}$$

**7.9.**

To keep the  $k_L a$  value constant,  $P_G/V$  and  $U_G$  should be kept constant. Under turbulent condition  $P_G/V$  is proportional to  $N^3 L^2$ . Therefore,

$$2^3 \times 0.3^2 = N^3 \times 0.6^2 \quad N = 1.26 \text{ s}^{-1} = 76 \text{ rpm}$$

$$\text{aeration rate } 0.6 \times 4 = 2.4 \text{ m}^3 \text{ min}^{-1}$$

(If we calculate  $k_L a$  by use of Equation 7.37

$$\begin{aligned} k_L a &= 0.060 (4.67 \times 10^5)^{1.5} (0.6 \times 1.26^2 / 9.8)^{0.19} [9.8 \times 10^{-4} / (1010 \\ &\quad \times 2.1 \times 10^{-9})]^{0.5} (9.8 \times 10^{-4} \times 0.0157 / 7.0 \times 10^{-2})^{0.6} \\ &\quad \times (1.26 \times 0.6 / 0.0157)^{0.32} \times 2.10 \times 10^{-9} / 0.6^2 = 0.0338 \text{ s}^{-1} = 121 \text{ h}^{-1} \end{aligned}$$

### 7.10.

$$(\text{Bo}) = gD^2 \rho / \sigma = 9.8 \times 0.3^2 \times 1000 / (7.5 \times 10^{-2}) = 11 \ 800$$

$$(\text{Ga}) = gD^3 / \nu^2 = 9.8 \times 0.3^3 / 10^{-12} = 2.65 \times 10^{11}$$

$$U_G = 10 / (3600 \times 0.15^2 \times \pi) = 0.0393 \text{ m s}^{-1}$$

$$(\text{Fr}), \text{Froude number} = U_G / (gD)^{1/2} = 0.0393 / (9.8 \times 0.3)^{1/2} = 0.0229$$

$$\begin{aligned} \varepsilon / (1 - \varepsilon)^4 &= 0.20 (\text{Bo})^{1/8} (\text{Ga})^{1/12} (\text{Fr})^{1.0} \\ &= 0.20 \times 3.23 \times 8.95 \times 0.0229 = 0.132 \\ \varepsilon &= 0.090 \end{aligned}$$

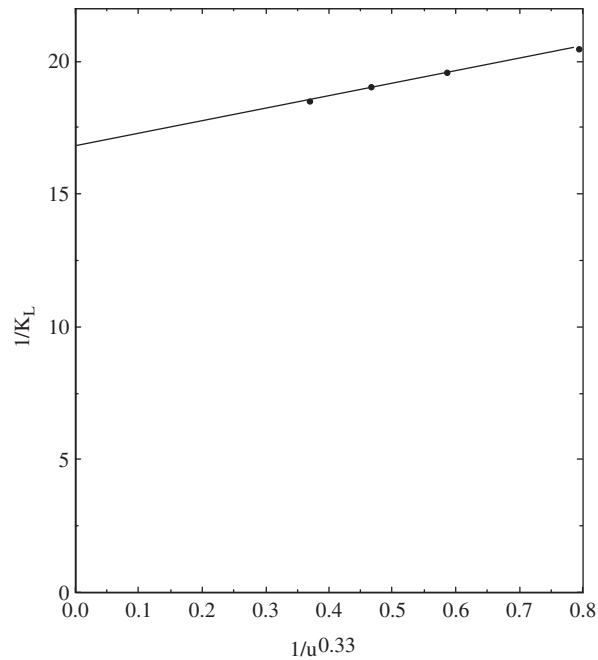
$$\begin{aligned} (k_L a D^2 / D_L) &= 0.60 (\text{Sc})^{0.50} (\text{Bo})^{0.62} (\text{Ga})^{0.31} \varepsilon^{1.1} \\ &= 0.60 \times 21.8 \times 334 \times 3.48 \times 10^3 \times 0.071 = 1.08 \times 10^6 \\ k_L a &= 1.08 \times 10^6 \times 2.10 \times 10^{-9} \times 0.3^2 \\ &= 0.0252 \text{ s}^{-1} = 91 \text{ h}^{-1} \end{aligned}$$

$$\begin{aligned} (d_{vs} / D) &= 26 (\text{Bo})^{-0.50} (\text{Ga})^{-0.12} (\text{Fr})^{-0.12} \\ &= 26 \times 0.00922 \times 0.0426 \times 1.57 \\ &= 0.0160 \\ d_{vs} &= 0.0160 \times 0.30 = 0.0048 \text{ m} = 4.8 \text{ mm} \end{aligned}$$

## Chapter 8

### 8.1.

$$1/K_L = 1/k_{L1} + 1/k_M + 1/k_{L2}$$



Since the liquid film mass transfer coefficients of both sides increase by one-third power of the averaged flow rate,

$$1/K_L = 1/(\text{cons} \times u^{1/3}) + 1/k_M$$

If  $1/K_L$  is plotted against  $1/u^{1/3}$ , a straight line will be obtained and the intersection with  $y$ -axis gives  $1/k_M$ .

Thus,  $1/k_M = 16.8 \text{ s cm}^{-1}$  is obtained as shown in the figure.

$$k_M = 0.060 \text{ cm s}^{-1}$$

## 8.2.

From Equation 8.5

$$J_F = k_L \ln(C_G/C) = k_L \ln C_G - k_L \ln C$$

Slopes of the straight lines in Figure 8.4 give  $-k_L$ .

Shear rate ( $\text{s}^{-1}$ )	$k_L$ ( $\text{cm min}^{-1}$ )
5000	$1.18 \times 10^{-2}$
2000	$0.94 \times 10^{-2}$
1000	$0.73 \times 10^{-2}$
500	$0.56 \times 10^{-2}$

These values are roughly proportional to one-third power of the shear rate, as expected by Equation 8.6.

**8.3.**

$$\gamma_w = 8v/d = 8 \times 5.5/0.03 = 1470 \text{ s}^{-1}$$

From Equation 8.7

$$J_F = 4.20(r^4/L)^{1/3} \gamma_w \ln(C_G/C) = 4.20[(2.57^4 \times 10^{-16})/20]^{1/3} \times 1470 \\ \times \ln(95/40) = 3.2 \times 10^{-2} \text{ cm min}^{-1}$$

**8.4.**

$$C_A = 1000 \times 0.035/342 = 0.10 \text{ g mol l}^{-1}$$

$$\Pi = RT C = 0.08206 \times 293 \times 0.10 = 2.4 \text{ atm}$$

**8.5.**

The solute concentration is

$$2 \times 20/58.4 = 0.685 \text{ mol l}^{-1}$$

The osmotic pressure is

$$\Pi = RT \times 0.685 = 16.75 \text{ atm}$$

$$J_F = 8 \times 10^{-4} \times (25 - 16.75)/25 = 2.65 \times 10^{-4} \text{ cm s}^{-1}$$

**8.6.**

See Section 8.5 as for the concentration polarization.

**Chapter 9**

**9.1.**

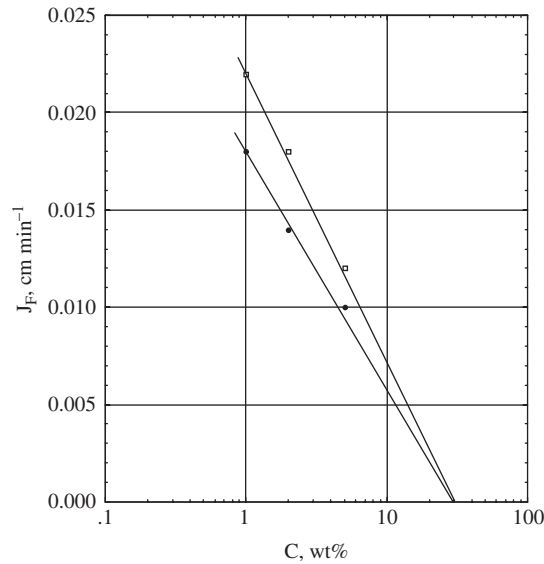
Since  $R_M$  can be neglected, substitution of  $R_C$  given by Equation 9.2 into Equation 9.1 and integration will give

$$V_f^2 = \frac{2\Delta P}{\mu\alpha\rho_C} t$$

Therefore,  $V_f$  is proportional to  $t^{1/2}$ .

$$\text{At 20 min} \quad 0.10 \times 2^{1/2} = 0.14 \text{ m}^3$$

$$30 \text{ min} \quad 0.10 \times 3^{1/2} = 0.17 \text{ m}^3$$

**9.2.**

The filtration fluxes ( $\text{cm min}^{-1}$ ) are plotted against  $\ln C$  similar to Figure 8.4. From the figure shown saturated albumin concentration 30 wt% liquid film mass transfer coefficient

$$\begin{aligned} \text{at } 5 \text{ cm s}^{-1} & \quad 5.3 \times 10^{-3} \text{ cm min}^{-1} \\ 10 \text{ cm s}^{-1} & \quad 6.4 \times 10^{-3} \text{ cm min}^{-1} \end{aligned}$$

**9.3.**

$$t = 294 \text{ s} = 4.9 \text{ min}$$

**9.4.**

Elimination of  $dt$  from Equations 9.10 and 9.11 gives

$$\frac{dz}{dt} = \frac{dz}{dr} \frac{dr}{dt} = \frac{dz}{dr} \frac{v_g \omega^2}{g} r = \frac{Q}{\pi(r_2^2 - r_1^2)}$$

Integration with boundary conditions ( $r=r_1$  at  $z=0$  and  $r=r_2$  at  $z=Z$ ) gives Equation 9.12.

**9.5.**

The terminal velocity under the influence of gravity is given by Equation 9.7

$$v_g = [(1.25 \times 10^{-4})^2 (1.1 - 1.0) \times 980] / (18 \times 0.01) = 8.5 \times 10^{-6} \text{ cm s}^{-1}$$

From Equation 9.12

$$\omega = 2\pi \times 15000 / 60 = 1570 \text{ rad s}^{-1}$$

By use of Equation 9.12

$$Q = [8.5 \times 10^{-6} \times \pi \times 100(100 - 25) \times 1570^2] / [980 \times \ln(10/5)] \\ = 726 \text{ cm}^3 \text{ s}^{-1} = 2.6 \text{ m}^3 \text{ h}^{-1}$$

### 9.6.

Since the sedimentation velocity is proportional to the square of the rotational speed, a rotational speed to obtain 10 times higher sedimentation is  $30\,000 \times 10^{1/2} = 95\,000 \text{ rpm} = 1580 \text{ rps}$

$$10 \times (2\pi \times 1580)^2 / 980 = 10^6$$

## Chapter 10

### 10.1.

By use of Equation 10.3

$$\ln(n_0/n) = k_d t \quad \ln(10^{10}/0.001) = 3.2 t \\ t = 10.1 \text{ min}$$

### 10.2.

For heating by direct sparging of steam in Table 10.1

$$T - T_0 = (H_{\text{steam}} - H_{\text{w at } 25^\circ\text{C}}) \times m \times t / [c_p(M + m t)] \\ 121 - 25 = (2723 - 105) \times 1000 \times t / [4.18(10\,000 + 1000 t)] \\ t = 1.81 \text{ h}$$

### 10.3.

The velocity distribution in a tube is given by Equation 2.8.

$$u = 2v[1 - (r^2 / r_i^2)] \\ u = v \text{ at } r = 0.707r_i$$

Thus, the flow rate  $Q_f$  that passes with a higher velocity than  $v$  is given

$$Q_f = 2\pi u \int_0^{0.707r_i} r dr = 4\pi v \int_0^{0.707r_i} \left[1 - \frac{r^2}{r_i^2}\right] r dr$$

Integration gives

$$Q_f = 4\beta v \left[ \frac{r^2}{2} - \frac{r^4}{4r_i^2} \right]_0^{0.707r_i} = 0.75\pi v r_i^2 \\ 0.75$$

**10.4.**

$$v = 1.0/(\pi \times 0.04^2) = 200 \text{ m h}^{-1}$$

$$(\text{Re}) = (0.08 \times 200 \times 950)/1 = 15\,000$$

From Figure 10.3 Pe is approximately 2.5, and thus

$$E_{Dz} = (200 \times 0.08)/2.5 = 6.4$$

Assuming a length of the holding section of 35 m

$$(\text{Pe}) = vL/E_{Dz} = 1100$$

$$(\text{Da}) = k_d L/v = 32$$

From Figure 10.4 the value of  $n/n_0$  is about  $10^{-13}$  is equal to that required in this problem.

$$35 \text{ m}$$

**Chapter 11****11.1.**

By use of Equation 11.7

$$1.0(q_A - 0) = 1.0(200 - C_A)$$

$$q_A = 120C_A^{0.11}$$

By repeated calculation

$$C_A = 27.3 \text{ g m}^{-3}$$

**11.2.**

For 95% recovery by single-stage adsorption,  $w$  of the adsorbent is required per  $1 \text{ m}^3$  of the solution

$$C_{A1}/C_{A0} = 0.05 = 1/(1 + 2.5w)$$

$$w = 7.6$$

By three-stage adsorption  $w'$  of the adsorbent is required for each adsorption

$$C_{A3}/C_{A0} = 0.05 = 1/(1 + 2.5w')^3$$

$$w' = 0.686$$

The ratio is  $7.6/(0.686 \times 3) = 3.7$

**11.3.**

Similar to the derivation of Equation 11.2

$$k_A p_A (N_s - N_a - N_b) = k_A' N_a$$

$$k_B p_B (N_s - N_a - N_b) = k_B' N_b$$



Therefore,

$$ap_A(N_s - N_a - N_b) = N_a$$

$$bp_B(N_s - N_a - N_b) = N_b$$

where  $a = k_A/k_A'$  and  $b = k_B/k_B'$ .

Addition of these equations gives

$$1 + ap_A + bp_B = N_s/(N_s - N_a - N_b)$$

Substituting this to

$$ap_A(N_s - N_a - N_b) = N_a$$

Thus,

$$N_a = N_s ap_A / (1 + ap_A + bp_B)$$

**11.4.**

$$\begin{aligned} t_B &= 1283 / (0.5 \times 1.6 \times 1.1) \times [0.50 - (0.5 \times 1.6 \times 2.26) / (2 \times 9.2)] \\ &= 586 \text{ h} \end{aligned}$$

$$\text{length of adsorption zone } (0.5 \times 1.6 \times 2.26) / 9.2 = 0.20 \text{ m}$$

**11.5.**

By Equation 11.18

$$V_R = V_0 + K(V_t - V_0)$$

$$20 = 10.60 + K(39.25 - 10.60)$$

$$K = 0.33$$

By Equation 11.19

$$Hs = 50 / [16(20/1.8)^2] = 0.025 \text{ cm}$$

**11.6.**

See Sections 11.6.3 and 11.6.4 for the resolution.

**11.7.**

See Section 11.7.2 for the sandwich method.

## Chapter 12

**12.1.**

$$3.5 \text{ kW} = 3.5 \text{ kJ s}^{-1} = 1.26 \times 10^4 \text{ kJ h}^{-1}$$

Therefore, heat to be removed is  $(1.26 + 2.5) \times 10^4 \text{ kJ h}^{-1}$

If we assume that cooling water leaves the coil at  $25^\circ\text{C}$ , the flow rate of cooling water is calculated by use of the following physical properties of water at  $20^\circ\text{C}$ ;  $\rho = 998 \text{ kg m}^{-3}$ ,  $c_p = 4.18 \text{ kJ kg}^{-1} \text{ K}^{-1}$ ,  $\mu = 0.001 \text{ Pa s}$ ,  $\kappa = 2.15 \text{ kJ h}^{-1} \text{ m}^{-1} \text{ }^\circ\text{C}^{-1}$ . The flow rate of cooling water is

$$4.18 \times 10 \times F = 3.76 \times 10^4 \quad F = 900 \text{ kg h}^{-1} = 0.90 \text{ m}^3 \text{ h}^{-1}$$

$$\text{velocity } v = 1270 \text{ m h}^{-1} = 0.354 \text{ m s}^{-1}$$

Reynolds number inside the tube

$$(\text{Re}) = (0.03 \times 0.354 \times 998)/0.001 = 1.06 \times 10^4$$

By use of Equation 5.8a

$$\begin{aligned} h &= 0.023 \times 2.15(\text{Re})^{0.8}(\text{Pr})^{1/3}/0.03 = 0.023 \times 2.15 \times 1660 \times 1.91/0.03 \\ &= 5200 \text{ kJ h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1} \end{aligned}$$

Heat transfer coefficient at the surface of coiled tubes is obtained by use of Equation 5.13. With physical properties of the broth

$$(\text{Re}) = (0.25 \times 3 \times 1050)/0.004 = 1.97 \times 10^5$$

$$(\text{Pr}) = (4.2 \times 0.004 \times 3600)/2.1 = 28.8$$

$$\begin{aligned} h &= 0.87 \times 2.1 \times (\text{Re})^{2/3}(\text{Pr})^{1/3}/1.5 = 0.87 \times 2.1 \times 3400 \times 3.06/1.5 \\ &= 12700 \text{ kJ h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1} \end{aligned}$$

The overall coefficient based on the outer surface area  $U_o$  is

$$1/U_o = 1/12700 + 0.005/55 + 4/(5200 \times 3)$$

$$U_o = 2350 \text{ kJ h}^{-1} \text{ m}^{-2} \text{ }^\circ\text{C}^{-1}$$

A mean temperature difference of  $10^\circ\text{C}$  is used.

The required surface area  $A$

$$A = 3.76 \times 10^4 / (2350 \times 10) = 1.6 \text{ m}^2$$

Tube length  $L = 1.6 / (3.14 \times 0.04) = 12.7 \text{ m}$

## 12.2.

The average shear rate  $du/dy = 11.5 \times 1 = 11.5 \text{ s}^{-1}$

By use of Equation 2.6

$$K(du/dy)^n = \mu_a(du/dy) \quad \mu_a = 0.39(du/dy)^{0.74-1} = 0.21 \text{ Pa s}$$

$$(\text{Re}) = 0.36 \times 1 \times 1020/0.21 = 1750 \quad (\text{Re})^{1.5} = 7.3 \times 10^4$$

$$(\text{dN}^2/g)^{0.19} = 0.588$$

$$(\text{Sc}) = 0.21/(1020 \times 2.1 \times 10^{-9}) = 98000 \quad (\text{Sc})^{0.5} = 313$$

$$(\mu U_G/\sigma)^{0.6} = 0.094$$

$$(\text{Nd}/U_G)^{0.32} = 4.16$$

$$\begin{aligned}
 k_L a d^2 / D_L &= 0.060 \times 7.3 \times 10^4 \times 0.588 \times 313 \times 0.094 \times 4.16 \\
 &= 3.15 \times 10^5 \\
 k_L a &= 3.15 \times 10^5 \times 2.1 \times 10^{-9} / 0.36 = 1.84 \times 10^{-3} \text{ s}^{-1} = 6.6 \text{ h}^{-1}
 \end{aligned}$$

**12.3.**

The effective shear rate is given by Equation 12.9.

$$S_{\text{eff}} = 50 U_G = 50 \times 150/36 = 208 \text{ s}^{-1}$$

By use of Equation 2.6

$$\begin{aligned}
 \tau &= K (du/dy)^n = \mu_a (du/dy) \\
 \tau &= 0.39(208)^{0.74} = \mu_a \times 208 \\
 \mu_a &= 0.097 \text{ Pa s}
 \end{aligned}$$

To calculate  $k_L a$  by use of Equation 7.45 we may use following values

$$\sigma = 0.070 \text{ N m}^{-1} \quad \text{and} \quad D_L = 2.1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$$

$$\begin{aligned}
 \text{Thus, (Bo)} &= gD^2 \rho / \sigma = 9.8 \times 0.8^2 \times 1020 / (7.0 \times 10^{-2}) = 91\,400 \\
 \text{(Ga)} &= gD^3 / \nu^2 = 9.8 \times 0.8^3 / (9.5 \times 10^{-5})^2 = 5.56 \times 10^8 \\
 \text{(Fr)} &= U_G / (gD)^{1/2} = 0.0417 / (9.8 \times 0.8)^{1/2} = 0.0149 \\
 \text{(Sc)} &= \nu / D_L = 9.5 \times 10^{-5} / 2.1 \times 10^{-9} = 45\,200 \\
 k_L a d^2 / D_L &= 0.09 \times 213 \times 5256 \times 2570 \times 0.0149 = 3.85 \times 10^6 \\
 k_L a &= 3.85 \times 10^6 \times 2.1 \times 10^{-9} / 0.64 = 0.0126 \text{ s}^{-1} = 45 \text{ h}^{-1}
 \end{aligned}$$

**12.4.**

$$(\text{Re}) = (1 \times 0.1^2 \times 1020) / 0.002 = 5100$$

We have almost turbulent condition.

If values of  $P_G/V$  and  $U_G$  are kept constant,  $k_L a$  becomes constant, according to Equation 7.36a.

If we can assume  $P_G/P_0$  is roughly constant at the same  $U_G$  by use of Equation 7.32,  $P_G/V$  is proportional to  $N^3 d^2$ .

$$N^3 d^2 = 1^3 \times 0.1^2$$

$\pi N d$  must be smaller than  $0.5 \text{ m s}^{-1}$

$$N \times (0.5/3.14)^2 = 0.01, \quad N = 0.39 \text{ s}^{-1}$$

Therefore,  $d = 0.5 / (3.14 \times 0.39) = 0.41 \text{ m}$

$$D = d \times 3 = 1.2 \text{ m}$$

(If it is allowed to increase  $U_G$ , we can design a larger size fermentor under the limitation of the allowable impeller tip speed of  $0.5 \text{ m s}^{-1}$ .)

**12.5.**

By use of Equation 12.17

$$\begin{aligned} t_b &= (1/\mu) \ln[1 + (C_{s0} - C_{sf})Y_{xs}/C_{x0}] \\ &= (1/0.24) \ln[1 + (20 - 1) \times 0.45/0.01] = 28 \text{ h} \end{aligned}$$

**12.6.**

The cell productivity in chemostat is given as

$$DC_x = Y_{xs}[C_{si} - K_s D / (\mu_{\max} - D)]$$

Differentiate this equation with  $D$  and set equal to zero.

When

$$\begin{aligned} D &= \mu_{\max} \left( 1 - \sqrt{1 - \frac{C_{si}}{C_{si} + K_s}} \right) \\ &= \mu_{\max} \left( 1 - \sqrt{\frac{K_s}{C_{si} + K_s}} \right) \end{aligned}$$

the productivity becomes maximum.

**Chapter 13****13.1.**

Summarize the difference between bioprocesses and chemical processes in terms of process control.

	Chemical processes	Bioprocesses
Type of process	Continuous	Batch/fed-batch
Size of plant	Large	Small
Catalyst	Inorganic catalyst	Enzyme/microorganism
Type of operation	Steady state	Nonsteady state
Operating conditions	High pressure, high temperature and acidic/basic condition	Normal pressure, moderate temperature, and neutral pH
Sterilization	Not required	Required
Sensors	Sufficiently available	Limited

**13.2.**

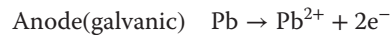
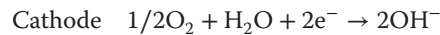
Give the operating principle of the following sensors in a few sentences.

## a) Resistance thermometer

A resistance thermometer is a temperature measuring device based on the principle that the resistance of metals proportionally increases as the temperature of a metal increases.

## b) Galvanic dissolved oxygen probe

A galvanic dissolved oxygen probe measures the partial pressure of oxygen by use of an electrode based on the following reactions.



The resulting electric current is proportional to the oxygen flux to the cathode through the oxygen permeable membrane that separates the electrode from a culture medium.

## c) pH electrode.

pH electrode measures hydrogen-ion activity based on an electric potential difference between a reference electrode and a glass electrode assembled in a sensor probe.

**13.3.**

Explain the reason why a proportional control (P action) results in the offset between the set point and the measured process variable at a steady state.

Suppose the temperature control of a bioreactor using heat supply with a proportional controller. When a proportional controller is tuned at a set point of 30 °C, as long as the set point remains constant, the temperature will remain at 30 °C successfully. Then, if the set point is changed to 40 °C, the proportional controller increases the output (heat supply) proportional to the error (temperature difference). Consequently, a heat supply will continue until the temperature gets to 40 °C and would be off at 40 °C. However, the temperature of a bioreactor will not reach 40 °C because a heat loss from the bioreactor increases due to the temperature increase. Finally, the heat supply matches the heat loss; at this point, the temperature error will remain constant, and therefore, proportional controller will keep its output constant. Now the system keeps at balance, but the temperature of a bioreactor is below its set point. This residual sustained error is called "Offset."

**13.4.**

Determine the parameters  $K_p$ ,  $T_I$ , and  $T_D$  in the PID controller by the ultimate gain method when the ultimate gain  $K_c$  and the oscillation cycle time  $T_c$  are 4.0 and 30 s, respectively.

According to Table 13.2,

$$K_p = 0.6 \times K_c = 2.4$$

$$T_I = 0.5 \times K_c = 15 \text{ s}$$

$$T_D = 0.125 \times T_c = 3.75 \text{ s}$$

**13.5.**

Calculate RQ value based on the analysis of the inlet air and the exhaust gas as follows.

Inlet air: O<sub>2</sub>, 20.90%, CO<sub>2</sub>, 0.01%  
 Outlet gas: O<sub>2</sub>, 18.30%, CO<sub>2</sub>, 2.90%

From the ideal gas law, the molar mass of the gas is proportional to the volume of the gas under constant pressure and constant temperature; therefore, the Equation 13.6 can be used as follows:

$$\begin{aligned} \text{RQ} &= \frac{\text{evolved CO}_2 \text{ (M)}}{\text{consumed O}_2 \text{ (M)}} = \frac{\text{evolved CO}_2 \text{ (\%)}}{\text{consumed O}_2 \text{ (\%)}} \\ &= \frac{2.9 - 0.01}{20.90 - 18.30} = \frac{2.89}{2.60} = 1.11 \end{aligned}$$

**Chapter 14****14.1.**

By use of Equation 14.1 and  $\alpha$  of  $7 \times 10^{11} \text{ m kg}^{-1}$ ,

$$(0.5/1)^2 = 2\Delta p \times 3600 \times 2.5 / (7 \times 10^{11} \times 20 \times 0.0012)$$

$$\Delta p = 2.3 \times 10^5 \text{ Pa} = 2.3 \text{ atm}$$

**14.2.**

From Equation 11.23, the resolution is proportional to  $Z^{1/2}$ , and thus it increase by  $2^{1/2}$ .

**14.3.**

According to the rate model, the effect of a sample volume on the dispersion of elution curve based on time is given

$$\sigma_t^2 = \sigma_{t_0}^2 + t_0^2/12 \text{ (Derivation is shown in Ref. [3] in Chapter 11)}$$

The retention time including the effect of the loading time of a sample is

$$t_R = t_{R0} + t_0/2$$

When  $Hs$  is defined based on the retention time, Equation 11.19 becomes

$$Hs Z \sigma_t^2 / t_R^2 = Z(\sigma_{t_0}^2 + t_0^2/12) / (t_{R0} + t_0/2)^2$$

**14.4.**

For the column packed with the small particles

$$Hs / (2r_0) = 0.22 / 0.044 = 5$$

From Figure 13.7  $2r_0 u / D_{\text{eff}} = 0.044 \times u / D_{\text{eff}} = 65$

For the column packed with the large particles

$$Hs = 0.22 / 0.075 = 2.9 \text{ and } 2r_0 u' / D_{\text{eff}} = 0.075 \times u' / D_{\text{eff}} = 25$$

We may assume a same value of  $D_{\text{eff}}$  for  $t$  two cases.

$$\text{Therefore, } u'/u = (0.044 \times 25)/(0.075 \times 65) = 0.23$$

#### 14.5.

From Equations 11.9 and 11.12

$$z_a = \frac{\epsilon u}{K_L a} \int_{C_B}^{C_E} \frac{dC_A}{C_A - C_A^*}$$

$$z_a = 0.196 \text{ m}$$

The residual capacity is

$$(0.196 \times 0.5) / 0.5 \times 100 = 19.6\%$$

### Chapter 15

#### 15.1.

By use of Equation 15.5

$$\text{pH} = 6.10 + \log\{[\text{total CO}_2 - \text{spCO}_2]/\text{spCO}_2\}$$

$$7.40 = 6.10 + \log\{[25.2 - \text{spCO}_2]/\text{spCO}_2\}$$

$$\text{spCO}_2 = 1.203 \text{ s} = 0.0314(\text{mmol l}^{-1})/(\text{pCO}_2 \text{ mmHg}).$$

Thus,  $\text{pCO}_2 = 38.3 \text{ mmHg}$

#### 15.2.

Number of follow fiber

$$n = 43000/(3.14 \times 0.025 \times 13) = 42000$$

Average blood velocity through the follow fiber

$$v = 4000/(60 \times 3.14 \times 0.01^2 \times 42000) = 5.06 \text{ cm s}^{-1}$$

Oxygen concentration difference in  $\text{mol l}^{-1}$

$$\begin{aligned} (C_w - C_s) &= [(710 - 36) \times 2.82 \times 10^{-5} \times 1000]/[22400 \times 310/273] \\ &= 7.47 \times 10^{-4} \text{ mol l}^{-1} \end{aligned}$$

Hemoglobin concentration

$$C_{\text{Hb}} = (150 \times 40/42)/68000 = 2.10 \times 10^{-3} \text{ mol l}^{-1}$$

Oxygen saturation of hemoglobin at 36 mmHg 70%

Unsaturated concentration of hemoglobin  $0.63 \times 10^{-3} \text{ mol l}^{-1}$

Oxygen diffusivity is given by Equation 15.10

$$D_{\text{O}_2} = 1.76 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1}$$

The dimensionless length given by Equation 15.7 is

$$\begin{aligned} z_+ &= (7.47 \times 10^{-4} \times 1.76 \times 10^{-5} \times 13) / (0.63 \times 10^{-3} \times 0.02^2 \times 5.06) \\ &= 0.134 \end{aligned}$$

By use of Equation 15.6 trial will give  $\eta$

$$\eta = 0.450$$

By use of Equations 15.8 and 15.9

$$f = 0.636 \text{ and } Q/Q_0 = 0.364$$

Oxygen saturation at the exit  $1 - 0.3 \times 0.364 = 0.891$

$$89\%$$

### 15.3.

Kinematic viscosity is obtained from Figure 15.4b.

$$\mu = 0.034 \times 1.05 \times 0.1 = 0.00357 \text{ Pa} \cdot \text{s}$$

Blood flow is laminar and by use of Equation 2.9,

$$\begin{aligned} \Delta P &= 32 \times 0.00357 \times 5.06 \times 13 / 0.02^2 \\ &= 1.88 \times 10^4 \text{ Pa} = 0.019 \text{ MPa} = 0.19 \text{ atm} \end{aligned}$$

### 15.4.

By use of Equation 15.27

$$\begin{aligned} C_1 &= w/C_{Bi} = Q_B(C_{Bi} - C_{Bo})/C_{Bi} = Q_D(C_{Do} - C_{Di})/C_{Bi} \\ &= 200(100 - 20)/100 = 500 \times 32/100 = 160 \text{ cm}^3 \text{ min}^{-1} \end{aligned}$$

### 15.5.

By use of Equations 15.31, 15.33, and 15.34

$$Z = 200 / 500 = 0.4$$

$$N_M = 0.030 \times 10\,000/200 = 1.5$$

$$E = (1 - \exp 0.9)/(0.4 - \exp 0.9) = 0.709$$

$$Dl = Q_B E = 200 \times 0.709 = 142 \text{ cm}^3 \text{ min}^{-1}$$